

Strange attractor in earthquake swarms near Valsad (Gujarat), India

H. N. SRIVASTAVA, S. N. BHATTACHARYA*, D. T. RAO** and S. SRIVASTAVA**

128, Pocket - A, Sarita Vihar, New Delhi, India

*Indian Institute of Technology, Kharagpur, India

**Gujarat Engineering Research Institute, Vadodara, Gujarat, India

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e mail : hn_srivastava@hotmail.com

सार — वर्ष 1986 के फरवरी माह के आरम्भ से ही प्रायद्वीपीय भारत के पश्चिमी तट के निकट दक्षिणी गुजरात के वलसाड जिले में भूकम्पों की अधिक मात्रा देखी गई है। सूक्ष्म भूकम्प लेखी के संजाल द्वारा किए गए भूकम्पीय मानीटरन से 1 से 15 कि. मी. तक की फोकी की गहराई सहित 7×10 वर्ग कि. मी. के क्षेत्र में एक सु-संकेंद्रित भूकम्पीय गतिविधि का पता चला है। मार्च 1986 से जून 1988 के दौरान आए कुल 21,830 भूकम्प अभिलेखित किए गए। असाधारण अट्रैक्टर और लयापूनोव घातांक के आयाम के आकलन के माध्यम से निर्धारणात्मक अव्यवस्था की जाँच करने के लिए इस अवधि के भूकम्पों की दैनिक आवृत्ति का उपयोग किया गया है। असाधारण अट्रैक्टर की 2.1 के न्यून आयाम और बृहत लयापूनोव घातांक के घनात्मक मान से भूकम्प की प्रागुक्ति करने के लिए कम से कम 3 प्राचलों सहित वलसाड में आए भूकम्पों की अधिक मात्रा से अव्यवस्थित गतिकों का पता चला है। इन परिणामों से भारत के अंतः पट्टिका और अंतः पट्टिका में निर्धारणात्मक अव्यवस्था के अभिलक्षणों में पाई गई भिन्नता की पुष्टि की गई है।

ABSTRACT. Valsad district in south Gujarat near the western coast of the peninsular India experienced earthquake swarms since early February 1986. Seismic monitoring through a network of micro earthquake seismographs showed a well concentrated seismic activity over an area of $7 \times 10 \text{ km}^2$ with the depth of foci extending from 1 to 15 km. A total number of 21,830 earthquakes were recorded during March 1986 to June 1988. The daily frequency of earthquakes for this period was utilized to examine deterministic chaos through evaluation of dimension of strange attractor and Lyapunov exponent. The low dimension of 2.1 for the strange attractor and positive value of the largest Lyapunov exponent suggest chaotic dynamics in Valsad earthquake swarms with at least 3 parameters for earthquake predictability. The results indicate differences in the characteristics of deterministic chaos in intraplate and interplate regions of India.

Key words – Chaos, Earthquake swarm, Strange attractor, Earthquake predictability.

1. Introduction

The term swarm is generally used to describe a group of related earthquakes concentrated in space and time without an obvious principal event. Such swarms commonly occur within the Peninsular India which dissipate after a few days or months. However, exceptions have been noted in a few cases when sudden increase in seismic activity is followed by a larger event in this region; we classify them as precursory swarms. Srivastava and Dube (1996) compared the characteristics of precursory and non-precursory types of swarms and found that the non-precursory swarms have focal depth down to only 2 km or so; but precursory swarms are associated with larger focal depths. The earthquake swarm of 1986 over the Valsad region is concentrated within an area of $7 \times 10 \text{ km}^2$ with focal depths ranging from 1 to 15 km² (Rao *et al.* 1991) with a larger number between 8 and 12 km. Better physical understanding of

such swarms requires dynamical approach which has not been attempted so far.

The objective of this paper is to examine the existence of deterministic chaos in Valsad earthquake swarms using two approaches namely through the fractal dimension of strange attractor (Grassberger and Procaccia 1983) and the Lyapunov exponent (Wolf *et al.*, 1985). The results have also been compared with those reported for Koyna region (Srivastava *et al.*, 1994) and other regions to understand the difference between interplate and intraplate earthquakes.

