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Statistical modelling of rainfall data using modified Weibull distribution

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सार – इस शोध–पत्र में वेबुल वितरण की संशोधित व्याख्या का उपयोग करके वर्षा के आँकड़ों के प्रतिरुपण के दृष्टिकोण को प्रस्तुत किया गया है । इस मॉडल से ऋतु के दौरान शून्य वर्षा के दिनों और शुष्क दिनों (अर्थात् जिस दिन वर्षा नहीं हुई) के आँकड़े लिए जाते हैं (लाइफ टेस्टिंग के संदर्भ में यह स्थिति तात्कालिक विफलता कहलाती है) । वर्षा न होने के दिनों में सिंग्युलर डिस्ट्रीब्यूशन और दो प्राचलीय वेबुल वितरण द्वारा ऐसी स्थितियों के लिए इसमें संशोधित वेबुल वितरण को ही उपयुक्त मॉडल माना गया है । इसमें प्राचलों और उनके संयुक्त उपगामी वितरणों के साथ–साथ वर्षा के औसत के अधिकतम संभावित आकलन (एम.एल.ई.) प्राप्त किए हैं । उक्त परिणामों के आधार पर मिश्र वितरण के औसत के लिए 95 प्रतिशत विश्वसनीयता (कान्फीडेंस इटरवल) प्राप्त की गई है । 1961 से 1970 तक की दस वर्षों की अवधि के दौरान भारत में दो स्थानों नामतः जलगाँव और कोयम्बटूर मंडलों में हुई वर्षा के लिए गए वास्तविक आँकड़ों के आधार पर इस पद्धति की उदाहरण सहित व्याख्या की गई है । इस शोध–पत्र में प्रत्येक वर्ष के प्राचलों, औसत वर्षा और विश्वसनीयता ठहराव के आकलन उपलब्ध कराए गए हैं ।

ABSTRACT. This article presents the aspects of modeling rainfall data using a modified version of weibull distribution. This model takes care of those days reported with zero rainfall corresponding to the dry days (*i.e.*, no rain day) during a season (in life testing context this situation is called instantaneous failures). We propose a modified weibull distribution as a suitable model to represent such situations by mixture of a singular distribution at zero and a two parameter weibull distribution. We derive Maximum Likelihood Estimates (MLE) of the parameters and their joint asymptotic distributions as well as that of the mean rainfall. Based on the above results a 95% confidence interval for the mean of the mixture distribution is obtained. The method is illustrated with actual measured data on rainfall from two stations namely, Jalgaon and Coimbatore divisions in India during a 10 year period from 1961 to 1970. Correspond to each year, the estimates of the parameters, average rainfall and confidence intervals are provided.

Key words – Fisher information, Instantaneous failures, Mixture distribution, Maximum likelihood estimator, Singular distribution at zero, Confidence interval, Weibull plot.

1. Introduction

Statistical modeling of rainfall data has been a major area of research for Climatologists and Meteorologists for quite some time. Often precipitation analysis is just an empirical analysis, see IPCC (2001) for examples. Obtaining an exhaustive model which incorporates rainfall occurrence, amounts and other factors associated with seasonality parameters and temporal dependence are a long pending problem and are very difficult to analyze as long as the data is sensitive with respect to its geophysical structure. Often the modeling of rainfall is done using first order Markov process. Under a normal rainfall situation this technique performs well with respect to its statistical properties are concerned. Other widely used models accepted within the statistical and meteorological fields for rainfall amounts are the gamma distribution. Whether this distribution provides a good fit to rainfall data has been explored and is of much current debate. For more and other details we refer to Barnett and Turkman (1994).

Smith (1994) showed for North Carolina data that, while the weibull distribution does a reasonable job, the gamma and the transformed gamma does equally well. Other questions being addressed in the statistical community is how best to model the spatial dependence between different precipitation stations. The author showed that one should not count a trace amount as a dry day. The implication is that how the data is recorded and taking care of the descritization of the data in its is recorded is important.

