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Letters to the Editor

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A STUDY OF MOUNTAIN WAVE ACROSS 2-D OROGRAPHIC BARRIERS FOR VARIABLE WIND

1. Teixeira *et al*. (2004) developed an analytical model to predict the surface drag exerted by gravity waves on an isolated axisymmetric mountain with velocity profile that varies slowly with height. Recently Teixeira and Miranda (2004) modified Teixeira *et al.* (2004) model to calculate the gravity wave drag and surface pressure perturbation analytically exerted by a stratified flow over a 2-D mountain ridge with same velocity profile. Therefore our motive of study is to extend this study to evaluate mountain drag, energy flux and surface pressure perturbation for orographic barriers of Indian region.

Mountain wave problem addressing properties of mountain waves over Indian region was firstly studied by Das (1964), Sarker (1965, 1966 & 1967), Sarker *et al*. (1978). Later Dutta (2001) have studied the mountain drag and energy flux across 2-D profile of Western Ghats of India, he has shown that plateau part of Ghats does not have any impact on the drag and flux. Dutta *et al.* (2002) analytically evaluated 3-D expression of streamline displacement and vertical velocity for Western Ghats and Khasi Jayantia hills. Dutta (2003) has developed model to compute mountain drag and energy flux for realistically varying wind and also computed these for some cases of Western Ghats and Khasi-Jayantia hills. Recently Dutta and Naresh (2005) studied fluxes of momentum and energy generated by mountain waves over Assam-Burma hills of India and shown the impact of valley between the ridges on the mountain drag and energy flux. Very recently Naresh *et al.* (2005) have shown the effect of Coriolis force on the mountain drag and energy flux across the profile of Khasi Jayantia hills of India.

The aim of the present study is to develop a mathematical model to obtain the analytical expressions for mountain drag, energy flux and surface pressure perturbation for wind which varies with height across Western Ghats as well as Assam- Burma hills of India using the analytical model of Teixeira and Miranda (2004).

2. *The mathematical model* - The surface pressure perturbation [Eqn. (10) of Teixeira and Miranda 2004] is given by

$$\hat{p}(z=0) = i\rho_0 N U_0 \left[1 + \frac{i}{2}\frac{U_0'}{N} - \frac{1}{8}\left(\frac{U_0'^2}{N^2} + 2\frac{U_0 U_0''}{N^2}\right)\right]\hat{h}(k) \tag{2.1}$$

where $N^2 = -\frac{g}{\rho_0}\frac{d\bar{\rho}}{dz}$ is Brunt-Väisälä frequency, ρ_0 is mean density , U_0 is the unperturbed surface wind velocity, U_0' and U_0'' are the first order and second order derivative of U_0, $\hat{h}(k)$ is the Fourier transform of the profile of orographic barrier $h(k)$.

By inverse Fourier transform of (2.1), we have

$$p'(z=0) = i\rho_0 N U_0 \left[1 + \frac{i}{2}\frac{U_0'}{N} - \frac{1}{8}\left(\frac{U_0'^2}{N^2} + 2\frac{U_0 U_0''}{N^2}\right)\right]\int_{-\infty}^{\infty}\hat{h}(k)\, e^{ikx}\mathrm{d}k \tag{2.2}$$

The expression of mountain drag [Eqn. (11) of Teixeira and Miranda 2004] is

$$D = 2\pi\rho_0 N U_0 \left[1 - \frac{1}{8}\left(\frac{U_0'^2}{N^2} + 2\frac{U_0 U_0''}{N^2}\right)\right]\int_{-\infty}^{\infty} k\hat{h}(k)\hat{h}^*(k)\mathrm{d}k \tag{2.3}$$

where $\hat{h}^*(k)$ is the complex conjugate of $\hat{h}(k)$.