2. Geotectonic set up

(i) *Geology* - The south Gujarat area around Valsad is occupied by flows and dykes of basaltic rocks. It forms a part of north western margin of the large Deccan trap formation of Cretaceous Eocene age. The Deccan traps

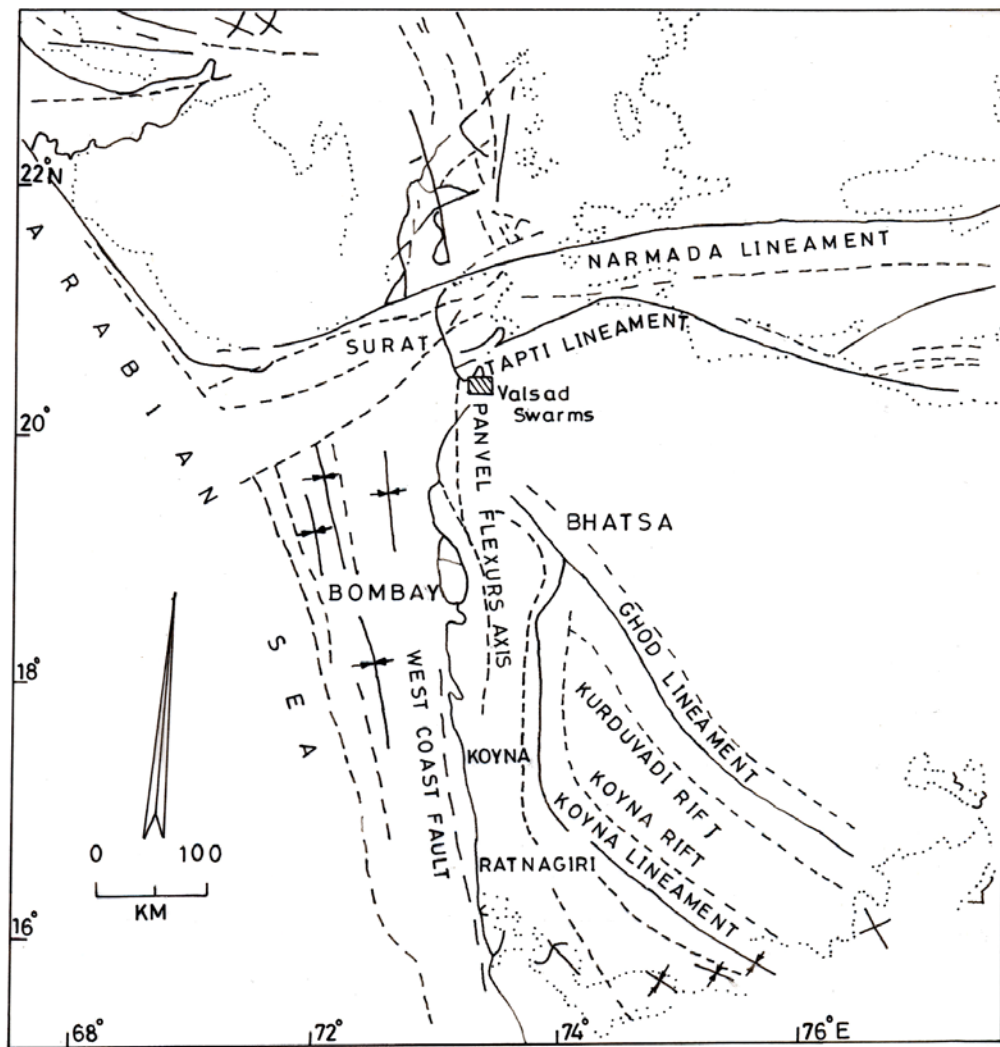


Fig. 1. Location of Valsad swarms and major tectonic features on the Peninsular India. The Koyna region is also shown

are overlain by older alluvium of late Pleistocene age and subsequent Holocene formation like newer alluvium blown sand and beach deposits.

(ii) *Tectonics* - Fig. 1 shows the location of Valsad swarms in relation to main tectonic features. The Deccan traps dip into the Arabian Sea at an angle of 7° to 10° as a monocline whose axis turns through Panvel and Kalyan to south of Surat. This monoclinical feature is termed the Panvel flexure. Along its axis, which is fractured, there are several hot springs namely Unai, Mola-Amla, Arnai and others. Another major fault is in the direction ENE-SSW along the Narmada river. The Tapti river runs parallel and close to this fault. Other major tectonic features trend along N-S and NW-SE.