In Indian situation though the effect of seasonality is not very serious, the occurrence of rainfall at various

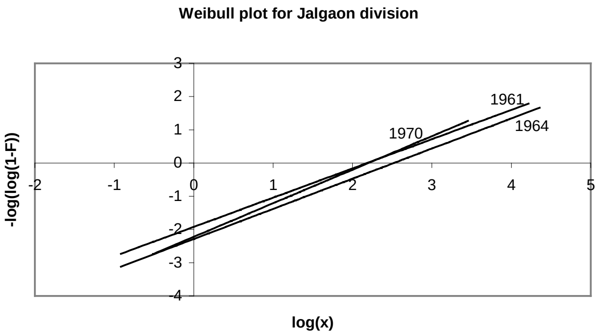

Fig. 1. Weibull plot for Jalgaon division

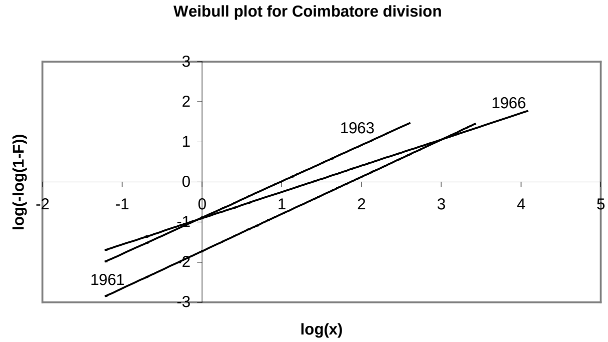

Fig. 2. Weibull plot for Coimbatore division

stations are not uniform because of its geophysical structure. Therefore, analyzing rainfall using a general statistical model has not received much attention. The commonly used methods for fitting and forecasting rainfall are either based on least squares regression techniques or time series analysis or trend analysis or Fourier series analysis etc. Sajnani (1964) has proposed a statistical technique for medium range forecasting of rainfall over Calcutta using the southwest monsoon in line with Jagannathan and Ramamurthy (1961). Another approach is based on stochastic modeling and forecasting. Thapliyal (1982, 1990) and Gowariker *et al*. (1989) have done large scale prediction of rainfall over India. The other models are due to De. *et al*. (1984) and Ramaswamy *et al*. (1986) etc.

Muralidharan and Kale (2002) have analyzed the rainfall occurrence based on modified version of gamma distribution for two meteorological stations in India. The authors have proposed the MLE approach to fitting parametric models. In this article, we present a similar approach for analyzing rainfall data using a modified version of Weibull distribution. We also give some simplified results for the information matrix elements, which are usually not available in the literature. We do admit that, the above model is not incorporating all physical components associated with the precipitation and the geophysical components of the stations. The object hence is to present a simple straight forward approach to model the daily rainfall occurrence during a season which may be useful for deciding future course of actions regarding scarcity of water.

Consider a model $\Im = \{F(x, \theta), x \geq 0, \theta \in \Omega\}$ where $F(x, \theta)$ is a continuous failure time distribution function (df) with $F(0) = 0$. To accommodate a real life situation, where instantaneous failures are observed at the origin (*i.e.*, $X = 0$), the model $\Im$ is modified to $\wp = \{G(x, \theta, \alpha), x \geq 0, \theta \in \Omega, 0 < \alpha < 1\}$ by using a mixture in the proportion $1-\alpha$ and α respectively of a singular random variable Z at zero and with a random variable X with df $F \in \Im$. Thus the modified failure time distribution is given by the df.

$$G(x, \theta, \alpha) = \begin{cases} 1-\alpha & , x = 0 \\ 1-\alpha+\alpha F(x,\theta), & x > 0 \end{cases} \tag{1.1}$$

The problem of inference about (α, θ) has received considerable attention particularly when X is exponential with mean θ. Some of the early references are Aitchison (1955), Kleyle and Dahiya (1975), Jayade and Prasad (1990), Vannman (1991, 1995), Kale and Muralidharan (1999), Muralidharan (1999, 2000) and references contained therein.

