A mathematical model for rotating stratified airflow across Khasi-Jayantia hills

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सार — इस शोध-पत्र में भारत की खासी और जयंतिया पर्वत श्रंखलाओं में पर्वत-वायुरोध, वायु-संवेग और ऊर्जा फलक्सों का अध्ययन करने के लिए एक ट्वि-आयामी मेसोस्केल निदर्श पर विचार किया गया है। स्तरित आवर्ती तरल पदार्थ के लिए पर्वत—वायरोध, वाय—संवेग और ऊर्जा फलक्सों की सामान्य अभिव्यक्तियाँ बताई गई है। इसमें यह देखा गया है कि पवन गति की तीव्रता में कमी के परिणामस्वरूप पर्वत—वायुरोध, वायू—संवेग और ऊर्जा फलक्स कम) हो जाते हैं जिनके परिणामस्वरूप यह वायू—प्रवाह लगभग भूविक्षेपी हो जाता है। इस अध्ययन से यह पता चलता है कि पर्वत का विस्तार अथवा अक्षांश में वृद्धि (अर्थात *f* में बढ़ोतरी) होने से पर्वत—वायुरोध, वायु—संवेग और ऊर्जा फलक्सों के परिमाण में कमी आती है। इन परिणामों की तुलना स्तरित अनावर्ती प्रवाह के साथ करने पर पर्वतीय वायू-प्रवाह में कोरिऑलिस बल का उल्लेखनीय प्रभाव देखा गया है।

ABSTRACT. A two-dimensional meso-scale model has been considered to study mountain drag, momentum and energy fluxes across Khasi-Jayantia hills of India. The general expressions for mountain drag, momentum and energy fluxes are derived for stratified rotating fluid. It is found that decrease of wind speed reduces the magnitude of mountain drag, momentum and energy fluxes. As a result, the flow becomes nearly geostrophic. The study suggests that broaden of mountain or increase in latitude (*i.e.*, increase in *f*) reduces magnitude of mountain drag, momentum and energy fluxes. When results are compared with stratified non-rotating flow, a significant impact of Coriolis force in the mountain wave is noticed.

Key words – Coriolis force, Energy flux, Mountain drag, Momentum flux and Mountain wave.

1. Introduction

For mountains wider than about 50 km (depending on wind speed) the Coriolis force begins to become important. For the smaller scale mountains airflow is characterized by generation and propagation of internal gravity waves, where as for the boarder mountains it is characterized by an anti-cyclonic vortex whose strength decays rapidly with height.

Mountain wave problem has been examined theoretically by a number of authors (Queney, 1947, 1948; Scorer, 1949; Sawyer, 1959; Eliassen and Palm, 1961; Blumen, 1965a, 1965b; Booker and Bretherton, 1967; Jone, 1967; Eliassen, 1968; Bretherton, 1969; Lilly, 1972; Buzzi and Tibaldi, 1977; Smith, 1978, 1979; Olafsson and Bougeault, 1997; etc.). Sawyer (1959) and Blumen (1965a,b) have shown that because of mountain waves, the pressure is systematically higher on the upwind slopes than on the downward ones. As a result, a net force is exerted on the ground. This force is known as pressure drag or mountain drag. Pressure drag or mountain drag is

one of the sinks in the atmospheric budget. Queney (1947) using a two-dimensional linearised model, showed that as the parameter *Lf* /*U* increases (*L* is mountain width, $f = 2\Omega \sin \phi$ is the Coriolis parameter and *U* is the mean wind speed), the flow gradually loses its wavelike character in vertical *x*, *z* plane. Eliassen and Palm (1961) showed that for 2-D linear gravity waves, the vertical flux of horizontal momentum is independent of height, when the waves are steady and non-dissipate in a non-rotating system. Bretherton (1969) reviewed theories concerning the propagation of internal gravity waves in a horizontally uniform shear flow. Smith (1978) showed that drag occurs due to the environmental wind perpendicular to ridge, and such a cross flow is necessary but not sufficient for production of drag.

Smith (1979) considered 2-D flow of a stratified rotating fluid over a ridge using linear theory model of Queney (1947). He calculated the influence of earth's rotation on mountain drag and showed that Coriolis force play an important role.