3. Seismic activity

The earthquake activity commenced in the first week of February 1986 when more than 50 shocks with magnitude 2.5 and above were felt. The largest earthquake of magnitude 4.6 occurred on 26 April 1986. Prior to these, two earthquakes both of magnitude 3.6 occurred on 16 and 17 February 1986. The locations of these three earthquakes were obtained through records of seismological observatories of India Meteorological Department and other stations operating around Kadana, Ukai and Kevadia projects. An observatory at Anklachh was established in the region from March 1986 which recorded the largest number of events in the networks of stations established by May 1986 (Srivastava 1991). The

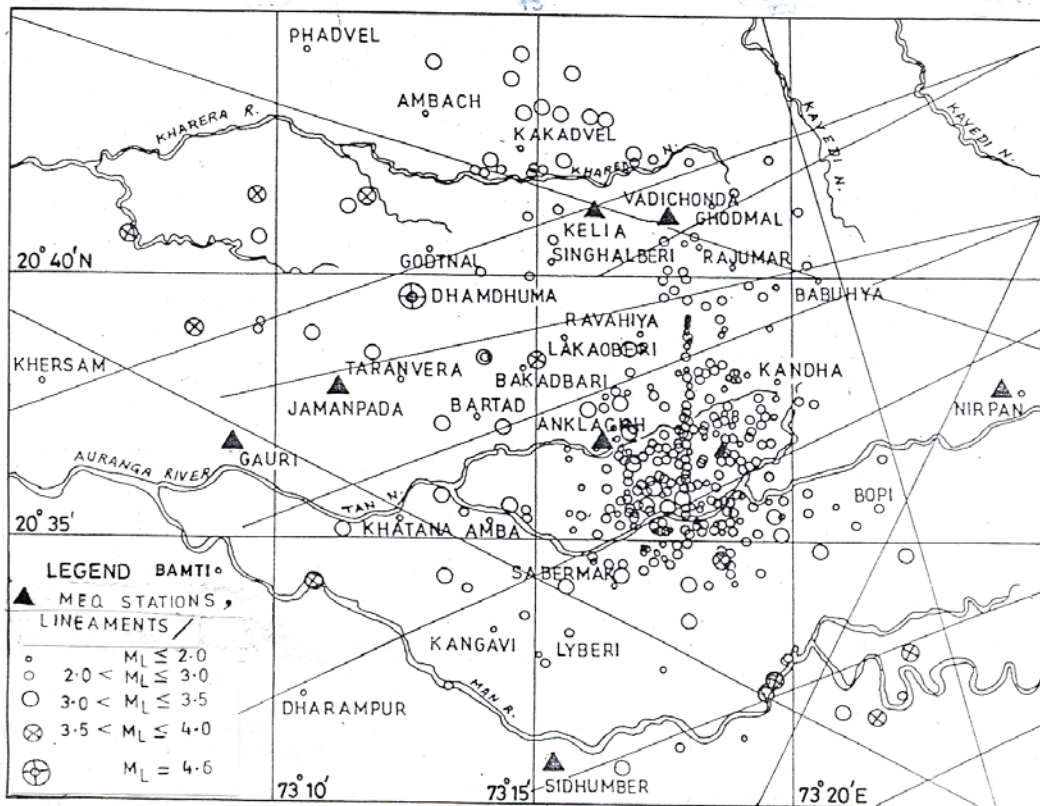


Fig. 2. Epicentral distribution of earthquake swarms

area remained active till end of 1987 and the activity started declining thereafter to 1 to 6 events per day in June 1988. From March 1986 to June 1988, a total number of 21,830 micro earthquakes were recorded at Anklachh. Fig. 2 shows the epicentral distribution of Valsad swarms in Gujarat. The epicenters of the earthquakes concentrated in an area of $7 \times 10 \text{ km}^2$ with focal depths ranging from 8 to 12 km for most of these earthquakes.

Detailed examination of the seismic activity has shown that the sequence broadly fits type III of Mogi's model which is characteristic of highly heterogeneous region with concentrated applied stress. Rao *et al.* (1991) have found that the epicenters of the microearthquakes were concentrated along a N-S axis which changed to NNE-SSW towards December 1986. The value of 'b' in Gutenberg Richter frequency magnitude relationship was found to vary between 0.78 and 1.03 during the period of observation.

4. Chaotic dynamics

A dynamical system whose equations and initial conditions are fully specified is called 'deterministic.'

