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MODELLING NAKSHATRA-WISE RAINFALL DATA OF THE EASTERN PLATEAU REGION OF INDIA

1. Statistical modelling of rainfall data has been a major area of research for Climatologists and Meteorologists for quite some time. Obtaining an exhaustive model which incorporates rainfall occurrence, quantum and other factors associated with seasonality parameters and temporal dependence are a long pending problem and are very difficult to analyze as long as the data is sensitive with respect to its geographical location. The rainfed agricultural operations are highly season dependent and in some parts of India particularly in eastern plateau region agricultural operations particularly rice sowing or transplanting depends on the appearance of Nakshatras in the sky. We fit different probability distribution models generally used to predict short-period rainfall and identify different best fit models for rainfall on different Nakshatra periods.

Several studies have been done on rainfall analysis and best fit probability distribution function such as gamma [Barger and Thom (1949), Mooley and Crutcher (1968)), log-normal (Kwaku and Duke (2007), Sharma and Singh (2010)], exponential [Duan *et al*. (1995)], generalized extreme value [Tao *et al*. (2002), Weibull (Duan *et al*. (1995), Burgueno *et al*. (2005)], Pearson Type-I [Banik *et al*. (1994), Mooley (1973)], Pearson Type-IX [Mooley (1973)], SB [Swift and Schreuder (1981)] , three-parameter mixed exponential [Woolhiser and Roldan (1982),Wilks (1998)], log-logistic [Shoukri *et al*. (1988), Sharda and Das (2005)] distributions were identified for rainfall on wet-days and long-term rainfall. In case of short-term rainfall such as weekly rainfall, for most part of the globe, there is a high chance of the observation being zero considering not only the wet-days but also the dry-days. Several studies have been conducted on such rainfall analysis (Thom 1951) and models like incomplete gamma distribution [Kulandaivelu (1984), Sarker *et al*. (1982)], incomplete Weibull distribution [Muralidharan and Lathika (2005)] were identified to fit the data best. De *et al*. (2004) performed a time-series analysis of rainfall on different Nakshatra periods covering Indian monsoon season.

Lot of work has already been done on daily, weekly or monthly rainfall data but the Nakshatra wise rainfall data analysis is really very scanty or basically nil. Farmers of eastern plateau region generally grow rainfed rice for their subsistence and the agricultural practices totally depend on the distribution of rainfall. Farmers initiate the agricultural operation particularly land preparation, sowing etc. depending on the appearance of Nakshatra on the sky. 'Rohini and Hasta' Nakshatras govern the start of aman rice and winter season cultivation respectively.

The 27 Nakshatras are having a mean period of 13 or 14 days. Eleven out of the 27 Nakshatras cover the Indian monsoon season. Nakshatra period differs in each year. The normal periods of these 11 Nakshatras (Rohini to Chitra) are presented in Table 1. In spite of being very important for agricultural purposes, statistical modelling of rainfall on different Nakshatra periods has not been done and we consider this topic in the paper.

The paper is organized as follows. In Materials and Methods section, we describe the dataset, different models we fit to the data, testing procedure. In Results and Discussion section, we mention the relevance of this study, tabulate the results of the goodness-of-fit test and compare the models and identify the best fit model and calculate different percentage probability level rainfall based on the best fit model. In Conclusion section, we include some concluding remarks.

2. *Methodology* – (*i*) *Data* - Daily rainfall data of Giridih (24º18' N, 86º30' E) for 43 years period 1969-2011 is used in this work out of which data has been collected by Damodar Valley Corporation for the period 1969-1989 and by Indian Statistical Institute, Giridih for the period 1990-2011 and rainfall data for 11 Nakshatra periods used in this paper, is prepared based on the normal periods. Rainfall data is collected using instruments conforming to the specified criteria of India Meteorological Department (IMD).

