Absolute angular momentum generation & its vertical transport in the field of mid-tropospheric cyclones

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सार – उत्तरी–पूर्वी अरब सागर और गुजरात तथा उत्तरी कोंकण के निकटवर्ती क्षेत्रों में दक्षिणी–पश्चिमी मानसून की अवधि में मौसम के चार्टों पर आमतौर पर लगभग स्थिर – सी सिनॉप्टिक प्रणाली देखी जाती हैं । वायुमंडलीय मौसम प्रणालियों की निर्बाध कोणीय संवेग की वहनीयता दक्षिणी–पश्चिमी मानसून की अवधि में महत्वपूर्ण भूमिका निभाती है ।

इस शोध–पत्र में औसत निर्बाध कोणीय संवेग और मध्य क्षोभमंडलीय चक्रवातों (एम. टी. सी.) से सम्बद्ध इसके ऊर्ध्वाधर अभिवहन पर विचार–विमर्श किया गया है । मध्य क्षोभमंडलीय चक्रवातों (एम. टी. सी.) के मध्य भाग के आस–पास 800 कि. मी. पूरे क्षोभमंडल में संवेग के संभावित संचयन का परिकलन करके इसको इस शोध–पत्र में दर्शाया गया है । धरातल के भीतरी भाग के कोणीय संवेग के अभिवहन तथा संवेग की कमी अथवा इसकी बढ़ोतरी पर भी विचार–विमर्श किया गया है । अभिवाहित कोणीय संवेग के माध्यम से सतही बल के आकलन तैयार किए गए हैं ।

इस अध्ययन से यह पता चला है कि कोणीय संवेग के फ्लक्स की अधिकांशतः धरातल और 500 हेक्टा पास्कल के मध्य प्रबलता पाई गई है तथा मध्य क्षोभमंडलीय चक्रवातों (एम. टी. सी.) के मध्य भाग से लेकर 500 कि. मी. तक अपरुपण प्रतिबल के मान इसके अनुकूल पाए जाते हैं ।

ABSTRACT. A quasi-stationary synoptic system is usually seen on the weather charts during southwest monsoon period over the north-east Arabian Sea and adjoining area of Gujarat and north Konkan. The transport of absolute angular momentum, for the atmospheric weather systems, plays a significant role during the southwest monsoon period.

In this paper, the mean absolute angular momentum and its vertical transport associated with MTC have been discussed. The expected accumulation of momentum throughout the troposphere upto 800 km around the centre of MTC has been calculated and displayed. The transport of angular momentum to the underlying surface and momentum loss or gain has also been discussed. Estimates have been made of surface stress by means of angular momentum transferred.

The study reveals that the flux of angular momentum found to be dominated mostly between surface and 500 hPa and the values of shearing stress are positive from the centre of MTC to 500 km.

Key words – Angular momentum, Flux of angular momentum, Shearing stress.

1. Introduction

Study of angular momentum has taken up by a wide group of research workers in meteorology holding widely divergent views (Starr 1948, Rossby & Starr 1949, Widgar (Jr) 1949, Palmen 1949 etc.). Some of these were of the opinion that transfer of momentum is like the heat transfer and as such has to be accompanied by eddy motion. On the other hand, Riehl & Yeh (1950), Riehl (1961) etc. argued that eddy transport is very small in comparison with the transport of mean motion and hence the eddy transport may be ignored.

The meridional transfer of average momentum associated with southwest monsoon for July and August in the year 1955 was calculated by Rao (1958). He concluded that the convergence of meridional transport required to balance the frictional torque of the surface west wind, is brought about by strong northward transport across the equator. He also pointed out that Indian monsoon areas become sink for westerly angular momentum on account of prevailing surface westerly winds. Keshavamurthy (1968) analyzed angular momentum balance in the region bounded by equator to 20° N and 50° E to 100° E and concluded that the

