Computation of synthetic seismogram in real earth model by eigen function expansion

AJIT DE, A. ROY*, M. MITRA* and R. K. BHATTACHARYA*

Department of Mathematics, Siliguri College, Siliguri – 734 001, West Bengal, India

**Department of Applied Mathematics, University of Calcutta, 92, A.P.C. Road, Kolkata – 700 009, India*

(*Received 12 November 2014, Accepted 29 June 2015*)

e mail : ajit_math@rediffmail.com

सार – इस शोध पत्र में वास्तविक भू मॉडल के स्तरीय इलास्टिक अर्द्ध अंतराल में सिंथेटिक या सैद्धांतिक भूकंपलेख का आकलन करने के लिए eigen इंजन फंक्शन एक्सपेंशन पद्धति का उपयोग किया गया। सरल विस्थापन स्रोत मॉडल पर विचार किया गया। विस्थापन क्षेत्र के अनुप्रस्थ (एस एच) अथवा रेडियल और उद्र्धवाधर (पी एस वी) अवयवों की समक्ष पद्धतियों के रूप में आकलन किया गया और सही तथा अंकीय तकनीकों दोनो का उपयोग करते हुए तुलना की गई। इस अध्ययन में प्रयुक्त की गई पद्धतियों में ऑर्डर 4 के रूंगे-कुटटा पद्धति का प्रयोग करते हुए अंकीय आकलनों के साथ-साथ प्रतिबिंब ट्रांसमिशन सहगुणांक का प्रयोग कर प्रोपेगेटर मैट्रिक्स अप्रोच द्वारा किया गया सटीक मूल्यांकन शामिल है। इस अध्ययन की विशेषता समरूपी और/अथवा असमरूपी स्तरों के साथ भू मॉडलों के लिए अधिकतम विस्थापन का आकलन करना है। अधिप्रवाह वाली त्रुटियों पर नियंत्रण करने के लिए इस अध्ययन में सामान्यीकृत तकनीक का प्रयोग किया गया है। इस अध्ययन में प्रतिकूल पुनरावृत तकनीक द्वारा भू-संरचना अथवा स्रोत मॉडल के बारे में जानकारी से लाभ मिलता है।

ABSTRACT. The method of eigen function expansion has been used in the present study to compute synthetic or theoretical seismogram in layered elastic half-space of real earth model. Simple dislocation source model has been considered. The transverse (SH) or radial and vertical (P-SV) components of displacement field have been computed as summed modes and compared by using both exact and numerical techniques. The methods used in the study, include exact evaluation by propagator matrix approach using Reflection-Transmission coefficients as well as numerical computations using Runge-Kutta method of order 4. The specialty of the present study is to evaluate approximate displacement field for the earth models with homogeneous and / or inhomogeneous layers. The normalization technique has been used in the study to control the overflow errors. The study has an advantage to get an idea of earth structure or source model by an inverse iterative technique.

Key words – Seismogram, Eigen function, Love wave, Rayleigh wave, Real earth.

1. Introduction

The structure of the earth's crust as determined from seismological studies exhibits a layered pattern. Thus earth can be modeled as a system of N parallel and vertically stratified media with the elastic parameters in each layer are either constant or some function of depth. Ingate *et al.* (1983) proposed model for calculation of theoretical or synthetic SH-seismogram in a laterally homogeneous layered medium for the buried source model, based on Harkrider (1964) and the extension of the reflectivity method of Kind (1978, 1979). The vertical inhomogeneity in the earth's parameters can also be modeled by introducing additional layers with constant elastic parameters, whose values are the average of the vertical inhomogeneity. Thus it is very hard to suggest any

general method to compute synthetic seismogram in a vertically inhomogeneous medium, like wave number integration or branch line integration as in the case of homogeneous medium, following Herrmann (1979). The wave number integration method [Apsel and Luco (1983)] body force equivalent technique [De and Roy (2012), De (2014)] and reflectivity method [Kind (1978)] are suitable for homogeneous elastic earth models. Apsel and Luco's (1983) algorithm uses reflection transmission matrix suggested by Kennett and Kerry (1979) and Kennett (1983) and avoids numerical instability that arises due to growing exponential terms. Takeuchi and Saito's (1972) numerical scheme avoids the necessity of sub-layering in case of vertically inhomogeneous medium and the use of shooting method for the eigen value problem gives a precise computational scheme. The classical propagator

matrix method [Haskell (1953)] was improved by Dunkin (1965) through compound matrix to avoid round-off error and growing exponential terms. Zhang *et al.* (2003) proposed the General Reflection and Transmission Method (GRTM) to compute synthetic seismogram. Das and Mitra (1998) computed the Love wave dispersion in vertically inhomogeneous media. Touhei (2003) presented the formulation and numerical examples of the analysis of scattering waves in an elastic layered half space based on the complete eigenfunction expansion form of Green's function [Touhei (2002)]. The spectral representation of the Love wave operator was studied by Kazi (1978). He obtained the exact form of the eigen values both for a finite layer and for the lowest layer extending to infinity. Harkrider (1970) presented the phase and amplitude spectra of Rayleigh and Love waves for continental and oceanic models with tabulated values of spectra of first three Rayleigh modes and first four Love modes for point sources at selected depths. Yang *et al.* (2010) presented a new derivation of the explicit expression of displacement in response to a point dislocation source in terms of the summation of the Earth's normal modes.

