Incomplete Gamma distribution of rainfall for sustainable crop production strategies at Palampur, Himachal Pradesh

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सार – वर्षा के पैटर्न (मात्रा और संभाव्यता) का ज्ञान होना क्षेत्र में उगाई जाने वाली फसलों की योजना तैयार करने में सहायक होता है। इस उद्देश्य से पालमपुर केन्द्र के 33 वर्षों (1974–2006) के साप्ताहिक, मासिक ऋतु संबंधी और वार्षिक वर्षा के आँकड़े एकत्रित किए गए और उनका विश्लेषण किया गया। इस शोध–पत्र में सूखा की सामान्य स्थिति और असामान्य स्थिति का पता लगाने के लिए वर्षा के वार्षिक और मासिक आँकड़ों का विश्लेषण किया गया। मुख्यतः 25.7 प्रतिशत परिवर्तनशीलता के साथ वार्षिक वर्षा 2343 मि. मी. तक ही सीमित रही। वार्षिक वर्षा का मानक विचलन 62.8 मि. मी. है। 26 वें से 35 वें तक के प्रत्येक मानक सप्ताह में 100 मि. मी. से अधिक वर्षा हुई जिससे फसल की अवधि का पता चला। पालमपुर क्षेत्र में पैडी नर्सरी में बीज बोने का कार्य 23वें से 25वें मानक मौसम विज्ञानिक सप्ताहों के दौरान मानसून के आरंभ होने के तत्काल बाद किया गया और प्रतिरोपण का कार्य लगभग 27वें अथवा 28वें मानक मौसम विज्ञानिक सप्ताह मे किया गया। दौजी लगना, 50 प्रतिशत फूल आना और डो स्टेज कमशः 32वें–33वें, 37वें–38वें और 40वें–41वें मानक मौसम विज्ञानिक सप्ताह के दौरान प्रेक्षित किए गए।

ABSTRACT. The knowledge of rainfall pattern (amount and probability) helps in planning of crops to be grown in a region. Therefore weekly, monthly, seasonal and annual rainfall data for 33 years (1974-2006) for the station Palampur have been collected and its analysis has been attempted. The annual and monthly rainfall data were analyzed for finding out drought normality and abnormality. The analysis indicated that the rainfall is mainly confined in annual rainfall 2343 mm with 25.7 per cent variability. The standard deviation of annual rainfall is 62.8 mm. Each standard week from 26th to 35th receive a rainfall of more than 100 mm, indicating the crop period. Seed sowing in paddy nursery in the Palampur region generally takes places immediately after initiation of monsoon during $23^{rd} - 25^{th}$ standard meteorological weeks and transplanting is carried out around 27^{th} or 28^{th} standard meteorological weeks respectively.

Key words – Incomplete Gamma probability, Rainfall, Drought, Seasonal rainfall, Kharif season, Rabi season, Monthly rainfall and weekly probability.

1. Introduction

The weather and its variability are well known to the farming community and it has great impact on crop production. The economy of the farmer is well influenced by weather. It also plays an important role in the economy of the state and even of the country. Moisture is the most limiting factor for crop production in semi-arid region, the greatest risk to crop yields in Indian agriculture is attributed to the variability of seasonal rainfall and the uncertainty in the amount and distribution of rainfall for a given season. Rainfall pattern largely decides the crop planning in dry farming tracts.

Amount, distribution and intensity of rainfall mainly determine the choice of any particular crop and agronomic practices. Scientific study on the quantum and distribution of rainfall if made would enable the farming community to adjust or modify the cropping programme as well as the cultural operations to utilize the actual moisture available in the field for profitable crop production (Parasuraman and Suresh, 2002). Hence, a study was undertaken at Palampur to understand the rainfall climatology for crop planning purpose. Crop productivity has increased in the past four decades. The reason being use of improved seeds, fertilizers and sustainable agro techniques. For maintaining food production in line with increasing population our agricultural research should integrate with the long-term technical policies (Parasuraman, 2003).