northward meridional motion in the lower part of the monsoon cell generates enough momentum through Ω transport to make up for frictional torque. Newton (1971) found that the transport of angular momentum across equator towards the summer hemisphere has a maximum value in June, July and August and of the order of 13 × 10^{25} gm /cm^2 /sec^2. He also concluded that momentum flows towards the hemisphere in which the rising branch of Hadley circulation (of winter hemisphere) is located. Sankar Rao (1962), Sankar Rao and Ramanadham (1963), Singh and Singh (1987) observed that the transient local eddies play a minor role in heat and momentum transport. Chowdhury (1982) computed mean angular momentum transport in the field of monsoon depressions and concluded that maximum extraction of angular momentum occurs where the strong wind exists. Singh and Singh (1988) observed that substantial change in periodicity with latitude in heat and momentum transport over east coast of India. Singh *et al*. (1990) observed that in the middle & upper troposphere the transport of angular momentum due to seasonal mean eddies dominates over seasonal mean circulation, whereas in the lower troposphere the momentum transport due to seasonal mean circulation exceeds that of several mean eddies transport of momentum over the west coast of India.

The angular momentum associated with synoptic system Mid - Tropospheric Cyclones (MTC) formed over northeast Arabian Sea does not seem to be much explored. In this paper, an attempt has been made to study the absolute angular momentum and its transport for well defined MTC. The eddy transport of absolute angular momentum has not been taken into consideration here (as suggested by Riehl 1961).

2.   Data and method of analysis

The cyclonic circulations formed over Gujarat & adjoining north-east Arabian Sea observed on synoptic charts have been identified in Weekly Weather Reports, issued by weather section, Pune. In the present study 27 cases of MTC's have been considered which formed during southwest monsoon season during 1985 to 1990 and months of June, July & August only. Upper air data of 0000 UTC have been collected from Mumbai, Ahmedabad, Aurangabad, Jodhpur, Karachi, Nagpur, Bhopal, Panjim and Hyderabad stations, having RS/RW facility and pilot balloon observations were also collected from Bhuj, Pune, Veraval, etc.

It is a well known fact that the MTC is not seen near the surface and upto 850 hPa and also beyond 400 hPa. The MTC is usually seen between 700 and 500 hPa. It is also known that this system shifts towards equator with height. Because of the subjectivity of the analysis a unique position of synoptic scale system cannot be fixed for each system. It may be mentioned here that the centre of MTC cannot be fixed at any unique position at any level because of the above reason. Thus, the position of the average MTC can be taken on the average as 20.8° N; 73° E ; 20° N, 72.3° E and 18.8° N, 72° E at 700 hPa, 600 hPa & 500 hPa respectively. These positions have been also considered by Miller and Keshavamurthy (1968) and Carr (1977) for their studies. For analytical study purpose the MTC is taken as axially symmetrical cylinder between 700 and 500 hPa and the average position of 600 hPa as the centre of the system, so that the system is divided into two equal halves. Thus, the average position of 600 hPa is taken as a fixed position and the analytical study is done here with respect to this only.

The radial velocity (*Vr*) and tangential velocity (*Vθ*) are resolved with the help of method suggested by Jordon (1952). The method of Composition of radial velocity (*Vr*) and tangential velocity (*Vθ*) have been adopted here as suggested by Ruprecht and Gray (1974), Sarkar and Chowdhury (1988).

3.   Absolute angular momentum transport and stress computation

The absolute angular momentum transport (*M*) is defined in cylindrical coordinates as,

$$M = r\, V\theta + \tfrac{1}{2} f r^2 \qquad (1)$$

where,   *Vθ* - tangential velocity.

   r - radial distance from the centre of MTC.

   f - Coriolis force.

The horizontal fluxes of angular momentum have been computed at 100 km interval radially and 100 hPa interval vertically upto 100 hPa.

The vertical transport term, arising due to the horizontal fluxes is given by,

$$\int_{\alpha} \gamma\left(\frac{\partial J\theta z}{\partial z}\right) d\alpha = -\int_{P_B}^{P_T}\int_{O}^{2\pi} r V_r M\, d\theta\, \frac{dp}{g} \qquad (2)$$

where,

α   is the volume of a cylinder and dα its element.