However in the present study, we have computed the eigen function and eigen solution in case of earth models with homogeneous layers or / and with inhomogeneous layer(s) The approximate displacement field associated with an assumed source model has been presented in the study at large distance as sum of normal modes for positive real eigen values. The computational techniques of the displacement-stress vector in the present study include, exact evaluation through propagator matrix approach with Reflection-Transmission coefficients [Haskell (1953); Harkrider (1964); Chen (1993)] as well as numerical computations [Takeuchi and Saito (1972)] using Runge-Kutta method of order 4. The advantage of the method of complete eigenfunction expansion over the Green's function approach, for the analysis, is that the formulation itself becomes independent of the number of layers and the scattering waves can be decomposed into the modes for the spectra in the layered medium. The specialty of the present study is to evaluate approximate displacement field for the layered homogeneous-inhomogeneous layered earth models as sum of eigen functions with either of the above mentioned exact and numerical approaches. Asymptotic expansion of Bessel function has been used to compute synthetic seismograms. The simple software programs have been used to compute the results are shown graphically. It is observed that the energy integrals associated with the surface waves - Love and Rayleigh, are useful in computation of synthetic seismogram at large distance from the source [Florsch *et al.* (1991)]. The present study can be extended to estimate ground motion from an earthquake.

2. Displacement field in terms of the eigen function expansion

In a multi-layered half space the eigen function equation satisfied by the Fourier-Bessel transformed displacement field [$U(\omega,k)$, $W(\omega,k)$, $V(\omega,k)$] n the cylindrical co-ordinate system (r. ɸ, z) is singular in nature, where the Fourier-Bessel transform of a function $g(r)$ is defined as :

$$G(s) = \int_0^\infty g(r) J_0(sr) r\, dr \qquad (1)$$

and the corresponding inverse transform as

$$g(r) = \int_0^\infty G(s) J_0(sr) s\, ds \qquad (2)$$

From the theory of the partial differential equation, it follows that the spectrum of surface wave dispersion equation consists of a finite number of real discrete eigen-spectrums $U_n(\omega,k,z)$ and $V_n(\omega,k,z)$ & continuous eigen spectrum, also called improper eigen spectrum, $\chi(v,z)$ due to branch cut for Rayleigh wave (Eqn. 4) and for Love wave discrete and continuous eigen spectrum are respectively $W_n(\omega,k,z)$ and $\psi(v,z)$. v represents the integration variable in continuous eigen displacement instead of ω^2. Orthogonal property holds between the discrete eigen displacement $U_n(\omega,k,z)$ and the continuous eigen displacement $\chi(v,z)$ and among themselves [Eqns. (1.45) & (1.46) of Andrianova *et al.* (1967)].

Thus,

$$< U_n(k,\omega,z), \chi(v,z) \ge \int_0^\infty \rho(z) U_n^t(z)$$
$$\chi(v,z)\, dz = 0$$
$$< \chi(v,z), \chi(v',z) \ge \int_0^\infty \rho(z) \chi^t(v,z) \chi^*(v',z)\, dz$$
$$= \delta(v - v')$$
$$(3)$$

where, χ^* is the complex conjugate of χ and the symbol $<\,,\,>$ denotes scalar product.

Similar type of relations between discrete and continuous eigen spectrum, $W_n(\omega,k,z)$ and $\psi(v,z)$ for Love wave can be deduced.

The discrete and continuous orthogonal eigen spectrum form a complete system while individually each one is not a complete set. The completeness property implies that the transformed radial and vertical displacement field for the Rayleigh wave [*i.e.*, $U(\omega,k,z)$ and $V(\omega,k,z)$] and also for the Love wave [*i.e.*, $W(\omega,k,z)$] can be expressed in terms of the complete set of eigen spectrum. Thus considering only the Rayleigh wave displacement,

we have

$$U(\omega,k,z) = \sum_n c_n U_n(z) + \int_\Gamma c(\nu)\chi(\nu,z)d\nu$$

(4)

where, $U_n(z)$ has been written instead of $U_n(\omega,k,z)$ and ν represents the integration variable in continuous eigen displacement instead of ω^2.

Now operating both sides of the above equation by the operator $\mathrm{L_R}$ [(Roy (2013)], defined as

$$\mathrm{L_R} \equiv \frac{d}{dz}\left(A\frac{d}{dz} + kB\right) - kB^t\frac{d}{dz} - k^2 C$$

(5)

where, B^t denotes the transpose of the matrix B and

$$A=\begin{pmatrix} \mu(z) & 0 \\ 0 & \lambda(z)+2\mu(z) \end{pmatrix}, B=\begin{pmatrix} 0 & -\mu \\ \lambda & 0 \end{pmatrix},$$

$$C=\begin{pmatrix} \lambda(z)+2\mu(z) & 0 \\ 0 & \mu(z) \end{pmatrix}$$

(6)

Now, using the results

$$LU(z) = -\rho\omega^2 U(z) + \rho F(\omega,k,z)$$

$$LU_n(z) = -\rho\omega^2 U_n(z)$$

$$L\chi(\nu,z) = -\rho\nu\chi(\nu,z)$$

(7)

the following relations are obtained

$$c_n(\omega^2 - \omega_n^2) < U_n(z), U_n(z) > = < F(\omega,k,z), U_n(z) >$$

$$c(\nu)(\omega^2 - \nu) = < F(\omega,k,z), \chi(\nu,z) >$$

(8)

where, $F(\omega,k,z)$ is the Fourier-Bessel transformed force field.