(*ii*) *The model* - Rainfall is a random variable which is non-negative and continuous on the set (0, ∞) and has a positive probability of having zero rainfall because of considering short-period rainfall over pre-monsoon and post-monsoon months. So, suppose Y = total rainfall on a specific Nakshatra period. We model Y as follows:

$$Y = \begin{cases} 0 & w.p. \quad q \\ X & w.p. \quad p \end{cases} \quad s.t. \; p+q=1$$

Where X is a positive continuous random variable with a density function and support (0, ∞). So, the cumulative distribution function (CDF) of Y is of the form

$$G(x) = q + pF(x) \;\; \forall x \,\varepsilon\, [0,\infty] \;\; s.t. p.+q=1$$

where $F(x)$ is the CDF of X and q is the probability of zero rainfall. Now, suppose $F(x)$ contains the parameter θ (θ is independent of p), then $G(x)$ contains the parameters, say $\phi = (p, \theta)$. Suppose the parameter space of

θ is Θ, then the parameter space of ϕ, $\Phi = [0, 1] \times \Theta$. For the random variable X we consider the following models for selection of the most appropriate distribution: (*i*) one-parameter exponential, (*ii*) two-parameter gamma, (*iii*) two-parameter Weibull, (*iv*) two-parameter log-normal, (*v*) two-parameter log-logistic.

(*iii*) *Estimation of parameters* - For a particular week, rainfall Y for n years can be considered as a sample $Y_1, Y_2, \ldots, Y_n \sim iid\ G$. We estimate the unknown parameters from different distributions using Maximum Likelihood Estimation (MLE). The likelihood function,

$$\mathrm{L}\left(\phi \mathrm{I} Y_1, Y_2, \ldots, Y_n\right) = q^{\sum_{i=1}^{n} I(Y_i = 0)} \ \Pi_{i=1}^{n} [pf(Y_i)]^{1 - \mathrm{I}(Y_i = 0)}$$

Log-likelihood function

$$= l\left(\phi \, I Y_1, Y_2, \ldots, Y_n\right)$$

$$= \left\{\sum_{i=1}^{n} I(Y_i = 0)\right\} \log(q) +$$
$$\sum_{i=1}^{n}\left\{1 - I(Y_i = 0)\right\} \left\{\log(p) + \log[f(Y_i)]\right\}$$

$$= \begin{bmatrix} \left\{ \{\sum_{i=1}^{n} I(Y_i = 0)\} \log(q) + \{\sum_{i=1}^{n} (1 - I(Y_i = 0)\} \log(p) \right\} \\ + \sum_{i=1}^{n} (1 - I(Y_i = 0) \ \log[f(Y_i)] \end{bmatrix}$$

$$= l_1 + l_2$$

where,

$$l_1 = \left[\left\{\sum_{i=1}^{n} I(Y_i = 0)\right\} \log(q) + \left\{\sum_{i=1}^{n} (1 - I(Y_i = 0)\right\} \log(p)\right]$$

and

$$l_2 = \left[\left\{\sum_{i=1}^{n} (1 - I(Y_i = 0)\right\} \log(f(Y_i)\right]$$

We can see that l_1 is dependent only on p and independent of θ and also l_2 is independent of p and dependent only on θ.

Using maximum likelihood estimation, we get,

$$\widehat{q_{mle}} = \frac{\sum_{i=1}^{n} I(Y_i = 0)}{n} \qquad \text{and so,}$$

$$\widehat{P_{mle}} = 1 - \widehat{q_{mle}}$$

We have $l_2 = \left[\sum_{i=1, Y_i > 0}^{n} \log(f(Y_i)\right]$

So, the m.l.e. of θ is same as the simple m.l.e. based on only non-zero data and the distribution whose cdf is given by F.

Testing of the goodness-of-fit

First, we claim that if we want to fit the data to the distribution with df G, we can simply fit the non-zero data to the distribution with df F.

If $Y \sim G(x) = q + pF(x); p + q = 1$,

then given $Y > 0, Y \sim F$ because

$$Pr[Y \in (y, y + \varepsilon) \mathrm{I} Y > 0]$$

where, $y \geq 0$ and ε is a very small positive quantity.

$$= \frac{Pr[Y \in (y, y + \varepsilon), Y > 0]}{Pr(Y > 0)}$$

$$= \frac{Pr[Y \in (y, y + \varepsilon),}{Pr(Y > 0)}$$

$$= \frac{G(y + \varepsilon) - G(y)}{1 - G(0)}$$

$$= \frac{[q + pF(y + \varepsilon)] - [q + pF(y)]}{1 - q}$$

$$= \frac{[pF(y + \varepsilon) - pF(y)]}{p}$$

$$= F(y + \varepsilon) - F(y)$$

So, given $Y > 0$, $Y \sim F$.