2. Material and methods

The basic data used comprises of daily rainfall data for period 1974-2006 collected from Deptt. of Agronomy,



Fig. 1. Relation between rainfall (mm) and coefficient of variation at Palampur region

Agromet Observatory, Palampur (Himachal Pradesh), which is located at 32.10° N Latitude, 76.05° E Longitudes with altitude of 1297 meters above mean sea level (MSL) were used for analysis of the probability and variability. The daily rainfall data was used to calculate annual, seasonal, monthly and weekly rainfall. The mean, standard deviation and coefficient of variation of annual, seasonal, monthly and weekly rainfall were worked out as described by Singh, 2005 and Deka and Nath, 2000.

The months were classified as drought, normal and excessive rainfall months. If *X* is the mean monthly rainfall, then a month receiving rainfall less than A1 is defined as drought month, between A1 and A2 is normal month and above A2 is excessive month where A1 = X/2 and A2 = 2X (Pimpale and Hiwase, 2001). Similar criteria were used to define weeks as normal excessive rainfall and drought as described for the month. The departure of annual rainfall and seasonal rainfall were found out using IMD (India Meteorological Department) specification, which categories normal (particular year that received +19 to -19 per cent of mean annual rainfall), excess (year that received more than 19 per cent of mean annual rainfall) and deficit (year that received less than -19 per cent of the mean annual rainfall).

Departure of rainfall from normal (%)	Classified as
>19	Excess
+19 to -19	Normal
< -19	Deficit

TABLE 1(a)

Initial probability analysis of rainfall at Palampur region

Probability of rainfall	Rainfall (mm)	Years
75 % and above	1126.4	2002
75%	1881.6	1993
50%	2321.9	1974
30%	2728.0	2006
25%	2900.3	1975

TABLE 1(b)

Percentage of rainfall under different categories at Palampur region

Categories/ rainfall	Excess	Normal	Deficit
Annual rainfall	27	46	27
Seasonal rainfall	49	27	24

The number of drought, normal and excessive rainfall or flood months and weeks in a year were listed in descending order of magnitude and probability analysis were carried out by using Weibull's formula, (Chow, 1964)

P = m/(n + 1)

Where P is plotting positions percent, m is the rank of magnitude and n is the total no. of years for which analysis was carried out.



Fig. 2. Relation between departure from normal of annual and seasonal rainfall at Palampur region

The incomplete Gamma distribution probability analysis was based as suggested by Victor, 2000 and Kar, 2002.

The incomplete Gamma distribution probability analysis was based as suggested by Victor, (2000).

The general formula for the probability density function of the gamma distribution is

$$f(x) = \frac{\left(\frac{x-\mu}{\beta}\right)^{\gamma-1} \exp\left(-\frac{x-\mu}{\beta}\right)}{\beta \Gamma(\gamma)} \qquad x \ge \mu; \gamma, \beta > 0$$

where γ is the shape parameter, μ is the location parameter, β is the scale parameter and Γ is the Gamma function which has the formula

$$\Gamma(a) = \int_0^\infty t^{a-1} e^{-t} \mathrm{d}t$$

The case where $\mu = 0$ and $\beta = 1$ is called the standard Gamma distribution. The equation for the standard Gamma distribution reduces to

$$f(x) = \frac{x^{\gamma - 1}e^{-x}}{\Gamma(\gamma)} \qquad x \ge 0; \ \gamma > 0$$

Since the general form of probability functions can be expressed in terms of the standard distribution, all subsequent formulas in this section are given for the standard form of the function.

Cumulative distribution function

The formula for the cumulative distribution function of the Gamma distribution is

$$F(x) = \frac{\Gamma_x(\gamma)}{\Gamma(\gamma)}$$
 $x \ge 0; \ \gamma > 0$

Where Γ the Gamma function is defined above and $\Gamma_{x}(a)$ is the incomplete Gamma function.