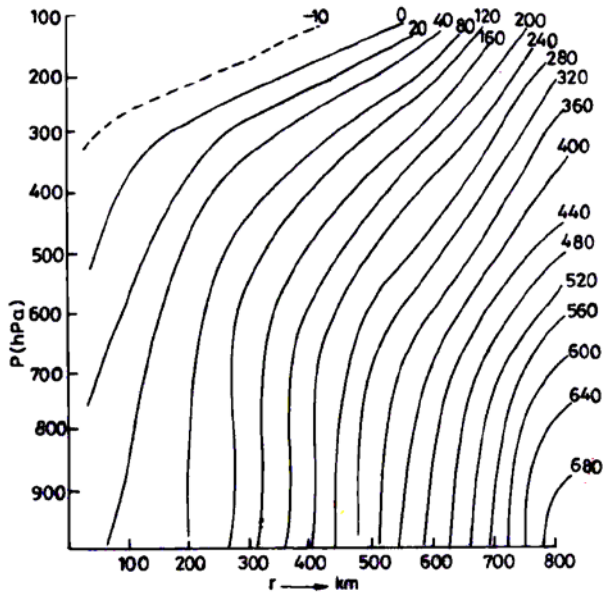P_B & P_T are pressure values at bottom & top of layer.

Fig. 1. Vertical cross section of absolute angular momentum (Unit : 100 km²/hr)

θ is angular displacement & $d\theta$ its element.

g   is acceleration due to gravity.

dp is pressure element.

Vr is the radial velocity.

The Eqn. (2) is similar to Miller (1962) and Hawkins & Rubsam (1968).

Using the values of absolute angular momentum, transfer of surface stress ($J\theta_0$) has been estimated. The expression for surface stress has been derived with the help of absolute angular momentum, radial velocity and tangential velocity. The expression for surface stress ($J\theta_0$) in θ - Z plane used here has been taken from Chowdhury (1982), given below,

$$J_{\theta_0} = \int_{P_B}^{P_T} \left[\frac{1}{r^2} \frac{\partial(r.Vr.M)}{\partial r} \right] \frac{dp}{g} \tag{3}$$

The surface stress ($J\theta_0$) has been computed from an absolute vorticity (ξ) following Hawkins & Rubsam (1968), which is given below :

$$J_{\theta_0} = \int_{P_B}^{P_T} V_r \xi \frac{dp}{g} \tag{4}$$

Where, $\xi = \eta + f$

where, η - relative velocity.

           f - Coriolis parameter.

The surface stress have been computed separately by these two methods and their comparison has been discussed.

4.   Results and discussion

4.1. *Distribution of absolute angular momentum (M)*

The vertical distribution of absolute angular momentum (*M*) is shown in Fig. 1. Absolute angular momentum is seen to decrease as the air parcel spiraled inward towards the centre of MTC. Complete momentum is lost at the radial distance of 50 km for 500 hPa, at 100 km for 450 hPa, at 200 km for 300 hPa, at 300 km for 250 hPa, at 400 km near 200 hPa and at the distance of 500 km for 150 hPa. The convergence of air mass in the MTC may be taken as the area obtained by the line joining the above points.

It is also seen that the isopleths in the MTC field are nearly vertical from surface to 500 hPa at 100 km radial distance and from surface to 900 hPa at 800 km radial distance. This suggests that more inflow of air mass is possible in the region near the centre, while the inflow gradually decreases radially outward. In the outer periphery of the MTC field the isopleths are more curved indicating inflow prominently decrease in this field.

4.2. *Momentum extraction*

The momentum extracted, by the underlying surface or given by the surface to the layers immediately above it, has also been calculated in this study. The values were derived using the net horizontal fluxes into and out of the annular volumes from the surface to 100 hPa level. The net difference of momentum (or residual) *i.e*., the import less the export per annulus represent the momentum gained or lost by it.