If given $Y > 0$, $Y \sim F$, then unconditionally

$Y \sim G = q + pF(x)$ for some

$p \in [0, 1], p + q = 1$

Suppose, $X \sim F$ has support $(0, \infty)$ and Y has support $(0, \infty)$ implies that Y is a mixture of two random variables X and degenerate at zero with proportion p unknown and $q = 1 - p$.

TABLE 1

Normal dates of Nakshatra

S. No.	Nakshatra	Period	No. of days
1	Rohini	25 May to 7 Jun	14
2	Mrigashira	8 Jun to 21 Jun	14
3	Ardra	22 Jun to 5 Jul	14
4	Punarvasu	6 Jul to 19 Jul	14
5	Pushya	20 Jul to 2 Aug	14
6	Ashlesha	3 Aug to 16 Aug	14
7	Magha	17 Aug to 30 Aug	14
8	Purva	31 Aug to 12 Sep	13
9	Uttara	13 Sep to 26 Sep	14
10	Hasta	27 Sep to 9 Oct	13
11	Chitra	10 Oct to 23 Oct	14

So, $Y \sim G(x) = q + pF(x)$ if and only if $Y > 0$, $Y \sim F$ and proves our claim.

So, we shall check the goodness-of-fit of the non-zero data to the distribution with df F and use Kolmogorov-Smirnov (K-S) test for this purpose. We consider the model with highest p-value to be the best-fit model.In case of this test, the parameters of the hypothesized distribution should be known to calculate the p-value theoretically. As the parameters of the model are unknown in our case and have to be estimated from the data, we have calculated the p-value based on the 100000 simulated samples from the corresponding probability model with the estimated parameter values.

100 r% probability rainfall is given by the $(1 - r)^{th}$ quantile of the best-fit probability distribution with parameters estimated from the data.

(*iv*) *Relevance of Nakshatra-wise rainfall modelling* - Preliminary statistics, *i.e.*, mean, standard deviation etc. do not depict the actual scenario and a proper probability model is required to calculate the amount of rainfall at different probability levels mainly in the situation of high value of the ratio of standard deviation and mean, *i.e.*, the co-efficient of the variation.

From Table 1, we can see that a Nakshatra period is approximately the combination of two weeks while a month consists of approximately 4 weeks. We define our variables of interest as X_i = total amount of rainfall in week i; i = 1, 2, 3, 4 and the monthly rainfall defined as say, $Y = X_1 + X_2 + X_3 + X_4$ and Nakshatra-wise rainfalls defined as say, $Z_1 = X_1 + X_2$ and $Z_2 = X_3 + X_4$. Based on the any underlying model, the expected monthly rainfall is algebraically equal to the sum of the expected weekly rainfalls. Similarly, the expected Nakshatra-wise rainfall is also algebraically equal to the sum of the two corresponding expected weekly rainfalls. As the preliminary statistics do not fulfill the purpose, some particular stochastic modelling is required. But, based on the underlying model, in general, the monthly rainfall at a particular probability level is not algebraically equal to the sum of the weekly rainfalls at that probability level insisting the modelling of monthly rainfall in spite of an already known good modelling strategy on weekly basis. Similar reason insists to model Nakshatra-wise rainfall data in spite of already existing strategies for weekly or monthly basis.

Suppose, F_Y gives the cumulative distribution function (CDF) of Y, Fx_i gives the cumulative distribution function of X_i ; i = 1, 2, 3, 4 and Fz_i gives the cumulative distribution function of Z_i; i = 1, 2; $F_{Y,p}$ gives the 100 p% probability rainfall for the model F_Y, *i.e.*, $F_{Y,p}$ gives the $(1- p)^{th}$ quantile of F_Y. $F_{Xi,p}$ and $Fz_{i,p}$'s are also similarly defined. Then, in general,

$$F_{Y,P} \neq F_{X_1,p} + F_{X_2,p} + F_{X_3,p} + F_{X_4,p}\,;$$

$$Fz_{1,p} \neq F_{X_1,p} + F_{X_2,p} \; and$$

$$Fz_{2,p} \neq F_{X_3,p} + F_{X_4,p}$$

To demonstrate this point, we choose gamma distribution with shape parameter α and scale parameter β for the modelling of non-zero part of weekly rainfall data. Thus, we model X_i as the following

$$X_i = \begin{cases} 0 & with\ prob. q_i \\ W_i & with\ prob.\ p_i \end{cases} ; q_i + p_i = 1;$$

$$W_i \sim \text{gamma}\,(\alpha_i, \beta_i)$$

with the assumption that the X_i's are independent and the choice of a set of combinations of the parameters