3. Results and discussion

The daily rainfall data for the period from 1974-2006 was analyzed and the results are presented under different heads for annual rainfall, mean annual rainfall and coefficient of variation (%) of annual rainfall and is presented in Fig. 1.

Monsoon rainfall deficiency might have caused a crop failure due to drought during the crucial monsoon months. That is why we have considered both situations in our study. During the study of the annual rainfall of 33 years, there were about (9 years) 27 per cent deficit, (9 years) 27 per cent excess and (15 years) 45 per cent

Normal, above normal and drought statistics based on 33 years rainfall data for monthly and seasonal rainfall analysis for Palampur region

Season	Month	Mean RF (mm)	Based on 33 years of rainfall data							
			SD (mm)	CV (%)	% of total	A1	A2	DM	EM	NM
	March	107.5	77.1	71.7	4.6	54	216	10	3	20
	April	55.8	44.9	80.5	2.4	28	112	11	3	19
	May	76.5	61.1	79.9	3.3	38.5	154	10	4	19
Summer		239.8	183.1	232.1	10.3					
	June	221.8	126.0	56.8	9.5	111	444	7	3	23
	July	655.1	319.7	48.8	28.1	327.5	1310	4	1	28
	August	670.0	155.2	23.2	28.8	335	1340	0	0	33
	September	244.2	141.7	58.0	10.5	122	488	8	3	22
Kharif		1791.2	742.6	186.8	76.9					
	October	29.9	40.7	136.1	1.3	15	60	15	4	14
	November	18.7	28.0	149.6	0.8	9.5	38	17	5	11
	December	96.3	71.4	74.1	4.1	48	192	10	4	19
	January	101.5	69.0	68.0	4.4	50.5	202	10	3	20
	February	52.1	60.4	115.9	2.2	26	104	15	6	12
Rabi		298.5	269.5	543.6	12.8					
Annual		2343	62.8	25.7	100.0	1165	4660			

CV = Coefficient of variation, SD = Standard deviation

A1 = Dry limit month, A2 = excessive rainfall month limit, EM = No. of years having excessive rainfall month, NM = No. of years having normal month, DM = No. of years having drought month.

TABLE 2(b)

Incomplete Gamma distribution of monthly rainfall at different probability levels at Palampur region

Month	Probability levels (%)							
	90	75	50	25	10	Mean (mm)		
1	21.2	42.9	82	140.6	212.1	101.9		
2	20.6	42	81	139.4	211	100.9		
3	17.5	39.8	83.4	152.1	238.8	109.3		
4	12.3	24.7	46.7	79.5	119.4	57.3		
5	16.6	33.9	65.1	111.9	169.2	80.9		
6	61	136.8	220.1	305.4	381.2	220.1		
7	257.1	450.5	664.3	880.2	173.5	664.3		
8	470.9	566.2	671.1	777.9	873.2	671.1		
9	88.2	138.3	214.3	314.3	426.8	239.5		
10	1.2	5.1	17.4	43.2	81.3	30.7		
11	0.6	2.9	10.7	27.6	53.2	19.4		
12	2.1	8.6	27.8	66.7	123	47.7		
Annual	1599.6	1906	2290.6	2723.9	3157.2	2343		