Expected accumulation of the angular momentum is shown in Fig. 2. The momentum lost to the surface is seen upto 500 km from the centre, which means the excess momentum is directed towards the Earth's surface. On the other hand, momentum is extracted from the Earth's surface and transported upwards beyond 500 km till the periphery of the MTC field. The values of vertically downward fluxes near the surface of the MTC field are lower in magnitude than those transported vertically
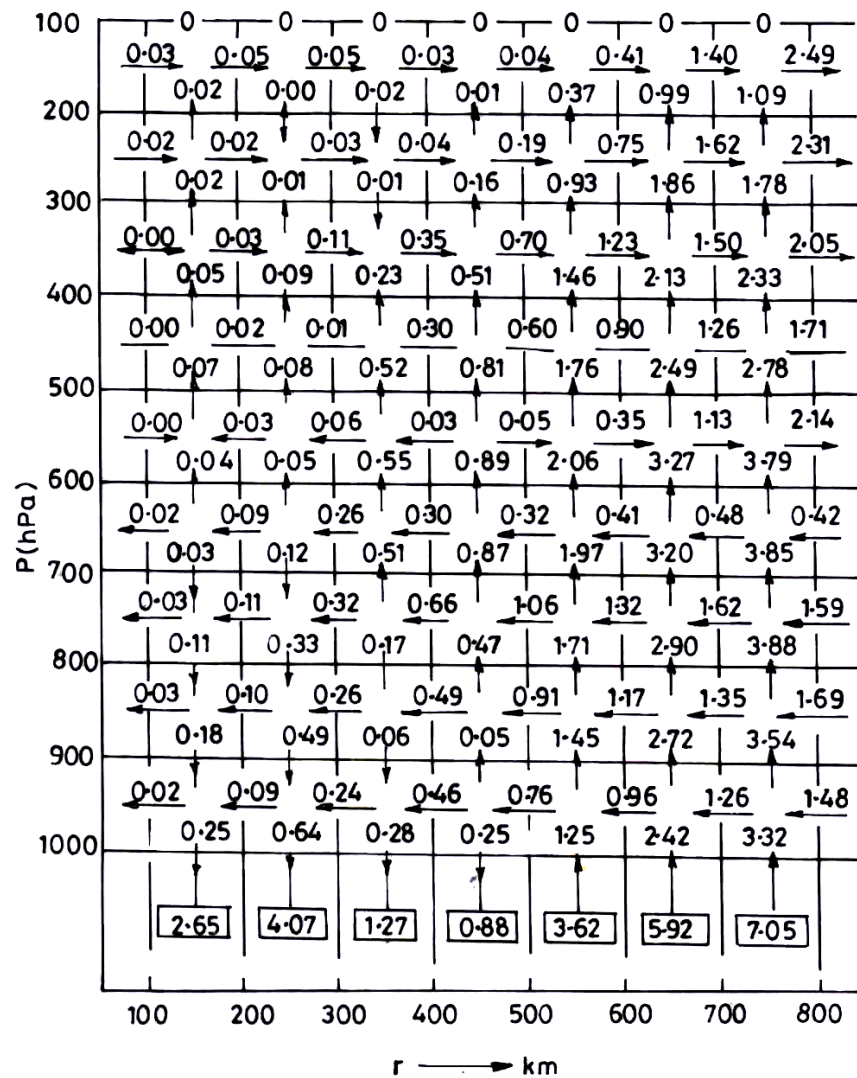
Fig. 2. Flux of angular momentum (Unit : 10^{24} gm cm^2 sec^{-2}) for "The average MTC" in the lower boxes are momentum lost to the underlying surface ($\downarrow$) or gained from it ($\uparrow$) computed to the top of the atmosphere (assumed 100 hPa) the lowermost small enclosed boxes contained the momentum loss or gain per unit area in the units of 10^8 gm sec^{-2}

upwards. The vertically transported angular momentum fluxes are greater in magnitude beyond 500 km radially outward. It means that to produce the necessary balance in the annular rings, the angular momentum is transported through friction to the winds above. Thus, the air penetrating in the lower levels radially outward from the centre pick up the momentum from the Earth's surface and is transported vertically upward. Thus, the vertical momentum fluxes are transported to greater height and the net balance is attained. The vertically downward fluxes near the centre upto 500 km shows that the air mass transport to the underlying surface, mostly covered by ocean surfaces, apparently compensates for increasing wind speed and a greater drag.