Olafsson and Bougeault (1997) considered a numerical model to investigate the form and magnitude of pressure drag created by elliptical mountains of various heights (*h*) and aspect ratios (*R*) in flows characterized by uniform upstream velocity (*U*) and stability (*N*). They showed that for lower value of the non-dimensional height Nh/U , the pressure drag is reduced by the effect of rotation and on the other hand, for the large value of *Nh* /*U* , the rotation has the opposite effect and increases the drag. Mountain wave problem addressing properties of mountain waves over Indian region was studied by many authors (Das, 1964; Sarker 1965, 1966, 1967; Sarker *et al*., 1978; De, 1973; Hatwar, 1982; Kumar *et al*., 1995 etc.). Kumar *et al.* (1995) has studied the effect of latent heat release on windward side of the mountain. Very recently Dutta (2001), Dutta *et al*. (2002) and Dutta & Naresh (2004) studied vertical velocity, fluxes of momentum and energy generated by mountain waves over India. But these studies did not consider the influence of earth's rotation on mountain wave. Therefore, the aim of the present paper is to develop a mathematical model for deriving pressure drag, momentum and energy fluxes taking into account the influence of earth's rotation for Khasi and Jayantia hills of India.

In this paper the mathematical approach to the problem is described in section-2. Section-3 contains the procedure for derivation of mountain drag and momentum flux and in section-4 the derivation of energy flux is given. The results obtained are discussed in section-5 and finally conclusions are given in section-6.

2. The mathematical approach to the problem

The basic equations of conservation of momentum, mass and density can be written as :

$$
\frac{\mathbf{D}U}{\mathbf{D}t} + f\hat{k} \times U + \left(\frac{1}{\rho}\right) \nabla p + g\hat{k} = 0 \tag{1}
$$

$$
\nabla.U=0\tag{2}
$$

$$
\frac{\mathbf{D}\theta}{\mathbf{D}t} = 0\tag{3}
$$

A stratified, steady, hydrostatic, frictionless, internally inviscid, rotating, adiabatic flow of a vertically unbounded Boussinesq fluid across a two-dimensional east-west oriented Khasi Jayantia hills is considered (De, 1973). With these assumptions and linearising the equations (1) to (3) , we get

$$
V\frac{\partial u'}{\partial y} - f v' = 0\tag{4}
$$

$$
\rho_0 V \frac{\partial v'}{\partial y} + \rho_0 f u' + \frac{\partial p'}{\partial y} = 0
$$
\n(5)

$$
V\frac{\partial w'}{\partial y} + \frac{1}{\rho_0} \frac{\partial p'}{\partial z} - g\frac{\theta'}{\overline{\theta}} = 0
$$
 (6)

$$
\frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0\tag{7}
$$

$$
V\frac{\partial \theta'}{\partial y} + \frac{d\overline{\theta}}{dz}w' = 0
$$
 (8)

In the above linearisation the basic flow has been taken $(0, V, 0)$ and perturbation (u', v', w') , where, *u*′,*v*′and*w*′ are components of velocity perturbation to the incoming flow due to orographic barrier in the *x*, *y* and *z* direction respectively. θ' , p' and ρ' are the deviation from the potential temperature, pressure and density respectively. θ is basic state potential temperature and ρ_0 is mean density.

As the earth rotates at a constant angular velocity ω, the rotation is characterized by Coriolis parameter, $f = 2\omega \sin \phi$, where ϕ denotes the latitude. The Coriolis parameter *f*, gravitational acceleration g and potential gradient $\frac{du}{dz}$ *θ* $\frac{d\theta}{dz}$ will be taken as constant. Also, it is assumed that basic flow is normal to ridge and is constant with height. The value of *V* is taken as mean of winds at different levels up to which southerly prevail. The gravitational stability of basic state is characterized by Brunt-Vaisala frequency $N^2 = \frac{g}{\overline{\theta}} \frac{d\theta}{dz}$ $e^{2} = \frac{g}{\theta} \frac{d\theta}{dz}$, which is assumed to be constant with height.