		Probability levels (%)							
Week	90	75	50	25	10	Mean (mm)			
1	0.2	1.5	7.7	24.4	52.2	17.9			
2	0.5	2.7	11	29.9	59.1	21.3			
3	0.5	2.5	8.7	22.1	42	15.2			
4	0.9	4.6	18.3	49.6	97.5	35.9			
5	0.7	3.1	10.8	27.3	51.8	19.1			
6	0	0	21.8	45.6	66.2	21.8			
7	0	11.4	37	64.5	88.4	37			
8	0	6.5	24.9	45.2	62.6	24.9			
9	0.5	2.9	12.6	35.9	72.4	25.9			
10	0.5	2.7	10.7	29.2	57.6	20.7			
11	1.3	5.2	16.6	39.9	73.7	28.1			
12	0.9	4.4	16.3	42.6	82.1	30.4			
13	1.1	3.6	10	21.9	38.5	14.9			
14	0.8	3.1	10.2	24.6	45.6	17			
15	0.7	2.7	8.6	20.4	37.3	13.8			
16	0.6	2.1	6.1	13.9	24.8	9.1			
17	0.5	2.2	7.9	20.1	38.2	13.8			
18	0.6	2.7	9.6	24.6	47	17.1			
19	0.9	3.6	11.3	26.6	48.7	18.4			
20	1.3	4.3	11.8	25.9	45.2	17.7			
21	1.1	3.8	10.9	24.5	43.5	16.7			
22	0.7	3	10.2	25.5	48	17.7			
23	1.5	5.3	15.2	34	60.2	23.6			
24	3.1	9.6	25.1	52.8	90.6	37.1			
25	6.2	17.1	41.9	84.5	141.2	60			
26	5.7	21.4	65.5	152.3	275.9	109.7			
27	16.5	39.5	86.2	161.3	257.4	115.7			
28	26.6	57.4	115.3	204.3	314.9	147.7			
29	34.2	70.1	135.4	233.6	354	169.8			
30	46.6	84.6	148.3	238.4	344.9	176			
31	59.9	103.3	173.4	270.2	382.7	201.4			
32	65.3	97.9	145.8	207.4	275.6	159.7			
33	49 1	78.6	123.9	184.2	252.5	139.2			

Incomplete Gamma distribution of weekly rainfall at different probability levels at Palampur region

			TABLE 3	(Conta.)		
Week			Proba	ability levels (%	6)	
	90	75	50	25	10	Mean (mm)
34	57.6	84.1	122.3	170.6	223.6	132.4
35	22.1	45	86.5	148.6	224.5	107.7
36	10.3	25.9	58.6	112.4	182.1	80.2
37	7.9	19.2	42.4	79.9	128.1	56.8
38	1.7	8	28.3	71.9	136.8	51.9
39	0.5	2.8	11.4	31	61.4	22.1
40	0.3	1.3	4.8	12.5	23.9	8.2
41	0.3	1.8	7.1	19.2	37.9	13.3
42	0.1	0.8	3.6	10.2	20.7	6.7
43	0.2	0.8	2.2	4.7	8.1	2.4
44	0.1	0.6	2.3	6	11.6	3.4
45	0.2	0.9	2.6	6	10.9	3.4
46	0.2	0.8	2.5	5.9	10.9	3.3
47	0.3	0.9	2.6	5.9	10.4	3.3
48	0.2	1	4	11	21.9	7.2
49	0.1	0.7	2.8	7.3	14	4.4
50	0.1	0.9	4.7	14.7	31.2	10.3
51	0.2	1.2	5.6	16.6	34.5	11.6
52	0.2	1.6	8.5	27.4	59.1	20.3
ANN	1599.6	1906	2290.6	2723.9	3157.2	2343

TABLE 3 (Contd.)

normal from annual years in Table 1(b). Also in the monsoon last 33 years, there were about 24 per cent of deficit, 48 percent of excessive and 27 per cent of normal occurrences in Table 1(b). The annual rainfall was lowest (1126.4 mm) in 2002 and rainy days were also lowest (70). The monsoon rainfall was lowest (938.9 mm) in 2002. The mean annual rainfall was 2343 mm with coefficient of variation of 25.7 per cent and standard deviation is 62.8 mm. The annual rainfall is highest of 3438 mm in the year 1977. A rainfall amount of 1126.4 mm would be expected with a high probability of 75 per cent or more while 1881.6 mm, 2321.9 mm, 2728 mm and 2900.3 mm of rainfall would be expected with 75, 50, 30 and 25 per cent probability in Table 1(a). The frequency of drought year for annual and monsoon period has also been worked out and presented in Fig. 2. The droughts are less frequent on annual basis as compared to those on seasonal basis.