Again the expression of Energy flux at surface is given by

$$E = -2\pi i U_0 \int_{-\infty}^{\infty} k\hat{p}(z=0)\hat{h}^*(k)\mathrm{d}k \quad \text{(Dutta, 2001)} \tag{2.4}$$

Finally substituting $\hat{p}(z=0)$ from Eqn. (2.1) into Eqn. (2.4) for real solution of energy flux

$$E = 2\pi\rho_0 N{U_0}^2\left[1-\frac{1}{8}\left(\frac{{U_0'}^2}{N^2}+2\frac{U_0U_0''}{N^2}\right)\right]\int_{-\infty}^{\infty} k\hat{h}(k)\hat{h}^*(k)\mathrm{d}k \tag{2.5}$$

3. *Surface pressure perturbation, mountain drag and energy flux across Assam-Burma hills of India* - The 2-D profile of Assam-Burma hills is

$$h(x)=\frac{a^2b_1}{a^2+x^2}+\frac{a^2b_2}{a^2+(x-d)^2} \qquad \text{(De, 1973)} \tag{3.1}$$

where, a = 20.0 km, b_1 = 0.9 km, b_2 = 0.7 km and d = 55.0 km.

The Fourier transform of Eqn. (3.1) is

$$\hat{h}(k)=ae^{-ak}\left(b_1+b_2e^{-i\mathrm{d}k}\right) \tag{3.2}$$

and

$$\hat{h}(k)\hat{h}^*(k)=a^2e^{-2ak}\left[{b_1}^2+{b_2}^2+2b_1b_2\cos(\mathrm{d}k)\right] \tag{3.3}$$

The expression of surface pressure perturbation [Eqn. (2.2)] using Eqn. (3.2) becomes

$$p_A'(z=0)=i\rho_0 aNU_0\left[1+\frac{i}{2}\frac{U_0'}{N}-\frac{1}{8}\left(\frac{{U_0'}^2}{N^2}+2\frac{U_0U_0''}{N^2}\right)\right]\int_{-\infty}^{\infty}\left(b_1+b_2e^{-i\mathrm{d}k}\right)e^{-(a-ix)k}\mathrm{d}k$$

Integrating $p'(z=0)$ for real solution, we have

$$p_A'(z=0)=-\rho_0 NU_0\left[1+\frac{i}{2}\frac{U_0'}{N}-\frac{1}{8}\left(\frac{{U_0'}^2}{N^2}+2\frac{U_0U_0''}{N^2}\right)\right]\left[\frac{b_1a(x-ia)}{a^2+x^2}+\frac{b_2a[(x-d)-ia]}{a^2+(x-d)^2}\right]$$

$$=-\rho_0 NU_0\left\{\left[1-\frac{1}{8}\left(\frac{U_0'^2}{N^2}+2\frac{U_0U_0'}{N^2}\right)\right]\left[\frac{ab_1x}{a^2+x^2}+\frac{ab_2(x-d)}{a^2+(x-d)^2}\right]+\frac{U_0'}{N}\left[\frac{a^2b_1}{a^2+x^2}+\frac{a^2b_2}{a^2+(x-d)^2}\right]\right\} \tag{3.4}$$

$p_A'(z=0)$ is the analytical expression for surface pressure perturbation, which contain two parts, first part is antisymmetric and second part is symmetric with respect to Assam - Burma hills. The symmetric part of the hills depends only the wind shear at surface.

Now to find analytical expression of mountain drag, substitute Eqn. (3.3) into Eqn. (2.3), we get

$$D_A=2\pi a^2\rho_0 NU_0\left[1-\frac{1}{8}\left(\frac{{U_0'}^2}{N^2}+2\frac{U_0U_0''}{N^2}\right)\right]\int_{-\infty}^{\infty}k\left(b_1^2+b_2^2+2b_1b_2\cos(\mathrm{d}k)\right)e^{-2ak}\mathrm{d}k$$

The mountain drag D_A for real solution becomes

$$D_A=\frac{1}{2}\pi\rho_0 NU_0\left[1-\frac{1}{8}\left(\frac{{U_0'}^2}{N^2}+2\frac{U_0U_0''}{N^2}\right)\right]\left[\left({b_1}^2+{b_2}^2\right)+8a^2b_1b_2\frac{\left(4a^2-d^2\right)}{\left(4a^2+d^2\right)^2}\right] \tag{3.5}$$

Eqn. (3.5) is the analytical expression for mountain drag across Assam- Burma hills, when wind varies with height.