Hence Eqn. (4) becomes

$$U(\omega,k,z) = \sum_n \frac{< F(\omega,k,z),U_n(z) > U_n(z)}{(\omega^2-\omega_n^2)<U_n(z),U_n(z)>}$$
$$+ \int_\Gamma \frac{< F(\omega,k,z),\chi(\nu,z)>}{(\omega^2-\nu)}\chi(\nu,z)d\nu$$

(9)

Similarly for $V(\omega,k,z)$. The corresponding transformed displacement field associated with the Love wave which is horizontal and having only the cross-radial component, can be expressed as

$$W(\omega,k,z) = \sum_n \frac{< F(\omega,k,z),W_n(z) > W_n(z)}{(\omega^2-\omega_n^2)<W_n(z),W_n(z)>}$$
$$+ \int_\Gamma \frac{< F(\omega,k,z),\psi(\nu,z)>}{(\omega^2-\nu)}\psi(\nu,z)d\nu$$

(10)

The time domain displacement field can be obtained on taking inverse Fourier-Bessel transform of the above equations.

The existence of continuous and discrete spectrum is supported by the basic property of the characteristic equation in the theory of partial differential equation. But the disturbance corresponding to a continuous spectrum decreases in strength at infinity as $r^{-3/2}$ [Andrianova *et al.* (1967)]. Now omitting the continuous eigen spectrum part and evaluating the residue term corresponding to the poles at the wave number $k = k_n$ [Eqn. (11.56) of Roy (2013)] in the first quadrant of the complex k-plane satisfying the radiation condition, the displacement field associated with the Love wave can be expressed Roy (2013); Aki and Richards (2002) at large distance for a point source as :

$$u^{love} = e^{-i\omega t}\sum_n \frac{i(f_y\cos\phi - f_x\sin\phi)W_n(\omega,k_n,h)}{8cU_L I_L^{(1)}}$$
$$\sqrt{\frac{2}{\pi k_n r}}\left[W_n(\omega,k_n,z)\hat\phi\right]\exp\left[i\left(k_n r + \pi/4\right)\right]$$

(11)

where, the source $(f_x, f_y, f_z)\exp(-i\omega t)$ is situated at a depth h below the surface, $\hat\phi$ is the unit vector in ϕ direction and $W_n(\omega,k_n,z)$ is the eigen function corresponding to the nth mode.

The parameters used in the last equation are defined as :

c = Phase velocity = ω / k

U_L = Group velocity = $\dfrac{\partial \omega}{\partial k} = \dfrac{I_L^{(2)}}{c I_L^{(1)}}$

$I_L^{(1)} = \dfrac{1}{2} \int\limits_0^\infty \rho(z)[W(\omega, k_n, z)]^2 dz$ and

$I_L^{(2)} = \dfrac{1}{2} \int\limits_0^\infty \mu(z)[W(\omega, k_n, z)]^2 dz$ (12)

where, $\rho(z)$, $\lambda(z)$ and $\mu(z)$ are respectively the depth dependent density and elastic parameters of the medium.

The similar expression for the Rayleigh wave can be expressed as

$$u^{Rayleigh} = e^{-i\omega t} \sum_n i \frac{f_z V_n(\omega, k_n, h) + (f_x \cos\phi + f_y \sin\phi) U_n(\omega, k_n, h)}{8 c U_R I_R^{(1)}}$$

$$x \sqrt{\frac{2}{\pi k_n r}} [U_n(\omega, k_n, z) \exp(-i\pi/4) \hat{r}$$

$$+ V_n(\omega, k_n, z) \exp(i\pi/4) \hat{z}] \exp(ik_n r)$$
(13)

where,

c = Phase velocity = ω / k

U_R = Group velocity = $\dfrac{\partial \omega}{\partial k} = \dfrac{I_R^{(2)} + I_R^{(3)}/2k}{c I_R^{(1)}}$

$I_R^{(1)} = \dfrac{1}{2} \int\limits_0^\infty \rho(z)[U^2(\omega, k_n, z) + V^2(\omega, k_n, z)] dz$

$I_R^{(2)} = \dfrac{1}{2} \int\limits_0^\infty \left\{ [\lambda(z) + 2\mu(z)] U^2(\omega, k_n, z) + \right.$

$\left. \mu(z) V^2(\omega, k_n, z) \right\} dz$

$I_R^{(3)} = \int\limits_0^\infty \left[\lambda(z) U(\omega, k_n, z) \dfrac{dV(\omega, k_n, z)}{dz} \right.$

$\left. - \mu(z) V(\omega, k_n, z) \dfrac{dU(\omega, k_n, z)}{dz} \right] dz$
(14)

The expressions as obtained in equations (11) & (13) are similar to the Eqns. (7.143) and (7.144) of Aki and Richard (2002) and also Eqns. (251) and (252) of Takeuchi and Saito (1972). The results as mentioned above have been represented in brief for the purpose to develop our computational scheme of theoretical or synthetic seismogram. In the present study, displacement field has been computed as sum of normal modes and two mathematical approaches have been adopted to compute the displacement-stress vector. The first one is the exact evaluation through propagator matrix approach with Reflection-Transmission coefficients [Appendix A] and the second one is the numerical integration using Runge-Kutta method (order 4) [Appendix B]

The energy integrals $I_L^{(j)}, (j = 1, 2)$ or $I_R^{(j)}, (j = 1, 2, 3)$ as mentioned in equations (12) and (14) are very useful in the computation of Love or Rayleigh wave displacement field and can be considered as response of the medium. The lower value of the energy integral gives significant rise of amplitude of these waves on the surface [Florsch *et al*. (1991)]. The eigen displacement as appearing in the energy integrals is dependent on depth and becomes very small at deeper depth. Thus the infinite integrals can be truncated to a finite depth and usual numerical integration formula gives us the value of the energy integrals. The eigen displacements are computed by either or both of the methods as mentioned in Appendix A and B.