Near the ground the vertical velocity must satisfy the boundary condition

$$
w'(y, z = 0) = V \frac{\partial h}{\partial y}
$$
 (9)

where, *h*(*y*) is the profile of Khasi -Jayantia hills and its expression given by Sarker *et al*. (1978) is

$$
h(y) = \frac{H}{1 + \frac{y^2}{a^2}}
$$
 (10)

where, $a = 25.0$ km and $H = 1.6$ km

Now, if $\hat{f}(k, z)$ be the Fourier transform of function $\hat{f}(y, z)$, then they are related by

$$
\hat{f}(k, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(y, z) e^{-iky} dy
$$
\n(10A)

$$
\hat{f}(y,z) = \int_{-\infty}^{\infty} \hat{f}(k,z)e^{iky}dy
$$
\n(10B)

Now performing Fourier transforms of Eqns. (4) to (8), we obtained

$$
ikV\hat{u} - f\hat{v} = 0\tag{11}
$$

$$
ik\rho_0 V \hat{v} + \rho_0 f \hat{u} + ik\hat{p} = 0
$$
\n(12)

$$
ikV\hat{w} + \frac{1}{\rho_0} \frac{\partial \hat{p}}{\partial z} - \frac{g}{\theta} \hat{\theta} = 0
$$
 (13)

$$
ik\hat{v} + \frac{\partial \hat{w}}{\partial z} = 0
$$
 (14)

$$
ikV\hat{\theta} + \frac{\mathrm{d}\overline{\theta}}{\mathrm{d}z}\hat{w} = 0\tag{15}
$$

where \hat{u} is the Fourier transform of u' and so on.

The system of Eqns. (11) to (15) reduces to a single equation for $\hat{w}(k, z)$, which is Fourier transform of the vertical velocity $\hat{w}(x, z)$ of a fluid parcel

$$
\frac{\partial^2 \hat{w}}{\partial z^2} + \frac{1}{\rho_0} \frac{d\rho_0}{dz} \frac{\partial \hat{w}}{\partial z} + k^2 \left(\frac{l^2 - k^2}{k^2 - k_f^2} \right) \hat{w} = 0 \tag{16}
$$

where, $l = \frac{N}{V}$ is the Scorer's parameter and $k_f = \frac{f}{V}$

let,

$$
\hat{w} = \sqrt{\frac{\rho_0(0)}{\rho_0(z)}} \hat{w}_0
$$
\n(16A)

Now, substituting Eqn. (16A) into Eqn. (16) and neglecting terms of second order of smallness, we get

$$
\frac{\partial^2 \hat{w}_0}{\partial z^2} + k^2 \left(\frac{l^2 - k^2}{k^2 - k_f^2} \right) \hat{w}_0 = 0
$$
 (16B)

The solution of Eqn. (16B) is

$$
\hat{w}_0(k,z) = Ae^{\mathrm{i}k\left(\frac{l^2-k^2}{k^2-k_f^2}\right)^{1/2}z} + Be^{\mathrm{i}k\left(\frac{l^2-k^2}{k^2-k_f^2}\right)^{1/2}z}
$$
(17)

For vertically propagating hydrostatic wave $k \ll 1 \approx 10^{-3}$ m⁻¹ equation (17) will reduce to

$$
\hat{w}_0(k, z) = Ae^{\frac{iklz}{(k^2 - k_f^2)^{1/2}}} + Be^{\frac{iklz}{(k^2 - k_f^2)^{1/2}}}
$$
\n(18)

Again, as energy is propagated at great height, B should be equal to zero and consequently Eqn. (18) reduces to

$$
\hat{w}_0(k, z) = Ae^{\frac{iklz}{(k^2 - k_f^2)^{1/2}}}
$$
\n(19)

Now, using Eqns. (9) and (10) in Eqn. (19), we obtain

$$
\hat{w}_0(k,z) = i\text{aH}Vke^{-ak}e^{\frac{iklz}{(k^2-k_f^2)^{1/2}}}
$$

Again using Eqn. (16A), we get

$$
\hat{w}(k, z) = i\text{aHV}\sqrt{\frac{\rho_0(0)}{\rho_0(z)}}\ ke^{-ak}e^{\frac{iklz}{(k^2 - k_f^2)^{1/2}}}
$$
(20)

3. Mountain drag

Consider the horizontal force exerted from below across the chosen orography *h*(*y*). Assume that perturbation vanish at $y = \infty$ or $y = -\infty$.