Now to find the energy flux across Assam-Burma hills substitute Eqn. (3.3) into Eqn. (2.5), the corresponding analytical expression for energy flux for real solution becomes

$$E_A=\frac{1}{2}\pi\rho_0 N{U_0}^2\left[1-\frac{1}{8}\left(\frac{{U_0'}^2}{N^2}+2\frac{U_0U_0''}{N^2}\right)\right]\left[\left({b_1}^2+{b_2}^2\right)+8a^2b_1b_2\frac{\left(4a^2-d^2\right)}{\left(4a^2+d^2\right)^2}\right] \tag{3.6}$$

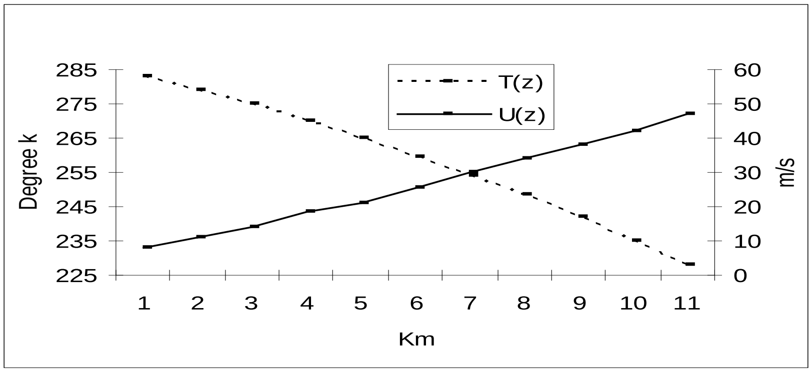

Fig. 1(a). Vertical profile of *U*(*z*) and *T*(*z*) on 09 January 1967 at Guwahati

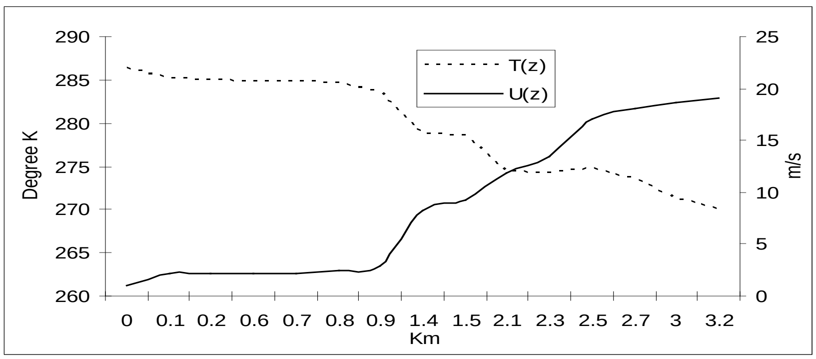

Fig. 1(b). Vertical profile of *U*(*z*) and *T*(*z*) on 24 December 2004 at Guwahati

Now mountain drag and energy flux due to valley between the ridges of Assam- Burma hills from equations (3.5) and (3.6) as done by Dutta and Naresh (2005) may be written as

$$D_{AV} = 4\pi\rho_0 N U_0 a^2 b_1 b_2 \left[1-\frac{1}{8}\left(\frac{U_0'^2}{N^2}+2\frac{U_0 U_0''}{N^2}\right)\right]\frac{\left(4a^2-d^2\right)}{\left(4a^2+d^2\right)^2} \tag{3.7}$$

and

$$E_{AV} = 4\pi\rho_0 N U_0^2 a^2 b_1 b_2 \left[1-\frac{1}{8}\left(\frac{U_0'^2}{N^2}+2\frac{U_0 U_0''}{N^2}\right)\right]\frac{\left(4a^2-d^2\right)}{\left(4a^2+d^2\right)^2} \tag{3.8}$$

De (1971) discussed the existence of lee waves on dated 9 January 1967 and 14 February 1967, whose profile are shown in Fig. 1(a) & Fig. 2(a) respectively. Now we have chosen similar type of profiles for dated 24 December 2004 and 26 December 2004, which may be favourable cases for the occurrence of lee waves, whose profiles are shown in Fig. 1(b) and Fig. 2(b) respectively.