3.   Discussions

An efficient computational technique to evaluate the transient response of layered half-space or inhomogeneous media, in cylindrical polar coordinate system, has been presented in the study as sum of normal modes with the help of both numerical and exact approaches. The source models of the study are of simple dislocation. The eigen displacements in an inhomogeneous medium are usually computed numerically using Runge Kutta method of order 4 and in exact approach the propagator matrix method is used. But the numerical method suffers from the defect of computational overflow error at high frequencies. To overcome the problem of numerical instability at high frequencies, the summation over '*n*' in (11) and (13) is restricted to its lower finite value which guarantees the convergence of the series as the solutions (11) and (13) are mainly dominated by the fundamental and few higher modes (Figs. 2 & 9). The technique of normalization of eigen functions has been used in the study to reduce the numerical instability upto certain limit. The use of R/T (Reflection and Transmission) coefficients [Apsel and Luco (1983); Hisada (1994)] has intrinsically excluded the growth terms as seen in Haskell's matrix [Haskell (1953);
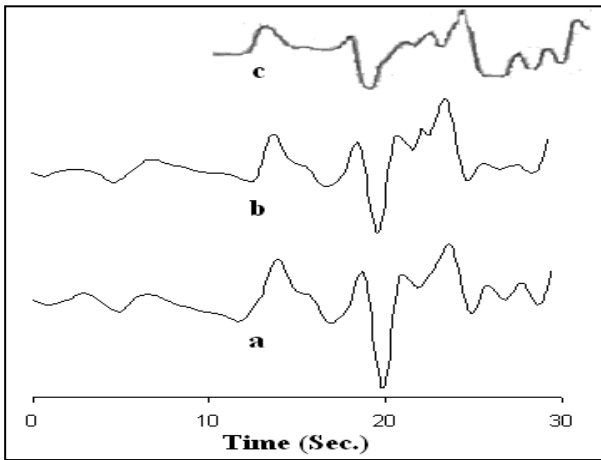
Fig. 1. Comparison of normalized transverse (SH-Love) component of displacement field at an epicentral distance 33 km by the present method with that of recorded displacement [Florsch *et al.* (1991)] due to the earth model as proposed by of Heaton and Helmberger (1978). Curve (a&b) respectively represent computations through exact method using R/T coefficients and numerical technique using Runge-Kutta method. Wave (c) represents the recorded displacement. All amplitudes are normalized to the source with scalar seismic moment. The maximum amplitude is 5.16 E-04
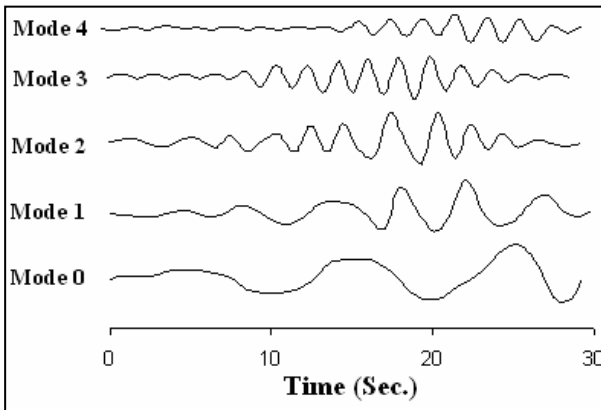
Fig. 2. The assembly of fundamental and first four modes of Love waves in the seismogram of Fig. 1 wave (a)
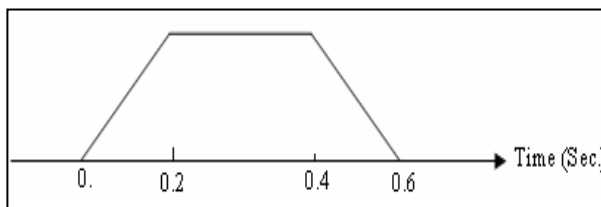

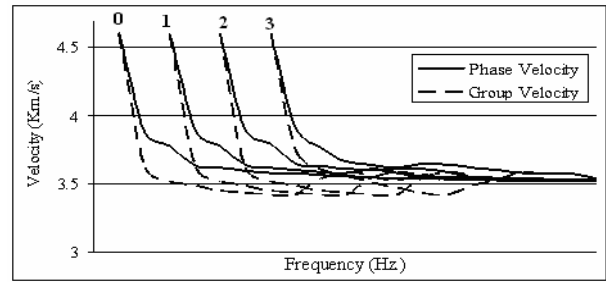**Fig. 3.** Derivative of source time dependence [Apsel and Luco (1983)]