The seasonal rainfall data on mean, standard deviation, coefficient of variation and per cent contribution of seasonal rainfall to total annual are presented in Table 2(a). The mean rainfall of 1791.2 mm is received in kharif season contributing 76.9 per cent to total annual rainfall with coefficient of variations of 28.5 per cent and standard deviation of 127 mm indicating its dependability. For summer season the mean rainfall is 239.8 mm and contribute 10.3 per cent to the total annual rainfall with coefficient of 53.2 per cent and standard deviation of 53.2 per cent and standard deviation of 298.5 mm and contributes substantial amount of 298.5 mm and contributes 12.8 per cent to the total with the coefficient of

variations of 50 per cent and standard deviation of 30 mm in Fig. 2.

Frequency of normal, surplus and deficient months and percentage of normal, surplus and deficient months is presented in Table 2 (a). It is evident from the Table 2 (a) that August is wettest month, which received on an average (670.0 mm) rainfall with low variation (CV=23.2 per cent). The wettest month next to August is July with average rainfall 655.1 mm, which has the minimum CV of 48.8 per cent, *i.e.*, has a consistent rainfall pattern. It is the main source of kharif rainfall in Palampur and brings rain from 1st week of June. Month of November is the driest month with meager amount (18.7 mm) of average rainfall. It has also the highest value of CV, which shows 149.6 per cent, which shows the erratic pattern of rainfall. The highest CV value in rabi season signified sporadic and highly variable rainfall during the winter month. During the last 33 years Palampur region had kharif season that started from the month of June and continued till September. However, the pre-monsoon showers normally start from mid May itself. During 396 months in 33 years, deficient occurred seven times in the month of June and eight in September (Table 3). During the study period deficient, normal and excess rainfall months were 117 (29.5 per cent), 240 (60.6 per cent) and 39 (9.8 per cent) respectively have been computed.

The monthly probability at 10, 25, 50, 75 and 90 per cent confidence level was computed using incomplete Gamma distribution and results are given in Table 2(b). At 75 per cent confidence level, 34 mm rainfall occurred during the month of May. This pre-monsoon rain at 75 per cent probability level can be utilized for summer ploughing or seedbed preparation for raising rice seedlings. At 90 per cent confidence level amount rainfall which occurs during July 257 mm. Therefore, the rainfall at least assured level should be utilized for growing kharif season crop like direct sown rice, maize, cowpea etc. can taken and rice nurseries can be prepared in second fortnight of June with the commencement of southwest monsoon in these region. The maximum amount of rainfall occurred during August. Higher amount of rainfall at 90 per cent confidence level can be utilized for rice transplanting starting from first fortnight of August. The transplanting of kharif rice in last week July will have additional advantage of almost assured water through rain during September and October. Since, the winter rainfall is uncertain and erratic, residual moisture in medium and low land region could be utilized for growing a second crop under rainfed condition.

In these regions, high value rabi crop like wheat, gram, pea etc., can be grown only with assured

TABLE	4
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Rainfall characteristics of Palampur region							
	Week No.						
	Mean	Earliest	Latest	Std. Dev.			
Start	24	22	27	1.2			
End	48	40	52	4.3			
	Mean	Minimum	Maximum	Std. Dev.			
Duration	24	15	29	4.7			
(No. of week)							

supplemental irrigation during rabi season starting from 1st week of November.

Mean and expected weekly rainfall at different probability level (Table 3) showed that from 25^{th} SMW onwards the rainfall was recorded within the range of 60 mm to 51.9 mm per week and contributed upto 38^{th} SMW. Table 3 indicates that 31^{st} week has the highest rain, contributing 201.4 mm and 38^{th} week recorded the lowest rainfall of only 51.9 mm. At 50 per cent chances rainfall is assured in 24^{th} week. A rainfall more than 50 mm per week occurs from 26^{th} to 36^{th} week at 50 percent probability.