The object of this paper is to consider the model G given by Eqn. (1.1) when $F(x, \beta, \theta)$ is a two parameter weibull distribution, β being the shape parameter and θ the scale parameter and thereby applying the same for rainfall data. Usually the rainfall data are statistically modeled using unimodal distributions like normal, gamma, extreme value distributions etc. This quite often discards the excessive number of observations with $X_i = 0$ (no rain days). In section 2, we first establish the suitability of the weibull distribution using a weibull plot and goodness-of-fit test. Then we obtained the MLE's of the parameters of the model. In section 3, the fisher information and asymptotic distribution of the over all mean and its asymptotic confidence intervals are derived. We illustrate the above technique on a real life data on rainfall at two different stations in India namely Jalgaon and Coimbatore treating the dry days as $X_i = 0$. This and other discussions are given in section 4. The conclusion is given in the last section.

2. Model fit and estimation

2.1. *Weibull plot*

First of all, we justify the use of weibull distribution for modeling the rainfall separately for Jalgaon division and Coimbatore division through weibull plot. We refer to Meeker and Escobar (1998) and plotted log {-log [1- $F(x_i)$]}, against log (x_i) and obtained the weibull plot. Fig. 1 is the plot for Jalgaon division and Fig. 2 for Coimbatore division. The figures show a good fit for weibull distribution. Further, we have also done the goodness-of-fit test for many data sets. If we assume, under H_0 : the data follows a modified weibull distribution, then for Jalgaon 1961 data, we have obtained the chi-square calculated value (χ^2_{cal}) as 0.990184, which is much less than the tabled value (χ^2_{α}) = 3.8414 for α = 5%. Hence the hypothesis is accepted.

2.2. *Maximum likelihood estimation*

Let $(X_1, X_2, ..., X_n)$ be a random sample of size n from $g \in \wp$. Then

$$L(x, \alpha, \beta, \theta) = \prod_{i=1}^{n} g(x_i, \alpha, \beta, \theta)$$

Define

$$Z(x) = \begin{cases} 1, x = 0 \\ 0, x > 0 \end{cases}$$

Then

$$L(x,\alpha,\beta,\theta) = \prod_{i=1}^{n} (1-\alpha)^{z(x_i)} \left[\alpha f(x_i, \beta, \theta)\right]^{1-z(x_i)}$$

$$= (1-\alpha)^{\sum z(x_i)} \left(\frac{\alpha\beta}{\theta}\right)^{n-\sum z(x_i)} \prod_{x_c > 0} \left[x_i^{\beta-1} e^{-x_i^{\beta}/\theta}\right]^{1-z(x_i)} \tag{2.1}$$

It is seen that $\left[\sum_{i=1}^{n} z(x_i), \quad \sum_{i=1}^{n}[1-z(x_i)]\ln x_i, \quad \sum_{i=1}^{n}[1-z(x_i)]x_i\right]'$ are jointly complete sufficient for $(\alpha,\beta,\theta)'$. If we denote $\sum z(x_i) = n_0$ then from Eqn. (2.1), the likelihood equations are given by

$$\frac{\partial \ln L}{\partial \alpha} = \frac{-n_0}{1-\alpha} + \frac{n-n_0}{\alpha} = 0 \tag{2.2}$$

$$\frac{\partial \ln L}{\partial \beta} = \frac{\sum_{x_i>0} x_i^{\beta} \ln x_i}{\sum_{x_i>0} x_i^{\beta}} - \frac{1}{\beta} - \frac{\sum_{x_i>0} \ln x_i}{n-n_0} = 0 \tag{2.3}$$

and

$$\frac{\partial \ln L}{\partial \theta} = \frac{-(n-n_0)}{\theta} + \frac{\sum_{x_i>0} x_i^{\beta}}{\theta^2} = 0 \tag{2.4}$$

then from Eqn. (2.2), we have $\hat{\alpha} = \frac{n-n_0}{n}$ and Eqn. (2.3) is solved using any numerical method to get $\hat{\beta}$ and then solve for Eqn. (2.4) to get $\hat{\theta}$.