It is also seen that from the surface to 500 hPa, the horizontal fluxes are mostly directed towards the centre of MTC. This suggests that the inflow of air mass between these levels takes place. As a result the outflow of air mass is above 500 hPa levels.

Fig. 2 also contains the momentum losses per sq. cm. ($\times$ 10^8gm/sec^2) within the annular rings. This is indicated in the rectangular boxes at the bottom of the figure. It is seen that the exchange of momentum from the centre up to 500 km is given below to the surface and beyond 500 km, it is extracted from the surface. The momentum from the surface has been observed to increase generally radially outward from the centre upto 400 km. Thus, the

TABLE 1

Tangential shearing stress (J_{θ_0}) [Unit : dynes/cm^2]

Range (km)	000-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800
From Eqn. 3	19.54	17.96	13.51	5.24	2.26	-6.76	-9.09	-9.16
From Eqn. 4	14.35	12.64	10.44	6.26	2.04	-1.74	-4.98	-6.92

rate at which angular momentum was extracted per unit area increased as the radial distance decreased upto 500 km. This rate also increases as the radial distance increases. This result in greater loss of momentum per unit area at higher wind speeds. Comparison of absolute angular momentum values for MTC with those of monsoon depression determined by Chowdhury (1982), shows that the two values are mostly comparable. However the absolute angular momentum flux entering the surface or given by the surface to the surrounding area revealed that these values in the MTC are much lower than that of monsoon depression. The reason for this is perhaps, the fact that monsoon depression extends upwards to the great height from the surface whereas MTC, as the name suggests, is observed only in mid-tropospheric levels from about 850 to 700 hPa.

4.3. *Absolute angular momentum and surface stress*

Numerical integration of Eqn. (3) and Eqn. (4) furnish value of the shearing stress by the two different methods. These results obtained are shown in Table 1.

Both methods gave negative shearing stress beyond 500 km, whereas positive values are observed upto 500 km from the centre. The values obtained by both methods are comparable. The values of shearing stress obtained angular momentum method appears to be longer (in absolute terms) than those by the other method. This shows that the influence of shearing stress in extraction of air mass from the surface and transportation vertically above is more predominant between 500 & 800 km. The positive values of shearing stress near the centre of MTC and upto 500 km indicates that the exchange coefficient is positive and so the accumulated air mass is transported towards Earth's surface. This is in conformity with the result obtained in the previous section.

Comparison of the angular momentum transport in the field of depression and MTC show that in the depression field the momentum is lost to the underlying surface upto 600 km from the centre, whereas in the MTC field the momentum loss is upto 500 km. Maximum extraction of momentum in the field of depression occurs in the 300 - 400 km annular zone, where wind speeds are the strongest. In the MTC field also maximum extraction of angular momentum is seen from centre upto 300 km. The exchange of angular momentum is restricted near the centre MTC, which indicates that the areal extent of MTC field is less compared to that of depression.

5. Conclusions

The following are the salient features that emerge out of the study :

(*i*) The angular momentum flux is directed towards the centre in the MTC field and generally observed to occurs between surface and 500 hPa.

(*ii*) From the centre upto 500 km. the shearing stress gives away air mass to the underlying surface and hence possesses positive value of exchange coefficient.

(*iii*) The magnitude of the tangential shearing stress has, comparatively stronger positive value upto 300 km.

(*iv*) The shearing surface stress extracts more momentum from the surface radially outward beyond 500 km which maintain the vertical transport.

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