We consider the quantity

$$
F = \int_{-\infty}^{\infty} p' d\eta' = \int_{-\infty}^{\infty} p' \frac{d\eta'}{dy} dy = -\int_{-\infty}^{\infty} \eta' \frac{dp'}{dy} dy \qquad (21)
$$

where $\eta'(y, z)$ is the height of the streamline above undisturbed level.

Near the ground $\eta'(y, z = 0) = h(y)$

Also,

$$
w' = V \frac{\partial \eta'}{\partial y} \tag{22}
$$

As the linearised momentum equation in the *y* direction is

$$
\rho_0 V \frac{\partial v'}{\partial y} + \rho_0 f u' + \frac{\partial p'}{\partial y} = 0
$$

So,

$$
F = \int_{-\infty}^{\infty} \rho_0 V \eta' \frac{\partial v'}{\partial y} dy + f \int_{-\infty}^{\infty} \rho_0 u' \eta' dy
$$

$$
= -\int_{-\infty}^{\infty} \rho_0 V v' \frac{\partial \eta'}{\partial y} dy + f \int_{-\infty}^{\infty} \rho_0 u' \eta' dy
$$
(23)

Using Eqn. (22) in (23)

$$
F = -\rho_0 \int_{-\infty}^{\infty} v' w' dy + f \rho_0 \int_{-\infty}^{\infty} u' \eta' dy
$$
 (24)

The first integral in Eqn. (24) is the mountain drag across a level surface, while the second is the Coriolis force acting in the region between undisturbed level and the vertically displaced streamline. Eqn. (24) is the correct form of wave drag in a rotating fluid.

In similar fashion, we can show that momentum flux (F_1) generated by mountain wave as

$$
F_1 = \rho_0 \int_{-\infty}^{\infty} v' w' dy - f \rho_0 \int_{-\infty}^{\infty} u' \eta' dy
$$
 (25)

From Eqns. (24) and (25), it is clear that mountain drag is equal to the negative of the momentum flux. Using Paraseval's theorem for Fourier integral, the mountain drag can be written as

$$
F = -2\pi \rho_0 \int_{-\infty}^{\infty} \hat{v} \hat{w}^* dk + 2\pi f \rho_0 \int_{-\infty}^{\infty} \hat{u} \hat{\eta}^* dk
$$
 (26)

where, \hat{w}^* and $\hat{\eta}^*$ are complex conjugates of \hat{w} and ηˆ respectively.

Now, using Eqns. (11) and (14) in Eqn. (26), we get

$$
F = -2\pi \rho_0 \left(i \int_{-\infty}^{\infty} \frac{1}{k} \frac{\partial \hat{w}}{\partial z} \hat{w}^* dk - Vk_f^2 \int_{-\infty}^{\infty} \frac{1}{k^2} \frac{\partial \hat{w}}{\partial z} \hat{\eta}^* dk \right)
$$
(27)

From Eqn. (22),

$$
\hat{w}(k, z) = ikV \hat{\eta}(k, z) \tag{28}
$$

Using Eqn. (28) in Eqn. (27), we get

$$
F = -2\pi i \rho_0 \int_{-\infty}^{\infty} \frac{1}{k} \frac{\partial \hat{w}}{\partial z} \hat{w}^* dk + 2\pi i \rho_0 k_f^2 \int_{-\infty}^{\infty} \frac{1}{k^3} \frac{\partial \hat{w}}{\partial z} \hat{w}^* dk
$$

= $-2\pi i \rho_0 \int_{-\infty}^{\infty} \frac{1}{k^3} (k^2 - k_f^2) \frac{\partial \hat{w}}{\partial z} \hat{w}^* dk$ (29)

