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APPLICATION OF MARKOV CHAIN MODEL TO DAILY RAINFALL OF JALPAIGURI STATION

1. Operational meteorological centers of this country were entrusted with the responsibility of issuing daily rainfall forecasts in public interest. In practice, these types of forecasts were issued depending upon the climatology of the place of interest, statistical methods and synoptic situations prevailing at particular point of time. Overall, forecaster's skill and experience coupled with his ability to assimilate facts before hand play an important role in formulation of such intended forecasts.

As on now, highly specific objective methods, truly appropriate and reliable, were not fully operational in these forecasting centers due to variety of reasons. Only few mathematical models that utilize the thermal and dynamical properties of atmosphere and radiation measurement are operative. In small scale, high spatial and temporal variation of daily rainfall limits the progress and evolution of suitable mathematical models which can be applied in day-to-day work with great confidence.

In present study, the Markov type mathematical model has been selected to assess the applicability and suitability of such model to predict the succeeding day situation when a particular day situation was known as Gabriel and Neumann (1962) Showed that Markov Chain Model holds well for daily rainfall occurrence at Tel Aviv and gave probability formulae for daily rainfall distribution. Medhi (1976) studied wet and dry days in pre-monsoon and monsoon seasons during the period 1967-1970 month-wise over Guwahati Airport and concluded that Markov chain of first order is adequate to represent each of the months. Fernandez *et al.* (1983) applied the same method to determine fine and bad weather days based on the sunshine hours at four cities.

In this work, firstly, the probability of occurrence of rainy day (non-rainy day) was studied and later on, the study has been extended to six-state Markov chain model and to uncertainty and redundancy.

2. Daily rainfall data for 10 years period 1991-2000 has been collected from the records of Flood Meteorological Office, India Meteorological Department, Jalpaiguri for each of the four months from June to September. Data for the purpose of study comprises four sets with 300, 310, 310 and 300 points for four months respectively.

3. The Markov chain model of Gabriel and Neumann (1962) was simple and takes into account the persistence as the next day's outcome needs to take into account the previous day's outcome. Chin (1977) suggested that higher order Markov processes are quiet natural in daily rainfall occurrences. However, here, two states, rainy day and non-rainy of the first order Markov chain model considered with maximum possible '$2n$' transitions generate a stochastic probability matrix.

$$[X] = \begin{bmatrix} P_{nn} & P_{nr} \\ P_{rn} & P_{rr} \end{bmatrix}$$

The 'Ps' indicate the transitional probabilities of one-state to another-state in a one-step. The stochastic matrix can also be represented using the one step transitional frequencies among possible states. Here 'r' represents rainy and 'n' represents non-rainy.

$$\begin{bmatrix} Fnn/Fnn + Fnr & Fnr/Fnn + Fnr \\ Frn/rn + Frr & Frr/Frn + Frr \end{bmatrix}$$

From the model, the probability that any sequence beginning with a non-rainy day will not deviate from the same state over a period of time say of specific length 'l' days is $(1-P_{nn})P_{nn}^{(l-1)}$. Similarly, the probability that any sequence beginning with a rainy day will not deviate from the state over a period of time of length 'l' days is $(1- P_{rr})P_{rr}^{(l-1)}$. The probability that a particular day will be a rainy day or non-rainy day after a lapse of fixed interval of time irrespective of the first day being rainy or non-rainy is $P_{nr}/P_{nr}+P_{rn}(P_{nr}+P_{rn})$. The mean length of such sequence is $1/(1-P_{nn})$ and $1/(1-P_{rr})$. The stochastic transitional matrix generated from the six categories of rainfall selected is

$$\begin{bmatrix} Pnn & Pl & Pnm & Pnr & Pnh & Pnv \\ Pln & Pll & Plm & Plr & Plh & Plv \\ Pmn & Pml & Pmm & Pmr & Pmh & Pmv \\ Prn & Prl & Prm & Prr & Prh & Prv \\ Phn & Phl & Phm & Phr & Phh & Phv \\ Pvn & Pvl & Pvm & Pvr & Pvh & Pvv \end{bmatrix}$$

Where 'n' denotes non-rainy, 'l' denotes light rain, 'm' denotes moderate rain, 'r' denotes rather heavy, 'h' denotes heavy and 'v' denotes very heavy.