Fig. 4. Dispersion curves (fundamental and next three higher modes) for the disturbances due to Love wave in the Bouchon's (1982) earth model with source in the half-space
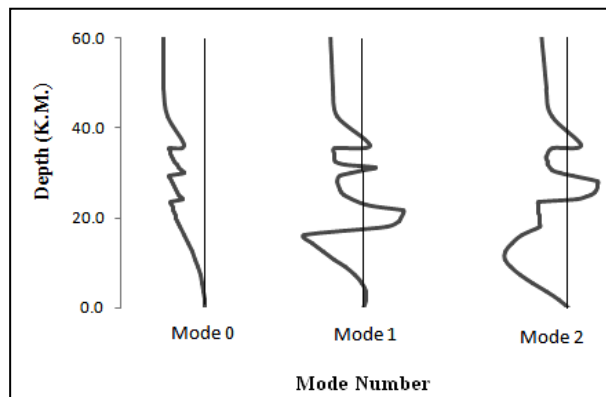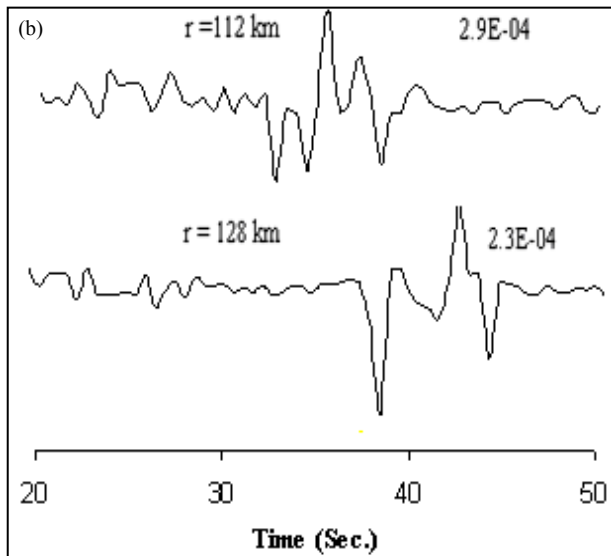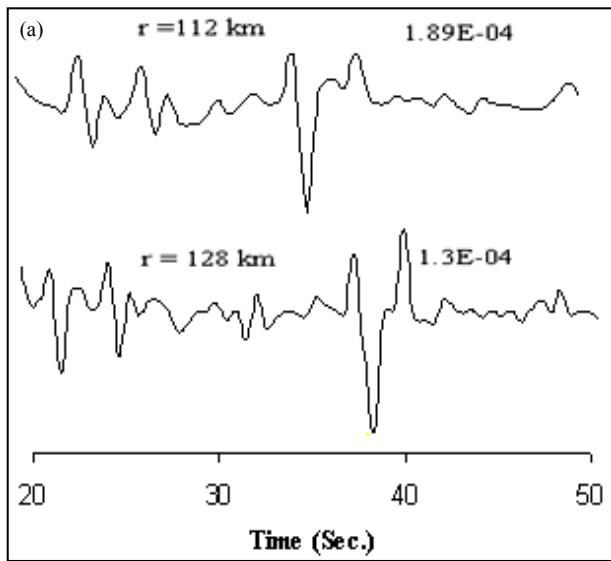

Fig. 5. Eigen-Displacement curves of the fundamental and next two higher modes at a circular frequency 1.25 Hz for Love wave in the Bouchon's (1982) earth model with source in the half-space at a depth 36 km

Watson (1970)] method. Thus, both the problems of overflow and underflow are over come in the present study. The novelty of our study is that our scheme can be used efficiently to the inhomogeneous media as well as inhomogeneous layer (s) inter-pressed between homogeneous layers by applying both the above mentioned mathematical techniques together - Matrix method for homogeneous layers and numerical solution using Runge-Kutta method (order 4) in case of inhomogeneous layers.

In the present study the source layer has been divided into two consecutive sub-layers through the source where a stress discontinuity exists due to the source (Harkrider, 1964). We apply shooting method which is very much similar with that of bisection or root bracketing method in numerical analysis to evaluate the eigen values. The roots of secular equation (*i.e.*, stress free boundary condition) are computed as wave numbers (k) for each given frequency ω. We compute the eigen functions on the surface by using layer matrix multiplication with R/T coefficients for homogeneous layers of the layered
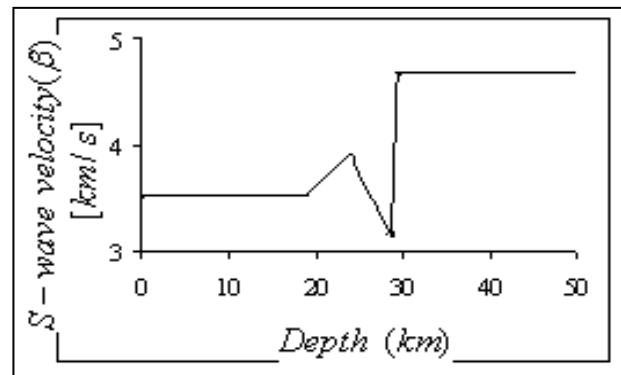
Fig. 7. Shear wave velocity (β)-depth graph for a modified three layered half-space earth model of Bouchon (1982) with inhomogeneous second and third layers and the source is in the half-space
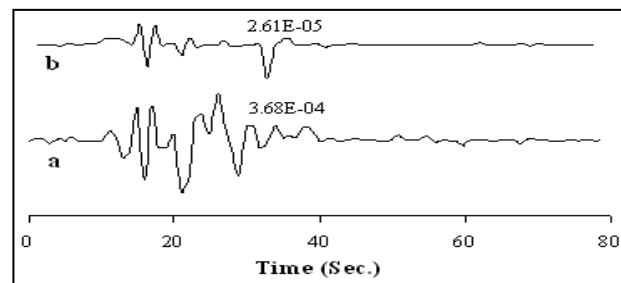

Fig. 8. Comparison of Normalized transverse displacement (SH-Love wave) at an epicentral distance 100 km due to a vertical strike slip dislocation (Fig. 3) for the (a) modified Bouchon's (1982) three layer half-space earth model (Fig. 7) and (b) Bouchon's (1982) model by modal summation