3. Fisher information and asymptotic distribution

Note that g (x, α, β, θ) given by

$$g(x,\alpha,\beta,\theta) = \begin{cases} 1-\alpha, \; x = 0 \\ \frac{\alpha\beta}{\theta} x^{\beta-1} e^{-x^{\beta}/\theta}, x > 0, \beta > 0, \theta > 0 \end{cases} \tag{3.1}$$

Then

$$I_{\alpha\alpha} = E\left(-\frac{\partial^2 \ln g}{\partial \alpha^2}\right) = \left[\alpha(1-\alpha)\right]^{-1} \tag{3.2}$$

$$I_{\alpha\beta} = E\left(\frac{\partial \ln g}{\partial \alpha}\frac{\partial \ln g}{\partial \beta}\right) = 0 \tag{3.3}$$

$$I_{\alpha\theta} = E\left(\frac{\partial \ln g}{\partial \alpha}\frac{\partial \ln g}{\partial \theta}\right) = 0 \tag{3.4}$$

$$I_{\theta\theta} = E\left(-\frac{\partial^2 \ln g}{\partial \theta^2}\right) = \alpha/\theta^2 \tag{3.5}$$

$$I_{\beta\theta} = E\left(\frac{\partial \ln g}{\partial \beta}\frac{\partial \ln g}{\partial \theta}\right) = -\frac{\alpha}{\theta} E\left[X^{\beta}\ln(X)\right]$$

$$= -\frac{\alpha}{\beta\theta}\{1-[\ln(1/\theta) - C]\} \tag{3.6}$$

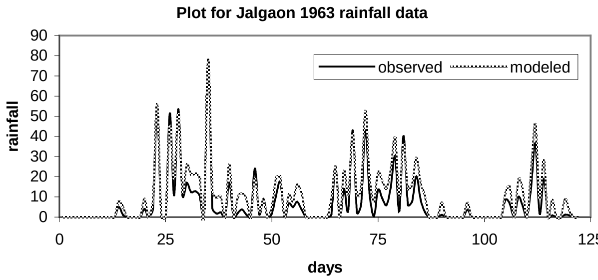

Fig. 3. Plot for Jalgaon 1963 rainfall data

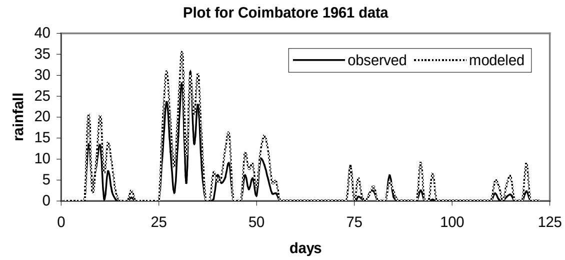

Fig. 4. Plot for Coimbatore 1961 rainfall data

and

$$I_{\beta\beta} = E\left(-\frac{\partial^2 \ln g}{\partial \beta^2}\right) = \frac{\alpha}{\beta^2} + \frac{\alpha}{\theta} E\left[X^{\beta}(\ln X)^2\right]$$

$$= \frac{\alpha}{\beta^2}[(\ln(1/\theta) + \mathrm{C})(\ln(1/\theta) + \mathrm{C} - 2) + \pi^2/6 + 1] \qquad (3.7)$$

where C is the Euler's Constant. Therefore, the Fisher information is

$$I_{\mathrm{g}}(\alpha,\beta,\theta) = \begin{bmatrix} I_{\alpha\alpha} & I_{\alpha\beta} & I_{\alpha\theta} \\ I_{\beta\alpha} & I_{\beta\beta} & I_{\beta\theta} \\ I_{\theta\alpha} & I_{\theta\beta} & I_{\theta\theta} \end{bmatrix}$$