As,
$$
\hat{w}(k, z) = i\text{aHV}\sqrt{\frac{\rho_0(0)}{\rho_0(z)}}ke^{-ak}e^{\frac{iklz}{(k^2-k_f^2)^{1/2}}}
$$

therefore,

$$
\frac{\partial \hat{w}}{\partial z} = -i\mathbf{a} H V \sqrt{\frac{\rho_0(0)}{\rho_0(z)}} k^2 \frac{l}{\left(k^2 - k_f^2\right)^{1/2}} e^{-ak} e^{\frac{iklz}{\left(k^2 - k_f^2\right)^{1/2}}}
$$
\n(30)

and

$$
\hat{w}^*(k, z) = -i\text{aHV}\sqrt{\frac{\rho_0(0)}{\rho_0(z)}}ke^{-ak}e^{\frac{iklz}{(k^2 - k_f^2)^{1/2}}}
$$
(30A)

Using Eqns. (30) and (30A) in Eqn. (29), it reduces to

$$
F = 2\pi \rho_0(0) N V a^2 H^2 \int_{-\infty}^{\infty} (k^2 - k_f^2)^{1/2} e^{-2ak} dk
$$
 (31)

As momentum flux is equal to the negative of the mountain drag, therefore, momentum flux can be written as

$$
F_1 = -2\pi \rho_0(0) N V a^2 H^2 \int_{-\infty}^{\infty} (k^2 - k_f^2)^{1/2} e^{-2ak} dk \qquad (32)
$$

Since, we are only interested in non-negative wave hence Eqn. (31) reduces to

$$
F = 2\pi \rho_0(0) N V a^2 H^2 \int_0^{\infty} (k^2 - k_f^2)^{1/2} e^{-2ak} dk
$$

Again to get real solution, we need to integrate only over range of *k*, where $k^2 > K_f^2$

Hence,

$$
F = 2\pi \rho_0(0) N V a^2 H^2 \int_{k_f}^{\infty} (k^2 - k_f^2)^{1/2} e^{-2ak} dk
$$
 (33)

For $f = 0 \Rightarrow k_f = 0$, Eqn. (33) becomes

$$
F_{f=0} = 2\pi \rho_0(0) N V a^2 H^2 \int_0^\infty k e^{-2ak} dk
$$

$$
F_{f=0} = \frac{1}{2} \pi \rho_0(0) N V H^2
$$
 (34)

Using Eqn. (34) in Eqn. (33), we get

$$
F = 4F_{f=0} \int_{k_f}^{\infty} \left(k^2 - k_f^2\right)^{1/2} e^{-2ak} dk
$$
 (35)

Substituting $p_f = 2a k_f = 2a \frac{f}{V}$ and $p = 2ak$ into Eqn. (35)

$$
F = F_{f=0} R(p_f) \tag{36}
$$

where,

$$
R(p_f) = \int_{p_f}^{\infty} (p^2 - p_f^2)^{1/2} e^{-p} dp
$$
 (37)

From Eqn. (36), we get

$$
R(p_f) = \frac{F}{F_{f=0}}\tag{37A}
$$

So $R(p_f)$ is the ratio of mountain drag in rotating stratified atmosphere to mountain drag in non-rotating stratified atmosphere (*i.e.*, for $f = 0$).

In similar fashion, we may get

$$
R(p_f) = \frac{F_1}{F_{1f=0}}\tag{37B}
$$

4. Energy flux

As shown by Eliassen and Palm (1961), the expression of vertical flux of wave energy is

$$
E = \int_{-\infty}^{\infty} p' w' dx
$$

= $2\pi \int_{-\infty}^{\infty} \hat{p} \hat{w}^* dx$ (38)

Using Eqns. (11), (12) and (14) in Eqn. (38), we get

$$
E = -2\pi i \rho_0 V \int_{-\infty}^{\infty} \frac{1}{k^3} \left(k^2 - k_f^2\right) \frac{\partial \hat{w}}{\partial z} \hat{w}^* dk
$$
 (39)

Substitute Eqns. (29) and (30) into Eqn. (39), we get

$$
E = 2\pi \rho_0 (0) N V^2 a^2 H^2 \int_{-\infty}^{\infty} (k^2 - k_f^2)^{1/2} e^{-2ak} dk \qquad (40)
$$

For non-negative and real solution Eqn. (40) becomes

$$
E = 2\pi \rho_0(0) N V^2 a^2 H^2 \int_{k_f}^{\infty} (k^2 - k_f^2)^{1/2} e^{-2ak} dk \qquad (41)
$$

It is clear from Eqns. (31) and (41) that mountain drag and energy flux is vertically upward for vertically propagating wave.