Next our aim is to calculate P_A, D_A and E_A for dated 24 December 2004 and 26 December 2004, so for dated 24 December 2004, we have

$$P_A'(z=0) = -\frac{(366x+6732)}{400+x^2} - \frac{(285x-10439)}{400+(x-55)^2}$$

$D_A = 67715$ N/m and $E_A = 69809$ W/m

and for dated 26 December 2004, we have

$$P_A'(z=0) = -\frac{(1879x+1461)}{400+x^2}$$

$D_A = 210328$ N/m and $E_A = 441690$ W/m

Fig. 2(a). Vertical profile of *U*(*z*) and *T*(*z*) on 14 February 1967 at Guwahati

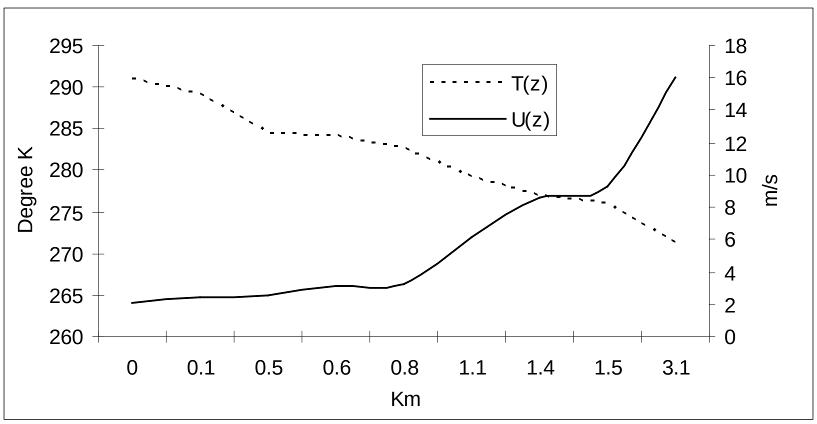

Fig. 2(b). Vertical profile of *U*(*z*) and *T*(*z*) on 26 December 2004 at Guwahati

4. *Surface pressure perturbation, Mountain drag and energy flux across Western Ghats* - Analytical expression for profile of Western Ghats is

$$h(x) = \frac{a^2 H}{a^2 + x^2} + b \tan^{-1}\frac{x}{a} \quad \text{(Sarker } et\ al.\text{, 1978)} \quad (4.1)$$

where, $a = 18.0\,\text{km}$, $H = .52\,\text{km}$, $b = \frac{2}{\pi} \times .35\,\text{km}$.

By Fourier transform of Eqn. (4.1)

$$\hat{h}(k) = \left[aH - i\frac{b}{k}\right]e^{-ak} \quad (4.2)$$

and

$$\hat{h}(k)\hat{h}^*(k) = \left[a^2 H^2 + \frac{b^2}{k^2}\right]e^{-2ak} \quad (4.3)$$

As done in case of Assam-Burma hills, similarly the surface pressure perturbation for Western Ghats of India [Eqn. (2.5)] using Eqn. (4.2) becomes

$$p'_W(z=0) = -\rho_0 N U_0 \left[1 + \frac{i}{2}\frac{U_0'}{N} - \frac{1}{8}\left(\frac{U_0'^2}{N^2} + 2\frac{U_0 U_0''}{N^2}\right)\right] \left[\frac{aH(x - ia)}{a^2 + x^2}\right]$$

$$= -\rho_0 N U_0 \left\{\left[1 - \frac{1}{8}\left(\frac{U_0'^2}{N^2} + 2\frac{U_0 U_0''}{N^2}\right)\right] \frac{aHx}{a^2 + x^2} - \frac{U_0'}{N}\frac{a^2 H}{a^2 + x^2}\right\} \quad (4.4)$$

$p'_W(z=0)$ is expression of surface pressure perturbation for Western Ghats of India, which is