Figs. 6(a&b). Comparison of normalized Radial [Fig. 6(a)] and Vertical [Fig. 6(b)] displacement components on the surface of a one layer half-space model of Apsel and Luco (1983) due to a vertical strike-slip dislocation (Fig. 3) at an epicentral distance r and 45º from the strike. The displacement components have been computed as sum of normal modes with propagator matrix method as discussed in Section 2

half-space media at the roots of the secular equation or solving a system of 2 (for SH-Love wave) or 4 (for PSV-Rayleigh wave) differential equations by Runge Kutta method of order 4 in case of inhomogeneous layers. The above mentioned scheme begins with stress discontinuity at the source layer and propagates towards the free surface. The displacement and stress field are continuous at the interface of two layers in the layered half-space models of the study, except at the source.

The normalized transverse (SH-Love wave) surface displacement field due to an earth model proposed by Heaton and Helmberger (1978) to model November 4, 1976 Brawley, California earthquake at an epicentral distance 33 km from the source has been presented by different methods in Fig. 1. A strike-slip point source which is situated at a depth 6.9 km below the earth's surface has been considered on a vertical plane. The eigen displacement has been computed by exact method using R/T coefficients [Fig. 1(a)] and numerically using Runge-Kutta method of order 4 [Fig. 1(b)]. The computed displacement fields [Figs. 1(a&b)] as obtained by the above methods are in agreement with the result of Florsch *et al.* (1991) and the recorded displacement [Fig. 1(c)] at the station, 33 km from the source. Fig. 2 represents the assembly of fundamental and first four modes of the seismogram in Fig. 1(a) and agrees with the result of Aki (1982, Fig. 4).
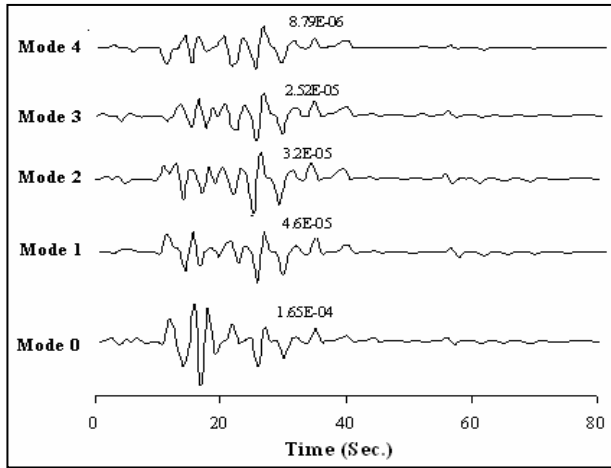
Fig. 9.   The assembly of fundamental and first four modes of Love waves in the seismogram of Fig. 8 wave (a).

Fig. 10.   Comparison of normalized Vertical displacement component in two models of earth at an epicentral distance 100 km due to a buried vertical strike slip dislocation (Fig. 3). Wave (a) represents computation in the modified Bouchon's earth model with second inhomogeneous layer has been placed between two homogeneous layers, while wave (b) represents computation in the Bouchon's earth model [Bouchon (1982)]

Layered - half space model of earth is the first step towards modeling of inhomogeneous earth. The inhomogenity can also be approximated, in a better way, by further layering of the layered media. But it increases the possibility of overflow error, even at low frequencies. The method involving R/T coefficients has its drawback in the application to purely inhomogeneous medium or inhomogeneous layer inter-pressed between homogeneous layers. To overcome this difficulty, the method involving R/T coefficients has been replaced in the present study by Runge-Kutta method (of order 4) for solution of simultaneous first order differential equation with stress free boundary condition. Thus we find numerical integral is better than propagator matrix method involving R/T coefficients. A comparative study of both exact and numerical approaches has been represented in Fig. 1 for the earth model as proposed by Heaton and Helmberger (1978). The normalized transverse displacement (SH-Love wave) field has been represented at an epicentral distance 100 km. due to a vertical strike - slip dislocation (Fig. 3) at a depth 36 km below the surface in Fig. 8 for the two earth models- one is homogeneous layered Bouchon's earth model [Bouchon (1982)] and another is the modified Bouchon's model with second and third inhomogeneous layers have been placed between homogeneous layers in a three layered half-space modified model (Fig. 7) of Bouchon (1982). It has been observed (Fig. 9) that fundamental mode and first few higher modes dominate the displacement field. Figs. 8 and 10 show the effect of introduction of inhomogeneity in the layered homogeneous earth models and serves to be a better representation of real earth over the layered models. Fig. 4 represents the dispersion curves of the phase and group velocities at an epicentral distance 100 km. Both the curves approach towards the minimum value of the
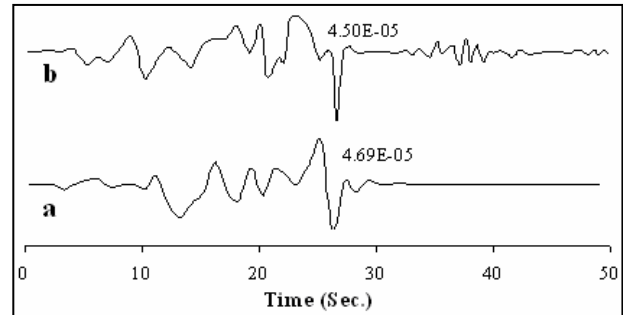
transverse wave velocity of the media at high frequencies. Fig. 5 shows the eigen displacement-depth graph of the fundamental and next two higher modes at an angular frequency 1.25 Hz. for the Bouchon's three layered half-space earth model [Bouchon (1982)]. The result shows that the effect dies away as depth increases. A comparative study of the normalized radial and vertical component of surface displacement field (PSV-Rayleigh wave) at two receiving stations [Figs. 6(a&b)] has been presented by the present method using R/T coefficients for the one layer half-space earth model, proposed by Apsel and Luco (1983). The results obtained by the present method are in close agreement with that of computed by Apsel and Luco (1983). It has been observed that the maximum value of the wave amplitude decreases with distance from the epicenter. The basic difference of our present scheme with that of Apsel and Luco's (1983) is that they evaluated the surface response by the method of quadrature using a quartic polynomial, while our present scheme computes the displacement field as sum of normal modes.