Also using the standard results on MLE, we have $(\hat{\alpha},\hat{\beta},\hat{\theta})' \sim \mathrm{AN}^{(3)}[(\alpha,\beta,\theta)', \frac{1}{n} I_G^{-1}(\alpha,\beta,\theta)]$, where

$$\frac{1}{n} I^{-1}(\alpha,\beta,\theta) = \begin{bmatrix} \frac{\alpha(1-\alpha)}{n} & 0 & 0 \\ 0 & \frac{I_{\beta\beta}}{n\Delta} & \frac{I_{\theta\beta}}{n\Delta} \\ 0 & \frac{I_{\beta\theta}}{n\Delta} & \frac{I_{\theta\theta}}{n\Delta} \end{bmatrix},$$

where $\Delta = \frac{\pi^2\alpha^2}{6\beta^2\theta^2}$

Using the above variances and covariance's, one can propose large sample tests and confidence intervals for α,θ and β.

If we take $\psi = \psi(\alpha,\beta,\theta) = \frac{\alpha}{\beta}\theta^{1/\beta}\Gamma(1/\beta)$, which is the mean of the mixture distribution, then by using δ-method, the variance of ψ is obtained as

$$\mathrm{Var}(\psi) = \left(\frac{\partial\Psi}{\partial\alpha}\right)^2 \mathrm{Var}(\hat{\alpha}) + \left(\frac{\partial\Psi}{\partial\beta}\right)^2 \mathrm{Var}(\hat{\beta}) + \left(\frac{\partial\Psi}{\partial\theta}\right)^2 \mathrm{Var}(\hat{\theta}) + 2\left(\frac{\partial\Psi}{\partial\alpha}\right)\left(\frac{\partial\Psi}{\partial\beta}\right)\mathrm{Cov}(\hat{\alpha},\hat{\beta}) + 2\left(\frac{\partial\Psi}{\partial\alpha}\right)\left(\frac{\partial\Psi}{\partial\theta}\right)\mathrm{Cov}(\hat{\alpha},\hat{\theta}) + 2\left(\frac{\partial\Psi}{\partial\beta}\right)\left(\frac{\partial\Psi}{\partial\theta}\right)\mathrm{Cov}(\hat{\beta},\hat{\theta}),$$

where $\frac{\partial\Psi}{\partial\alpha}, \frac{\partial\Psi}{\partial\beta}, \frac{\partial\Psi}{\partial\theta}$ are the partial derivative of Ψ with respect to α, β and θ respectively and Γz is the usual gamma function. The asymptotic distribution of ψ is then given by

$$\hat{\Psi} \sim \mathrm{AN}^{(1)}[\Psi, \mathrm{Var}(\Psi)]$$

Therefore,

$$Z = \frac{\sqrt{n}}{\sqrt{\mathrm{Var}(\Psi)}}(\hat{\Psi} - \Psi) \sim N(0,1).$$

Here Z is a pivotal quantity and can be used for constructing confidence interval for the mean . The 95% confidence interval for Ψ is $\hat{\Psi} \pm 1.96\sqrt{\mathrm{Var}(\Psi a}/\sqrt{n}$.

4. Application

We now apply the above method to estimate mean rainfall $\psi = \frac{\alpha}{\beta}\theta^{1/\beta}\Gamma(1/\beta)$, in the rainy season at a given place. In India, normally the rainy season (monsoons) cover a period of four months from June to September each year. Since the crop-yield depends crucially on rainfall the prediction of the mean rainfall in the rainy season is of interest. If deficient rains are predicted alternative sources of water such as tankers, wells etc. can