$(p_i, q_i, \alpha_i, \beta_i)$; i=1, 2, 3, 4 as follows:

$(p_1, \alpha_1, \beta_1) = (0.8, 1.5, 20)$;

$(p_2, \alpha_2, \beta_2) = (0.7, 1.75, 25)$;

$(p_3, \alpha_3, \beta_3) = (0.6, 2.0, 20)$;

$(p_4, \alpha_4, \beta_4) = (0.5, 2.25, 30)$;

TABLE 2

Weekly, monthly and Nakshatra-wise rainfall at different probability level for the chosen model

%-age prob. (p)	$F X_{1,p}$	$F X_{2,p}$	$F X_{3,p}$	$F X_{4,p}$	$F_{Y,p}$	$Fz_{1,p}$	$Fz_{2,p}$
10%	57.394	76.999	64.704	99.654	204.775	111.858	132.904
20%	41.083	55.056	45.786	68.842	166.210	86.208	98.804
30%	31.098	41.299	33.567	48.035	140.213	69.917	76.715
40%	23.660	30.745	23.777	29.860	119.740	57.437	59.633
50%	17.540	21.676	14.621	0	101.831	46.536	45.037
60%	12.125	12.965	0	0	85.187	36.864	32.310
70%	6.924	0	0	0	68.816	27.563	19.913
80%	0	0	0	0	51.905	17.961	2.175
90%	0	0	0	0	32.013	7.097	0

TABLE 3

Preliminary statistics for different Nakshatra periods

	Rohini	Mrigashira	Ardra	Punarvasu	Pushya	Ashlesha	Magha	Purva	Uttara	Hasta	Chitra
Mean rainfall	45.337	81.577	154.656	138.226	134.266	134.858	126.889	120.545	120.600	75.972	26.428
S. D. of rainfall	48.174	87.236	123.982	81.721	90.836	83.874	102.707	83.714	108.936	70.515	38.119
Mean rainy days	2.744	4.1395	6.093	6.791	6.767	6.651	6.256	5.558	5.070	3.465	1.256
S. D. of rainy days	2.216	2.436	2.776	2.493	2.580	2.581	2.226	2.383	2.109	2.529	1.329

TABLE 4

Maximum Likelihood Estimates of model parameters for different Nakshatra-wise rainfall

Maximum likelihood estimates	Rohini	Mrigashira	Ardra	Punarvasu	Pushya	Ashlesha	Magha	Purva	Uttara	Hasta	Chitra
mle of p	0.860	0.953	1	1	1	1	1	1	0.977	0.930	0.605
mle of exponential_mean	52.689	85.556	154.656	138.226	134.266	134.858	126.889	120.545	123.472	81.670	43.708
mle of gamma_shape	1.300	1.160	1.576	2.599	1.865	2.744	2.105	2.087	1.782	1.046	0.978
mle of gamma_scale	40.539	74.013	98.118	53.186	71.974	49.148	60.273	57.767	69.307	78.065	44.714
mle of Weibull_scale	55.523	87.513	167.891	155.747	148.608	151.938	140.632	134.219	134.796	83.481	43.775
mle of Weibull_shape	1.154	1.054	1.308	1.785	1.499	1.728	1.409	1.524	1.299	1.061	1.004
mle of log-normal_location	3.533	3.958	4.692	4.724	4.608	4.711	4.587	4.534	4.510	3.854	3.185
mle of log-normal_scale	1.033	1.064	0.947	0.685	0.863	0.657	0.717	0.772	0.794	1.237	1.310
mle of log-logistic_location	3.603	3.994	4.776	4.773	4.688	4.748	4.585	4.590	4.523	3.980	3.310
mle of log-logistic_scale	0.577	0.598	0.518	0.401	0.483	0.363	0.412	0.441	0.448	0.727	0.732

These values are chosen close to the m.l.e.s. we obtained from our original data and the assumption of independence is tested.