Solutions of deterministic equations become chaotic if adjacent solutions diverge exponentially in phase space. The evolution of dynamical systems can be represented by trajectories in the state space from some initial condition. For periodic systems that develop deterministically, all trajectories initiated from different initial conditions stay on low dimensional smooth topological manifolds, called attractor. These attractors are characterised by an integer dimension, equal to the topological dimension of the submanifold. An important property of these attractors is that trajectories converging on them do not diverge implying long term predictability of the system. It has been found for many dynamical systems, that the trajectories stay on an attracting submanifold which is not topological. These submanifolds are called 'fractal' sets and are characterized by a dimension which is not an integer. The corresponding attractors are called 'strange' attractors. An important property constraints could be developed for attractors of low dimension in spite of the unpredictability of chaotic system in the long run. The determination of the dimension of an attractor requires a number of constraints that should be satisfied by a model used to predict the evolution of a system. The higher the value of the fractal dimension, the more complex is the

system. The fractal dimension also gives the minimum and the maximum number of independent parameters required for modeling the system. Being fractal in nature, more details are revealed as they are increasingly magnified.

Instead of taking recourse to the mathematical formulation of a nonlinear system through differential equations, an alternate method is adopted in practice by replacing the state space by the so called phase space which is a co-ordinate space defined by the state variable of a dynamical system. The phase space may be produced using a single record of observable variable $x(t)$ from the system. The physics behind such an approach is that a single record from a dynamical system is the outcome of all interacting variables and thus information about the dynamics of that system should, in principle, be included in an observable variable.

It is assumed that variables present in the evolution of the system in question satisfy a set of n first-order differential equations:

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2, \dots, x_n) \\ \dot{x}_2 &= f_2(x_1, x_2, \dots, x_n) \\ \dots\dots\dots\dots\dots\dots \\ \dot{x}_n &= f_n(x_1, x_2, \dots, x_n) \end{aligned} \tag{1}$$

where the prime indicates the first derivatives with respect to time. In such a case, the co-ordinates of the state space are (x_1, x_2, \dots, x_n) . In this work, we have attempted to obtain the fractal dimension of the strange attractor (Grassberger and Procaccia, 1983) and Lyapunov exponents (Wolf *et al.*, 1985). These two methods which characterize chaos, involve the geometry of the motion in the phase space of the system.

(i) *Fractal dimension of strange attractor*

The system Eqn. (1) can be reduced to a single differential equation of one of the variables $x_j(t)$, say, $x(t)$, if all others are eliminated by differentiation. This gives n^{th} order nonlinear differential equation

$$x^{(n)} = f(x, x', \dots, x^{(n-1)}) \tag{2}$$

where $x^{(i)}$ denotes is the derivative with respect to time. Ruelle (1981) suggested that instead of a continuous variable $x(t)$ and its derivatives, it will be easier to work with $x(t)$ and the set of variables obtained from it, by shifting its values by a fixed delay parameter τ .

We may begin the commutation with a time series of dependent or independent variable $x(t)$ of the system. (Ruelle 1981). We construct points X_i in an m -dimensional space :

$$X_i = \{x(t_i), x(t_i+\tau), \dots, x[t_i + (m-1) \tau]\} \tag{3}$$

with $i = 1, 2, \dots, N - m + 1 (= k, \text{ say})$,

where N is the number of data in the time series of x . Here, t_i is the initial time and $t_i = t_1 + (i-1) \tau$. Thus one discretizes the orbit to a set of k points X_i in the state space. The distance $s_{ij} = |X_i - X_j|$ between the pair of points X_i and X_j is calculated as the Euclidian norm. We look for these points for which their distance is less than a given correlation distance r . A correlation integral is then obtained as (Grassberger and Procaccia, 1983)

$$C_m(r) = \frac{1}{K^2} \text{ [number of pairs } (i, j) \text{ with } s_{ij} < r]$$

where r is correlation length. $C_m(R)$ may be calculated effectively using the relation (Abraham *et al.* 1986 and Theiler, 1988)

$$C_m(r) = \frac{1}{K} \sum_{i=1}^k \sum_{j=i+1}^k H(r - s_{ij}) \tag{4}$$

where,

$$\begin{aligned} H(x) &= 1, \text{ for } x > 0 \\ &= 0, \text{ for } x < 0 \end{aligned}$$

and $K = k(k-1)/2$ with $k = N-m + 1$. K gives the number of distinct pair of points.