4. Crop planning

The major crop of the region is paddy, which is about 120-150 days duration. Lowland of this region is characterized by having better moisture holding capacity and fertility. The crop planning of this area cab is done at 50 per cent probability level so that early sowing of paddy crop for nursery can be done in 24th and 25th week for better germination. Therefore, germination will be difficult after these weeks. Thus, the growth of standing crop may be such that they can resist severe water logging expected in 30th weeks. At 50 per cent probability total rainfall received at end of 29th (before transplanting) could be 135.4 mm, which is sufficient to flush out salts by rainwater and create favourable soil conditions for transplanting. As regards to rahu planning (broadcasting of sprouted seed), 27th and 28th weeks are suitable better logging expected 30th and 31st weeks. The 31st, 34th and 39th weeks would be tillering, panicle initiation and maturity respectively. The transplanting of paddy should be completed in the last month of July. Farmers may use salt tolerant paddy varieties, *i.e.*, Santosh, Satyam and Rajshree with stand salinity variation in soil sandy clam loam during intermittent dry spells. These varieties mature

by the end of September can be harvested at end of October. There is scope for water harvesting in on farm reservoir during the 27^{th} to 35^{th} week.

Early start of length of growing season occurs during 22nd week (Table 4). During the years with delay in onset of monsoon, the length of growing season may start as late as 27th week (2 July - 8 July) in the region. Under normal condition, it starts by 24th week (11 June - 17 June). During the years with early withdrawal of southwest monsoon, the length-growing season ends by 40th week (1 October - 7 October). Under normal conditions, it ends by 48th week (26 November - 2 December). During some of the years, the post monsoon rain may extend the length of growing season upto 50^{th} to 52^{nd} week (10 December -17 December to 24 December - 31 December). The maximum lengths of growing season of 29th weeks are recorded in Table 4. Under average conditions, the length of growing season was recorded as 24th week. Therefore, short duration maize, i.e., Navjot and Mahi Kanchan, kharif pulses and sesame are suitable in the region.

To make the use of residual moisture direct sowing of rabi immediately after the harvest of kharif crop could be tried upon. However, rabi crops are possible where assured irrigation facilities are available in northwest alluvial plain zone.

References

- Chow, V. T., 1964, "Statistical and probability analysis of hydrological data", Hand Book of Applied Hydrology. V. T. Chow editor, Mc.Graw Hill, New York, 8.1-8.97.
- Deka, R. L. and Nath, K. K., 2000, "Rainfall analysis for rainfall crop planning in the upper Brahmputra valley zone of Assam", *Journal of Agrometeorology*, 2, 1, 47-53.
- Kar, G., 2002, "Rainfall probability analysis for sustainable crop production strategies in coastal Orissa", *Journal of Agrometeorology*, 4, 2, 181-185.
- Parasuraman, P., 2003, "Agro-meteorological planning for sustainable development at Denkanikottai taluk", *Madras Agric. J.*, 90, 10-12, 726-728.
- Parasuraman, P. and Suresh, M., 2002, "Study on rainfall climatology of regional research station, Paiyur for crop planning", *Madras Agric. J.*, 89, 7-9, 469-473.
- Pimpale, A. R. and Hiwase, S. S., 2001, "Probability analysis of rainfall at Nagpur", *Maharashtra Journal of Water Management*, 9, 1&2, 9-12.
- Singh, P. K., 2005, "Rainfall variability and crop planning in Sabour region of Bihar", *Journal of Agrometeorology*, **7**, 2, 284-290.
- Victor, U. S., 2000, "Characterisation of rainfall distribution and its variability", Summer school on Methods in Agroclimatic Analysis lecture notes on 22 May – 01 June, 2000 at CRIDA, Hyderabad.