Though it is very easy to find percentiles from the CDF of X_i, but due to non-existence of any closed form expression in case of $X_1 + X_2 + X_3 + X_4$, $X_1 + X_2$ and

TABLE 5

K-S test statistic, p-values and best fit model for different Nakshatra-wise rainfall

K-S stat	Rohini	Mrigashira	Ardra	Punarvasu	Pushya	Ashlesha	Magha	Purva	Uttara	Hasta	Chitra
exponential	0.076	0.110	0.128	0.202	0.144	0.240	0.209	0.200	0.190	0.144	0.068
gamma	0.050	0.128	0.080	0.098	0.100	0.086	0.108	0.089	0.097	0.141	0.065
Weibull	0.055	0.124	0.071	0.085	0.078	0.106	0.096	0.079	0.105	0.136	0.069
log-normal	0.082	0.094	0.124	0.141	0.139	0.108	0.071	0.134	0.077	0.181	0.085
log-logistic	0.060	0.071	0.101	0.110	0.112	0.089	0.078	0.096	0.062	0.153	0.071
p-value											
exponential	0.908	0.608	0.440	0.051	0.302	0.012	0.040	0.055	0.072	0.263	0.863
gamma	0.999	0.420	0.925	0.767	0.752	0.880	0.661	0.854	0.768	0.283	0.895
Weibull	0.996	0.460	0.970	0.892	0.943	0.682	0.786	0.935	0.670	0.326	0.854
log-normal	0.855	0.784	0.481	0.328	0.341	0.658	0.973	0.387	0.933	0.084	0.640
log-logistic	0.988	0.963	0.739	0.631	0.618	0.855	0.935	0.790	0.992	0.204	0.830
best fit	gamma	log-logistic	Weibull	Weibull	Weibull	gamma	log-normal	Weibull	log-logistic	Weibull	gamma

TABLE 6

Nakshatra-wise rainfall at different probability level

Probability	Rohini	Mrigashira	Ardra	Punarvasu	Pushya	Ashlesha	Magha	Purva	Uttara	Hasta	Chitra
10%	107.070	195.532	317.690	248.529	259.188	243.984	246.110	232.037	243.335	177.760	78.893
20%	75.941	119.925	241.583	203.341	204.117	194.420	179.561	183.428	169.019	125.174	48.226
30%	57.205	86.432	193.497	172.819	168.193	163.065	143.050	151.610	132.536	93.799	30.360
40%	43.508	65.898	157.035	148.302	140.191	139.067	117.796	126.734	108.477	71.149	17.752
50%	32.503	51.192	126.856	126.833	116.381	118.877	98.238	105.521	90.147	53.273	8.053
60%	23.097	39.554	100.451	106.895	94.947	100.765	81.928	86.365	74.774	38.391	0.306
70%	14.608	29.569	76.325	87.407	74.720	83.584	67.465	68.225	60.790	25.525	0
80%	6.327	20.227	53.322	67.207	54.650	66.200	53.746	50.148	46.851	14.044	0
90%	0	10.038	30.040	44.137	33.131	46.567	39.213	30.644	30.597	3.358	0

$X_3 + X_4$, we find out quantiles using Monte Carlo approach with 100000 simulations.The results we obtained are presented in the Table 2.

From the above table it is clear that,

$F_{Y,p} \neq F_{X_1,p} + F_{X_2,p} + F_{X_3,p} + F_{X_4,p}$;

$Fz_{1,p} \neq F_{X_1,p} + F_{X_2,p}$ and $Fz_{2,p} \neq F_{X_3,p} + F_{X_4,p}$ in general and so, even the number of variables increases than in case of monthly study, it is actually very important to consider the modelling of Nakshatra-wise rainfall.

3. *Results and discussions* - Means and standard deviations of the rainfall and number of rainy days for different Nakshatra periods are presented in Table 3. The co-efficient of the variation is very high for all the cases mainly in case of pre-monsoon and post-monsoon periods for the rainfall as well as the number of rainy days. For rainfall, it ranges from 59.12% for Punarvasu to 144.23% for Chitra and for rainy days, it ranges from 35.59% for Magha to 105.83% for Chitra (Table 3).

The maximum likelihood estimates of the model parameters are provided in Table 4. MLEs of p's for Adra to Purba Nakshatra periods are 1, *i.e.*, all the years considered in our study received positive amount of rainfall. For the rest of periods, except for Chitra period (MLE of p is 0.605), the MLEs of p's are close to 1. The estimates of the other model parameters,required for modelling of the positive part of the rainfall, are also provided.