TABLE 1

Estimates and confidence intervals for Jalgaon division rainfall data

Year	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\theta}$	Actual mean rainfall (mm)	Estimated mean rainfall (mm)	95% confidence interval
1961	0.58197	0.88024	6.83847	5.50984	5.50453	(3.6101,7.3989)
1962	0.42623	0.73619	4.97058	4.63361	4.54981	(2.5605,6.5391)
1963	0.45906	0.79238	5.61827	4.60114	4.62272	(2.7458,6.4996)
1964	0.54098	0.90709	9.84454	6.80738	7.04861	(4.5071,9.5901)
1965	0.36885	0.80481	6.62215	4.34590	4.35662	(2.3561,6.3572)
1966	0.36885	0.78841	6.16844	4.35229	4.24369	(2.2827,6.2047)
1967	0.48361	0.78559	6.33689	6.36148	5.82148	(3.5238,8.1192)
1968	0.40164	0.66676	4.52493	5.79344	5.13504	(2.6758,7.5943)
1969	0.50000	0.92949	11.4602	7.18852	7.12785	(4.4221,9.8336)
1970	0.40164	1.01037	9.22427	3.60492	3.60295	(2.0039,5.2019)

TABLE 2

Estimates and confidence intervals for Coimbatore division rainfall data

Year	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\theta}$	Actual mean rainfall (mm)	Estimated mean rainfall (mm)	95% confidence interval
1961	0.39344	0.92605	5.61288	2.63115	2.62528	(1.4766,3.7739)
1962	0.26229	0.87913	3.96845	1.35082	1.34061	(0.6049,2.0763)
1963	0.28685	0.90478	2.43447	0.80574	0.80421	(0.3841,1.2248)
1964	0.31148	0.53568	3.19874	6.67541	4.84377	(1.6972,7.9903)
1965	0.23770	0.85336	2.93103	0.90984	0.90916	(0.3814,1.4369)
1966	0.31148	0.65483	2.47379	1.81314	1.68259	(0.7427,2.6225)
1967	0.27869	0.67705	2.31808	1.04016	1.26164	(0.5281,1.9952)
1968	0.36066	1.02105	2.41375	0.84672	0.84688	(0.4451,1.2487)
1969	0.22131	0.69714	2.65925	1.19754	1.14343	(0.4027,1.8842)
1970	0.27049	1.09241	1.72554	0.43278	0.43062	(0.1808,0.6805)

be used to minimize losses due to crop failure. The rainfall data (mm) for the months of June to September (for 122 days) for the years 1961-70 at the meteorological stations in Jalgaon and Coimbatore in India were made available to us by Prof. S.A. Paranjpe, Department of Statistics, University of Pune, Pune, through the courtesy of Indian Meteorological Department, Pune. The stations Jalgaon and Coimbatore were selected as Jalgaon is classified as moderate rainfall area whereas Coimbatore is classified as scanty rainfall area. Muralidharan and Kale (2002) have used the above data with modified gamma distribution as the underlying model. Fig. 3 plots the modeled and observed rainfall data for Jalgaon 1963 data and Fig. 4 plots the same for Coimbatore 1961 data on rainfall.

Using the above techniques we obtained the estimates of (α, β, θ) for each year separately and 95% confidence intervals for mean rainfall for each year and compared with the observed mean rainfall. The Table 1 gives the estimates of (α, β, θ), actual mean rainfall, estimated mean rainfall and the confidence intervals for each of ten years for Jalgaon division and the Table 2 gives the same for Coimbatore division.

It is worth noting that the actual average rainfall and the estimated average rainfall are almost the same for both the stations throughout. Also all the confidence interval includes the actual average rainfall. If the confidence intervals for ψ are used to predict the mean rainfall for next year, then we see that at Jalgaon station 7 out of 9 confidence intervals contain the observed mean rainfall in the following year. For Coimbatore station only 4 out of 9 confidence intervals contain the observed rainfall in the following year. Though there is considerable variation in the estimates of $(\alpha_i, \beta_i, \theta_i)$ from year-to-year, our proposed model gives a good statistical conclusion with regards to prediction of rainfall even for moderate to dry rainfall stations also.

5. Conclusion

We have proposed a statistical model involving three parameters, one being the mixing parameter. This model is actually a mixture of two distributions, *i.e*., a mixture of degenerate (degenerated at zero) and two parameter weibull distribution and takes care of the dry days during a rainy season. Though the estimates are not expressible in closed form, it can be tractable using computer oriented numerical methods. The 95% confidence interval provided separately for both the divisions very much include the actual rainfall of each year. From the inferences developed it is found that the suggested model is an alternative for describing rainfall data with dry spells during a season.