Generally, uncertainty in the transitional probabilities of all the states of occurrences are determined as this method is useful for judging the suitability of Markov chain models to represent rainfall data. The uncertainty of the Markov model is the average of all entropy values which are weighted in accordance

TABLE 1

Showing probabilities that a non-rainy (rainy) day remains in the same state during given number of days

	June				July				August				September			
	Number of days				Number of days				Number of days				Number of days			
Day	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5
Non-rainy	0.24	0.14	0.08	0.05	0.25	0.12	0.06	0.03	0.25	0.11	0.05	0.02	0.23	0.15	0.10	0.06
Rainy	0.20	0.14	0.11	0.08	0.17	0.13	0.10	0.08	0.22	0.15	0.10	0.08	0.24	0.15	0.09	0.06

with their individual probabilities of occurrence. The uncertainties are measured as the multiplication of logarithmic probabilities and the individual probabilities for each of the states by $P_{lk} \log 10\ P_{lk}$ for all $l = 1$, n. ultimately the favorableness and un-favorableness of the Markov chain system is assessed by the redundancy values given as $R = (1 - E/E_M)$ where $E_M = \log_{10} N$. 'N' being the number of states. The theory tells that when the value of redundancy reaches one, the event is almost certain. A measure of uncertainty of the random model over the chosen Markov chain model is equal to difference in the respective entropies of two states.

4. For Jalpaiguri, based on the past decade, the average rainfall, standard deviation, average number of rainy days for the month of June were 687.25 mm, 312.31 mm and 18.3 respectively, for the month of July 989.50 mm, 332.45 mm and 21.9 respectively, for the month of August 692.14 mm, 313.28 mm and 19.4 respectively and for the month of September 509.26 mm, 304.48 mm and 14.4 mm respectively. The monthly long period averages are 668.0 mm, 826.0 mm, 649.0 mm and 535.6 mm respectively for all four months of monsoon season starting with the month of June. The study also indicated that out of 300 occasions, 10 to 20 cm rainfall occurred 12(12) times, greater than 20 cm occurred once (2) in the month of June (September). Out of 310 occasions, 10 to 20 cm rainfall occurred 20(8) times, greater than 20 cm rainfall occurred 5(2) times in the month of July (August). Heavy to very heavy rainfalls occurred 33, 45, 32, 22 occasions respectively from June to September during the period of study.

The frequencies of occurrence of rainfalls were categorized using the following criterion for each of the four months of the monsoon season starting from June.

0.0 mm to 2.4 mm   non rainy day

2.5 mm to 7.5 mm   light rainy day

7.6 mm to 34.5 mm   moderate rainy day

TABLE 2

Showing month wise mean length of non-rainy and rainy days

Month	Mean length of days	
	Non-rainy	Rainy
June	3.74	2.39
July	4.76	1.98
August	3.08	1.84
September	2.62	2.84

35.0 mm to 65.4 mm   rather heavy rainy day

65.5 mm to 125.4 mm   heavy rainy day

Greater than 125.5 mm   very heavy rainy day

The frequencies of transition from one state to another has been obtained taking into consideration the preceding day of the first day of the month and also the succeeding day of the last day of the month. The first order Markov chain model with four possible transitions, non-rainy to non-rainy, non-rainy to rainy, rainy to non-rainy and rainy to rainy yielded stochastic matrices.