A comparative study of the vertical component of displacement field has been placed in Fig. 10 at an arbitrarily chosen representation point, at a distance 100 km from the epicenter. It has been observed that amplitude in Fig. 10(a) for the modified Bouchon's earth model with two homogeneous and an inhomogeneous layers, decreases rapidly after 28 second in comparison with that of given in Fig. 10(b) for the Bouchon's earth model and it has been observed that inhomogeneous layer with decreasing values of shear and compressional wave velocity, very close to zero, reduces the amplitude of the wave field for the vertical component. Similar type of comparison also holds in case of radial component of displacement field.

A systematic and efficient computational scheme has been developed for computation of synthetic seismogram associated with realistic seismic source model of earthquake and we feel that our technique plays an important tool in the study of wave propagation in a vertically inhomogeneous medium. The computed theoretical seismograms are in close agreement with previously computed seismograms. The advantage of the present study is that the comparison of our theoretical seismogram with the observed ones can give an idea of proper earth structure or source model by an inverse iterative technique.

References

Aki, K. and Richards, P. G., 2002, "Quantitative Seismology Second Edition", University Science Books, Sausalito, California.

Aki, K., 1982, "Strong motion prediction using mathematical modeling techniques", *Bull. Seis. Soc. Am.*, **72**, 6, S29-S41.

Andrianova, Z. S., Keilis-Borok, V. I., Levshin, A. L. and Neigauz, M. G., 1967, "Seismic Love Waves", Translated by Goodspeed F. M. C., Consultants Bureau, New York, 16-27.

Apsel, R. J. and Luco, J. E., 1983, "On the Greens function for a layered half space Part-II", *Bull. Seis. Soc. Am.*, **73**, 930-953.

Bouchon, M., 1982, "The complete synthesis of seismic crustal phases at regional distances", *J. Geophys. Res.*, **87**, 1735-1741.

Chen, X., 1993, "A systematic and efficient method of computing normal modes for multilayered half space", *Geophys. J. Int.*, **115**, 391-409.

Das, S. P. and Mitra, M., 1998, "Hybrid method for Love wave dispersion in vertically inhomogeneous media", *Pure Appl. Geophys.*, **152**, 523-538.

De, A. and Roy, A., 2012, "Transient response of an elastic half space to normal pressure acting over a circular area on an inclined plane", *J. Eng. Math.*, **74**, 119-141.

De, A., 2014, "Two dimensional self-similar expanding crack problems in elastic half-space", *Wave Motion*, **51**, 852-864.

Dunkin, J. W., 1965, "Computation of modal solution in layered, elastic media at high frequencies", *Bull. Seis. Soc. Am.*, **55**, 335-358.

Florsch, N., Fah, D., Suhadolc, P. and Panza, G. F., 1991, "Complete Synthetic Seismogram for High-Frequency Multimode SH-waves", *PAGEOPH*, **136**, 4, 529-560.

Harkrider, D. G., 1964, "Surface waves in multilayered elastic media I. Rayleigh and Love waves from buried sources in a multi-layerd elastic half-space", *Bull. Seis. Soc. Am.*, **54**, 627-679.

Harkrider, D. G., 1970, "Surface waves in multi-layered elastic media, Part II. Higher mode spectra and spectral ratios from point sources in plane layered Earth models", *Bull. Seis. Soc. Am.*, **60**, 1937-1987.

Haskell, N. A., 1953, "The Dispersion of surface waves in multi-layered media", *Bull. Seis. Soc. Am.*, **43**, 17-34.

Heaton, T. H. and Helmberger, D. V., 1978, "Predictability of strong ground motion in the Imperial Valley: Modeling the m4.9, November 4, 1976 Brawley earthquake", *Bull. Seis. Soc. Am.*, **68**, 1, 31-48.

Herrmann, R. B., 1979, "SH-wave generation by dislocation sources - A numerical study", *Bull. Seis. Soc. Am.*, **69**, 1-15.

Hisada, Y., 1994, "An efficient method for computing Green's functions for a layered half-space with sources and receivers at close depths", *Bull. Seis. Soc. Am.*, **84**, 5, 1456-1472.

Ingate, S. F., Bock, G. and Kind, R., 1983, "Synthesis of complete SH seismograms", *Geophys. J. Roy. Astron. Soc.*, **75**, 261-274.

Kazi, M. H., 1978, "The Love wave scattering matrix for a continental margin", *Geophys. J. Roy. Astron. Soc.*, **52**, 25-44.

Kennett, B. L. N., 1983, "Seismic wave propagation in stratified media", Cambridge University Press, 103-128.

Kennett, B. L. N. and Kerry, N. J., 1979, "Seismic waves in a stratified half space", *Geophys. J. Roy. Astron. Soc.*, **57**, 557-583**.**

Kind, R., 1978, "The reflectivity method for a buried source", *J. Geophys.*, **44**, 603-612.