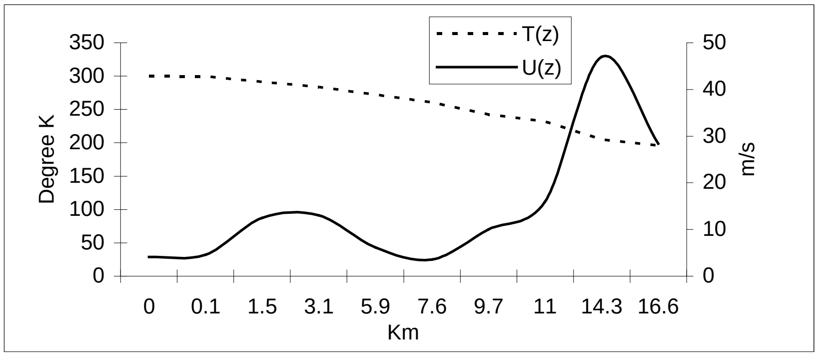

Fig. 3. Vertical profile of $U(z)$ and $T(z)$ on 05 August 2005 at Santacruz

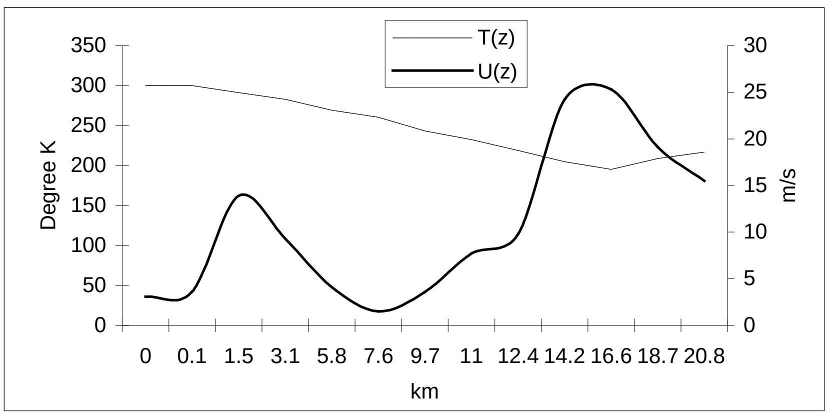

Fig. 4. Vertical profile of $U(z)$ and $T(z)$ on 08 August 2005 at Santacruz

independent on the plateau part of the Western Ghats. Both the parts of the normalized surface pressure perturbation for Western Ghats independent on the plateau part of the Western Ghats.

Similarly mountain drag across Western Ghats by substitute Eqn. (4.3) into Eqn. (2.3) becomes

$$D_W = \frac{1}{2}\pi\rho_0 N U_0 H^2 \left[1 - \frac{1}{8}\left(\frac{U_0'^2}{N^2} + 2\frac{U_0 U_0''}{N^2}\right)\right] \quad (4.5)$$

Eqn. (4.5) is expression of mountain drag, which is independent on the plateau part as well as half width of the Western Ghats.

For energy flux, substituting Eqn. (4.3) into Eqn. (2.5)

$$E_W = \frac{1}{2}\pi\rho_0 N H^2 U_0{}^2 \left[1 - \frac{1}{8}\left(\frac{U_0'^2}{N^2} + 2\frac{U_0 U_0''}{N^2}\right)\right] \quad (4.6)$$

If we assume that wind is constant with height in equations (4.5) and (4.6), then our results will reduce to similar results as obtained by Dutta (2001).

Now our aim is to evaluate P_W, D_W and E_W using realistic wind profile of Santacruz of dated 05 August 2005 and 08 August 2005 as given in Fig. 3 and Fig. 4 respectively.