June	Non-rainy	Rainy
Non-rainy	0.5812	0.4188
Rainy	0.2667	0.7323

July	Non-rainy	Rainy
Non-rainy	0.4945	0.5055
Rainy	0.2100	0.7900

August	Non-rainy	Rainy
Non-rainy	0.4569	0.5431
Rainy	0.3247	0.6753

September	Non-rainy	Rainy
Non-rainy	0.6474	0.3526
Rainy	0.3819	0.6181

TABLE 3

Showing month wise transitional probabilities of six state model

	Transitional Probability																							
	Jun						Jul						Aug						Sep					
	N	L	M	R	H	V	N	L	M	R	H	V	N	L	M	R	H	V	N	L	M	R	H	V
N	0.58	0.08	0.23	0.08	0.02	0.01	0.49	0.09	0.30	0.09	0.01	0.02	0.46	0.09	0.33	0.05	0.07	0.00	0.65	0.10	0.17	0.05	0.01	0.02
L	0.40	0.20	0.22	0.05	0.12	0.01	0.33	0.08	0.31	0.18	0.05	0.06	0.55	0.05	0.17	0.14	0.07	0.02	0.43	0.19	0.31	0.00	0.06	0.01
M	0.29	0.14	0.28	0.12	0.12	0.05	0.20	0.16	0.30	0.13	0.15	0.06	0.34	0.14	0.25	0.16	0.08	0.02	0.44	0.10	0.27	0.07	0.09	0.03
R	0.22	0.28	0.22	0.13	0.09	0.06	0.18	0.13	0.33	0.22	0.13	0.01	0.24	0.26	0.24	0.16	0.05	0.05	0.26	0.05	0.37	0.21	0.05	0.06
H	0.17	0.08	0.33	0.25	0.08	0.08	0.17	0.20	0.13	0.20	0.20	0.10	0.08	0.22	0.26	0.22	0.18	0.04	0.23	0.15	0.23	0.08	0.08	0.23
V	0.00	0.09	0.36	0.18	0.19	0.18	0.23	0.13	0.25	0.13	0.13	0.13	0.33	0.00	0.17	0.17	0.33	0.00	0.20	0.10	0.40	0.10	0.10	0.10
P	0.39	0.13	0.25	0.11	0.08	0.04	0.29	0.13	0.28	0.14	0.10	0.05	0.37	0.13	0.28	0.12	0.09	0.02	0.52	0.11	0.23	0.06	0.04	0.03
Log P	0.41	0.88	0.60	0.97	1.10	1.43	0.53	0.90	0.54	0.84	1.01	1.29	0.43	0.89	0.57	0.91	1.06	1.71	0.28	0.97	0.63	1.20	1.36	1.48
E	0.51	0.62	0.72	0.73	0.71	0.66	0.55	0.56	0.73	0.67	0.77	0.71	0.56	0.57	0.69	0.71	0.66	0.58	0.47	0.53	0.63	0.66	0.74	0.70
Ew	0.62						0.65						0.62						0.55					
P log P	0.67						0.61						0.66						0.58					
E_M	0.05						-0.04						0.04						0.04					
R	0.20						0.16						0.20						0.30					

N-No rain; L-Light rain; M-Moderate rain; R-Rather heavy rain; H-Heavy rain; V-Very heavy rain;
P – Probability; E-Entropy; E_w-Weighted Entropy; E_M-Log of number of states 'N'; R- Redundancy

It can be seen from the above stochastic matrices that the rainy day preceded by rainy day is more probable than other transitions for all four months of the monsoon season.

The probability of rainy day is 0.61 for the month of June, 0.71 for the month of July, 0.63 for the month of August and 0.48 for the month of September. The probability of non-rainy day is 0.39 for the month of June, 0.29 for the month of July, 0.37 for the month of August and 0.52 for the month of September. The probability that any sequence beginning with the non-rainy day (rainy day) will not deviate from the same state over a period of time, say, 'l' days can be arrived at by the stochastic matrices of individual months (Table 1). The figures in Table 1 indicate that the probability is not more than 25% for a length of two days and the probability is decreasing with increasing length of days in all the months considered. The mean length of non-rainy (rainy) sequences were 3.74 (2.39), 4.76 (1.98), 3.08 (1.84) and 2.62 (2.84) from June to September respectively as given in Table 2. The probability that a particular day will be rainy day after a lapse of fixed interval of time is 0.6101 for June, 0.7065 for July, 0.6258 for August and 0.4801 for September. The probability that a particular day will

be non-rainy day after a lapse of fixed interval of time irrespective of first day is rainy day or non-rainy day is 0.3899 for June, 0.2935 for July, 0.3742 for August and 0.5199 for September. The calculated probabilities for the six-state model for individual months were given in Table 3. From Table 3, it is seen that higher probabilities existed for one step transitions non-rainy to non-rainy, light to non-rainy, heavy or very heavy to moderate in all the four months considered. Further, firstly, a day is rainy or non-rainy and secondly, how much rain would be if the day is rainy can be answered probabilistically by way of multiplication of probabilities given in two-state and six-state models studied in this paper.

5.   Analysis of daily rainfall data of Jalpaiguri with Markov chain model has given all non-zero transitional probabilities which suggest dependency of future values on the past and present values to certain extent. Estimated transitional probabilities indicate rainy day to rainy day transitions were more probable in all the months followed by non-rainy to non-rainy day transitions. In case of transitions from one state to another out of 36 maximum possible transitions, one step transitions non-rainy to non-rainy, light to non-rainy, heavy or very heavy to moderate in all the four months considered. The uncertainty values

were close to four percent and redundancy values were nearly 0.2 for the months of June, July and August. The redundancy value was 0.3 for the month of September. The observation of transition probabilities may yield an idea on how the future weather conditions differ from the past and present weather conditions.

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