Kind, R., 1979, "Extensions of the reflectivity method", *J. Geophys.*, **45**, 373-380.

Roy, A., 2013, "Waves in Elastic Solids", First edn, Levant Books, Kolkata, 367-391.

Takeuchi, H. and Saito, M., 1972, "Seismic surface waves, Methods in Computational Physics", Ed. Bolt, B. A., Academic Press, New York., 217-295.

Touhei, T., 2003, "Analysis of scattering waves in an elastic layered medium by means of the complete eigenfunction expansion form of the Green's function", *Int. J. Solids Struct.*, **40**, 3347-3377.

Touhei, T., 2002, "Complete eigen function expansion form of the Green's function for elastic layered half space", *Arch. Appl. Mech.*, **72**, 13-38.

Watson, T. H., 1970, "A note on fast computation of Rayleigh wave dispersion in the multilayered elastic half-space", *Bull. Seis. Soc. Am.*, **60**, 1, 161-166.

Yang, H-Y, Zhao, L. and Hung, S-H, 2010, "Synthetic seismograms by normal-mode summation a new derivation and numerical examples", *Geophys. J. Int.*, **183**, 1613-1632.

Zhang, H. M., Chen, X. F. and Chang, S., 2003, "An efficient numerical method for computing synthetic seismograms for a layered half-space with sources and receivers at close or same depths", *Pure and Applied Geophysics*, **160**, 3, 467-486, http://dx.doi.org/10.1007/PL00012546.

Appendix A

Exact computation through propagator matrix method

A vertically stratified n-layered media overlying a half-space is first considered and origin of the reference system is on the surface of the media with *z*-axis directed inside it.

The displacement vectors *u* in an inhomogeneous layer satisfy the differential equation [Roy, (2013)]

$$\nabla[(\lambda+2\mu)\nabla.u] - \nabla\times[\mu\nabla\times u] + 2[(\nabla\mu\nabla)u + \nabla\mu\times(\nabla\times u)] = \rho\frac{\partial^2 u}{\partial t^2} \qquad (A.1)$$

where, layer parameters λ, μ and density ρ are depth dependent.

A source has been considered at a depth '*h*' below the surface as a time dependent stress discontinuity $\Delta(t)$ at the source layer S as [Harkrider, (1964)]

$$\begin{pmatrix} U_p^{S+}(h) \\ D_p^{S+}(h) \end{pmatrix} = \begin{pmatrix} U_p^{S-}(h) \\ D_p^{S-}(h) \end{pmatrix} + \begin{pmatrix} 0 \\ \Delta(t) \end{pmatrix}, \quad (p = PSV \text{ or } SH) \qquad (A.2)$$

where, S+ and S- are respectively the sub-layers below and above the source. The displacement-stress vector is continuous at the other layer boundaries.

The dynamic displacement-stress vectors $[U_p^j(\omega,k,z;h), D_p^j(\omega,k,z;h)]$, $p = PSV$ or SH in the jth homogeneous layer of a layered-half space media can be expressed in terms of down and up going *P* and *S* waves by using modified R/T coefficients as [Apsel and Luco (1983); Chen (1993) and Hisada (1994)]

$$\begin{pmatrix} U_p^j(\omega,k,z;h) \\ D_p^j(\omega,k,z;h) \end{pmatrix} = \begin{pmatrix} E_{11}^j & E_{12}^j \\ E_{21}^j & E_{22}^j \end{pmatrix} \begin{pmatrix} \Lambda_d^j(z) & 0 \\ 0 & \Lambda_u^j(z) \end{pmatrix} \begin{pmatrix} C_d^j(h) \\ C_u^j(h) \end{pmatrix} \qquad (A.3)$$

where, $C_d^j(h)$ and $C_u^j(h)$ are respectively the down & up going coefficients and E^j as layer matrix in the jth layer.

Appendix B

Numerical computation by Runge Kutta method of order 4

The displacement-stress vectors $[U_p^j(\omega,k,z), D_p^j(\omega,k,z)]$, $p = PSV$ or SH at the *j*th layer in a N-layered half space media, satisfies the differential equation [Takeuchi and Saito (1972); Aki & Richards (2002)].

$$\frac{d}{dz}\begin{pmatrix} U_p^j \\ D_p^j \end{pmatrix} = A\begin{pmatrix} U_p^j \\ D_p^j \end{pmatrix} \qquad (B.1)$$

where, A is either a 4 × 4 matrix (PSV case) or a 2 × 2 matrix (SH case).

The displacement-stress vectors $[U_p^j(\omega,k,z), D_p^j(\omega,k,z)]$, $p = PSV$ or SH in the jth layer satisfies the continuity condition at the jth interface and the stress vanishes on the free. Thus, $[U_p^j(\omega,k,z_j),$ $D_p^j(\omega,k,z_j)]^{\text{t}} = [U_p^{j+1}(\omega,k,z_j), D_p^{j+1}(\omega,k,z_j)]^{\text{t}}$, except at the source and $D_p^1(\omega,k,0) = 0$. The seismic wave field also

satisfies the radiation condition at infinity. The vanishing of stress on the free surface is used to evaluate a set of finite number of wave numbers [k_n , $n = 0, 1, 2,…,$ L(ω)] for a given frequency ω by shooting method which is similar to that of bisection method and the system (B.1) is numerically evaluated by Runge Kutta method of order 4.

———————