The values of the Kolmogorov-Smirnov test statistics, the corresponding p-values and the best fit model among those we considered in our study are presented in Table 5. Out of 11 Nakshatra periods, Weibull distribution gives the best fit for 5 periods (p-value) - Ardra (0.9693), Punarvasu (0.8917), Pushya (0.9425), Purba (0.9347) and Hasta (0.3263); gamma distribution gives the best fit for 3 periods- Rohini (0.9988), Ashlesha (0.8790) and Chitra (0.8951); log-logistic distribution gives the best fit for 2 periods- Mrigashira (0.9627) and Uttara (0.9924); log-normal distribution gives the best fit for Magha (0.9730) Nakshatra period (Table 5). Except for some cases for mixed-exponential model, for all other models the p-values are quite large enough implying that anyone of the four models fits the data for tests with level 5%.

The completely specified best fit models (only for the non-zero part, zero part is obtained from MLE of q, *i.e.*, 1-MLE of p) for the Ardra, Punarvasu, Pushya, Purva, Hasta - Nakshatra periods are Weibull distribution with scale and shapeparameters which are 167.891 and 1.308, 155.747 and 1.785, 148.608 and 1.499, 134.219 and 1.524, 83.481 and 1.061 respectively whereasfor Rohini, Ashlesha, Chitra – Nakshatras are gamma distribution with parameters (shape and scale 1.300 and 40.539, 2.744 and 49.148, 0.978 and 44.714 respectively).Log-logistic distribution fits best for Mrigashira, Uttara - Nakshatra and on the contrary, for Magha Nakshatra log-normal distribution shows its efficiency with parameters (location, scale – 4.587, 0.717).

Amount of rainfall at different probability levels (10% - 90%) based on the best fit model are summarised in Table 6.Two prominent breaks of rainfall can be identified. Least assured rainfall suddenly jumped up in Ardra Nakshatra period and again suddenly jumped down in Hasta Nakshatra period. At 30% probability level, first two periods (25th May to 21st June) and last two periods (27th September to 23rd October) are almost dry periods. Noticeable amount of rainfall occurs only in the remaining 7 periods (22nd June to 26th September). At 70% probability level, we observe similar trend except the amount of rainfall drops and the 7 periods-Ardra to Uttara receive more than 60 mm least assured rainfall for each period. It is clear from the data (Table 6) that with 50 per cent probability level Rohini period can be considered as suitable for preparation of rice nursery bed whereas at Migashira period rice seedlings can be transplanted in main plot of the eastern plateau region. There is a possibility to get the rainfall at Hasta Nakshatra so second crop (winter crop) can be sown in eastern plateau area.

4. *Conclusion* - We have proposed a mixture model of two distributions, *i.e.*, degenerate at zero and a positive continuous distribution with one extra parameter for mixing proportion for each Nakshatra period which takes care of the dry spells. MLEs of the parameters of continuous distribution can be independently obtained only from the non-zero data and for the parameter of mixing proportion, MLE is just the proportion of non-zero data. Fitting the whole data (including zeroes) to the mixture distribution, we can simply fit the non-zero data to the positive continuous part.

The co-efficient of variation is very high for the rainfall as well as the number of rainy days and so, not only the basic statistics but also a proper modelling is necessary. For the pre-monsoon and the post-monsoon periods, the estimate of the mixing proportion is less than one and so, mixture model is necessary and the p-values for the best fit models are very high ranging between 0.3255 and 0.9988. Except a very few cases, all the p-values are large enough implying that not only the best fit model but also other models give a good enough fit. For the monsoon period, mixed-Weibull distribution gives the best fit for most of the periods (Ardra, Punarvasu, Pushya, Purba and Hasta) while mixed-gamma distribution gives the best fit mainly for the pre-monsoon and the post-monsoon period (Rohini and Chitra) and log-logistic model fits the data best for two periods (Mrigashira and Uttara) and Log-normal model fits the data best for one period (Magha).We can conclude that the first and the last Nakshatra of the monsoon period (*i.e.*, in case of pre-monsoon and post-monsoon), gamma distribution gives the best fit while Weibull distribution gives the best fit for many of the Nakshatra periods in peak monsoon period.

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