So for dated 05 August 2005, we have

$$P_W'(z=0) = -\frac{8.6238x + 116.6}{324 + x^2}$$

$$D_W = 45820\text{N/m} \text{ and } E_W = 188780\text{W/m}$$

and for dated 08 August 2005, we have

$$P_W'(z=0) = -\frac{8.08x + 112.6}{324 + x^2}$$

$$D_W = 32460\text{N/m} \text{ and } E_W = 100626\text{W/m}$$

5. *Results and discussions* - We have derived the analytical expressions for surface pressure perturbation (Eqn. 3.4), mountain drag (Eqn. 3.5) and energy flux (Eqn. 3.6) for 2-D profile of Assam-Burma hills for variable wind. Similar analytical expressions from Eqn. (4.4) to Eqn. (4.6) also have been obtained for western ghats.

For constant wind velocity, mountain drag (Eqn. 3.5) and energy flux (Eqn. 3.6) reduce into following results

$$(D_A)_0 = \frac{1}{2}\pi\rho_0 N U_0 \left[\left(b_1^2 + b_2^2\right) + 8a^2 b_1 b_2 \frac{\left(4a^2 - d^2\right)}{\left(4a^2 + d^2\right)^2}\right]$$

$$(E_A)_0 = \frac{1}{2}\pi\rho_0 N U_0^2 \left[\left(b_1^2 + b_2^2\right) + 8a^2 b_1 b_2 \frac{\left(4a^2 - d^2\right)}{\left(4a^2 + d^2\right)^2}\right]$$

These results are the same as obtained by Dutta and Naresh (2005)

Using above results into equations (3.5) and (3.6) respectively, we get

$$\frac{D_A}{(D_A)_0} = \frac{E_A}{(E_A)_0} = \left[1 - \frac{1}{8}\left(\frac{U_0'^2}{N^2} + 2\frac{U_0 U_0''}{N^2}\right)\right]$$

This implies that normalized mountain drag is equal to normalized energy flux and independent on the orographic barrier.

Similarly using equations (3.7) and (3.8), we may have following expression for normalized mountain drag and normalized energy flux across valley of Assam-Burma hills

$$\frac{D_{AV}}{(D_{AV})_0} = \frac{E_{AV}}{(E_{AV})_0} = \left[1 - \frac{1}{8}\left(\frac{U_0'^2}{N^2} + 2\frac{U_0 U_0''}{N^2}\right)\right]$$

Thus normalized mountain drag and normalized energy flux for realistic wind profile of Guwahati become

$$\frac{D_A}{(D_A)_0} = \frac{E_A}{(E_A)_0} = \frac{D_{AV}}{(D_{AV})_0} = \frac{E_{AV}}{(E_{AV})_0} = .55$$

for 24 December 2004

$$\frac{D_A}{(D_A)_0} = \frac{E_A}{(E_A)_0} = \frac{D_{AV}}{(D_{AV})_0} = \frac{E_{AV}}{(E_{AV})_0} = .98$$

for 26 December 2004

In Similar way normalized mountain drag for realistic wind profile of Santacruz may be written as

$$\frac{D_W}{(D_W)_0} = \frac{E_W}{(E_W)_0} = .92135$$

for 05 August 2005

$$\frac{D_W}{(D_W)_0} = \frac{E_W}{(E_W)_0} = .863362$$

for 08 August 2005

Thus values of normalized mountain drag and energy flux is near to one for both the profiles of hills in SW monsoon.

As in equations (4.5) and (4.6) a factor '*b*' for plateau part does not appear, so we may say that plateau part of the Western Ghats does not contribute towards the generation of the mountain drag and energy flux, which is conformity with the earlier findings of Dutta (2001). Also normalized pressure perturbation (Eqn. 4.4) is independent on the plateau part of the Western Ghats.

Normalized pressure perturbation (equations 3.4 and 4.4) contains two parts, first part is antisymmetric with respect to hills and second part is symmetric with respect to hills. The symmetric parts of normalized pressure perturbation depend on the shear of surface wind. Thus for the constant wind velocity case Eqn. (3.4) reduces to

$$P_0'(z=0) = \frac{p_A'(z=0)}{\rho_0 N U_0} - \frac{a b_1 x}{a^2 + x^2} - \frac{a b_2 (x-d)}{a^2 + (x-d)^2}$$

Thus in $P_0'(z=0)$, there is no symmetric part in the above expression.

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