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References

Aitchison, J., 1955, "On the distribution of a positive random variable having a discrete probability mass at the origin", *Jour. Amer. Stat. Assn*., **50**, 901-908.

Barnett, V. and Turkman, K. F., 1994, "Eds. Statistics for the Environment 2: Water Related issues, John Wiley, Chichester, 19-42.

De, U. S., Sundri, G., Vaidya, D. V., Pillai, P. V., Das, H. P. and Rao, G. S. P., 1984, "Dynamical parameters associated with medium range oscillations of the summer monsoon rainfall over India", *Mausam*, **35**, 331-336.

Gowariker, V., Thapliyal, V., Sarker, R. P., Mandal, G. S. and Sikka, D. R., 1989, "Parametric and power regression models: New approach to long-range forecasting of monsoon rainfall in India", *Mausam*, **40**, 1, 115-122.

IPCC, 2001, "Climate change 2001 the scientific basis", Houghton, J. T., Ding, Y., Griggs, D. J., Nogur, M., Van der Linden, P. J., Dai, X., Maskell, K. and Johnson, C. A., , (eds)., Cambridge Press, Cambridge.

Jagannathan, P. and Ramamurthy, K. M., 1961, "Contingency technique applied to medium range forecasting of rainfall during the monsoon season in India, *Aust. Met. Mag*., **41**.

Jayade, V. P. and Prasad, M. S., 1990, "Estimation of parameters of mixed failure time distribution", *Comm. Statist. – Theory and Methods*, **19**, 12, 4667-4677.

Kale, B. K. and Muralidharan, K., 1999, "Optimal estimating equations in mixture distributions accommodating instantaneous or early failures", *J. Indian Statistical Asociation*, **38**, 317-329.

Kleyle, R. M. and Dahiya, R. L., 1975, "Estimation of parameters of mixed failure time distribution from censored data", *Comm. Statist.- Theory and Methods*, **4**, 9, 873-882.

Meeker, W. Q. and Escobar, L. A., 1998, "Statistical Methods for reliability data, John Wiley & Sons, New York.

Muralidharan, K., 1999, "Tests for the mixing proportion in the mixture of a degene-rate and exponential distribution", *J. Indian Stat. Assn*., **37**, 2.

Muralidharan, K., 2000, "The UMVUE and Bayes estimate of reliability of mixed Failure time distribution", *Comm. Statist-Simulations & computations*, **29**, 2, 603-619.

Muralidharan, K. and Kale, B. K., 2002, "Modified gamma distribution with singularity at zero", *Comm. Statist.-Simulation and Computations*, **31**, 1, 143-158.

Ramasastry, A. A., De, U. S., Vaidya, D. V. and Sundri, G., 1986, "40-day mode and medium range forecasting", *Mausam*, **37**, 305-312.

Sajnani, P. P., 1964, "A statistical technique for medium range forecasting of rainfall over Calcutta during the southwest monsoon season., *Indian J. Met. & Geophys*., **15**, 2, 149-162.

Smith, R. L., 1994, "Spatial modeling of rainfall data. In, Statistics for the Environment 2: Water related issues, edited by V. Barnett and F. Trukman, John Wiley, Chichester, 19-42.

Thapliyal, V., 1982, "Stochastic dynamic forecast model for monsoon rainfall in Peninsular India", *Mausam*, **33**, 399-404.

Thapliyal, V., 1990, "Large scale prediction of summer monsoon rainfall over India : Evolution and development of new models", *Mausam*, **43**, 339-346.

Vannman, K., 1991, "Comparing samples from non-standard mixtures of distributions with Applications to quality comparison of wood", Research report 1991:2 submitted to Division of Quality Technology, Lulea University, Lulea, Sweden.

Vannman, K., 1995, "On the distribution of the estimated mean from the non-standard mixtures of distribution", *Comm. Statist.–Theory and Methods*, **24**, 6, 1569-1584.