For
$$
f = 0 \Rightarrow k_f = 0
$$
, Eqn. (41) becomes

$$
E_{f=0} = 2\pi \rho_0 (0) N V^2 a^2 H^2 \int_{k_f}^{\infty} k e^{-2ak} dk
$$

= $\frac{1}{2} \pi \rho_0 (0) N V^2 H^2$ (42)

Fig. 1. Relationship between $R(p_f)$ and $V(ms^{-1})$

Using Eqn. (42) in Eqn. (41), so Eqn. (41) becomes

$$
E = 4E_{f=0} \int_{k_f}^{\infty} \left(k^2 - k_f^2\right)^{1/2} e^{-2ak} dk
$$
 (43)

Again substituting $p_f = 2a k_f = 2a \frac{f}{U}$ into Eqn. (43), we get

$$
E = E_{f=0} R(p_f) \tag{44}
$$

From Eqn. (44),

$$
R(p_f) = \frac{E}{E_{f=0}}\tag{44A}
$$

Equations (37A), (37B) and (44A) show that respective ratios of mountain drag, momentum and energy flux between stratified rotating flow and that of stratified non-rotating flow are same.

5. Discussion

Mountain drag, momentum and energy fluxes are investigated for stratified rotating fluid over Khasi-Jayantia hills. The analytical expressions for drag and

fluxes are obtained for stratified non-rotating flow (*i.e.*, for $f = 0$). For rotating stratified flow (*i.e.*, for $f \neq 0$), the expressions of mountain drag, momentum flux and energy flux are obtained in the form of integral, which are difficult to evaluate analytically. So, we evaluated these expressions numerically by using Gaussain-Legurre method in term of $R(p_f)$ for different values of mean wind, as shown in Fig. l. From Fig. 1, it appears that $R(p_f)$ asymptotically approaches to the value 1. Thus for higher mean wind $(> 25 \text{ms}^{-1})$ the Coriolis force has practically no influence on the flow generated by a bell shaped mountain with half-width 25 km. But for the light wind (*i.e.*, wind speed less than 10 ms⁻¹ the contribution of *f* is significant. In the case of light wind speed, the contribution of rotation is very strong and magnitudes of drag and fluxes become negligible. The resultant flow becomes nearly geostrophic. From the study, it can be seen that as the mean wind decreases or as latitude increases (*f* increases) the magnitude of mountain drag, momentum flux and energy flux decreases from its value at $f = 0$.

Equations (24) and (25) show that both mountain drag and momentum flux are equal in magnitude and opposite in sign. Equations (37A), (37B) and (44A) show that respective ratio of mountain drag, momentum flux and energy drag in rotating stratified atmosphere to nonrotating stratified atmosphere is same. This indicates that variation of drag and fluxes for rotating stratified and nonrotating stratified flow are same.

6. Conclusions

Following conclusions can be drawn from this study:

(*i*) Increase of latitude causes decrease of mountain drag, momentum flux and energy flux from its value at $f = 0$.

(*ii*) The impact of Coriolis force on mountain drag, momentum flux and energy flux is to reduce them for light wind.

(*iii*) Decrease of wind speed reduces magnitude of mountain drag, momentum flux and energy flux from its value at $f = 0$ and the resultant flow becomes nearly geostrophic.

(*iv*) As *f* increases or as mountain become broadens, the magnitude of mountain drag, momentum flux and energy flux decreases from its $f = 0$ value.

(*v*) For a vertically propagating mountain wave, energy flux is vertically upward and momentum flux is vertically downward.

 (vi) For wind speed more than 15 ms⁻¹ the magnitude of mountain drag, momentum flux and energy flux for stratified rotating flow becomes nearly equal to that of stratified non-rotating flow.

(*vii*) Ratios of mountain drag, momentum flux and energy flux between stratified rotating flow and that of stratified non-rotating flow are same.

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