The dimension D of the attractor is related to $C_m(R)$ by the relation

$$C_m(R) = r^D,$$

where r is small; or,

$$\log C_m(R) = D \log(r) \tag{5}$$

Hence, the dimension D of the attractor is given by the slope of the $\log C_m$ versus $\log(r)$. The scaling region is obtained for various embedding dimensions. As we increase the embedding dimension m , the slope saturates at a limiting value which is considered as fractal dimension of the strange attractor. The delay time is slowly increased until the same fractal dimension is obtained for two consecutive delay times.

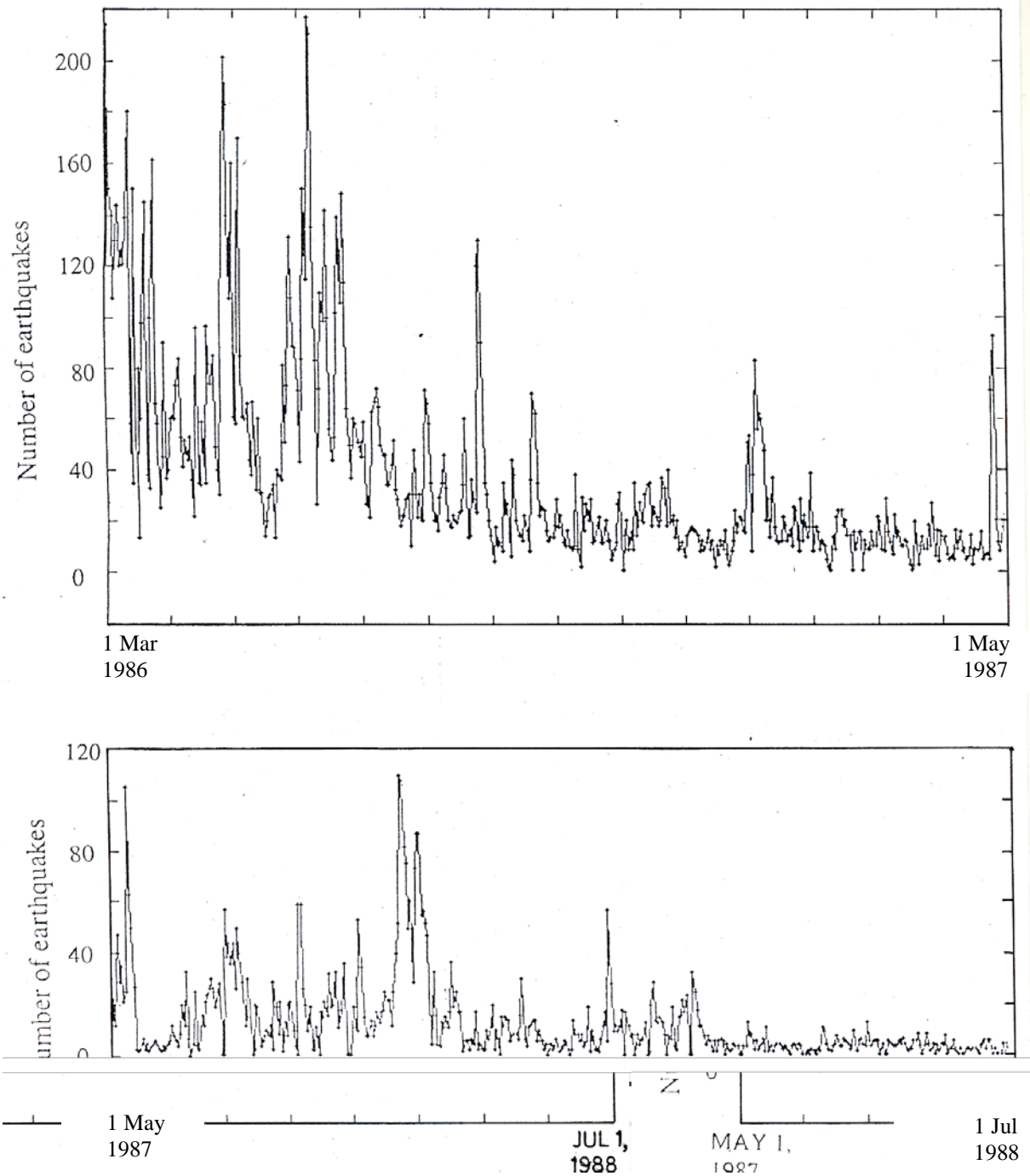
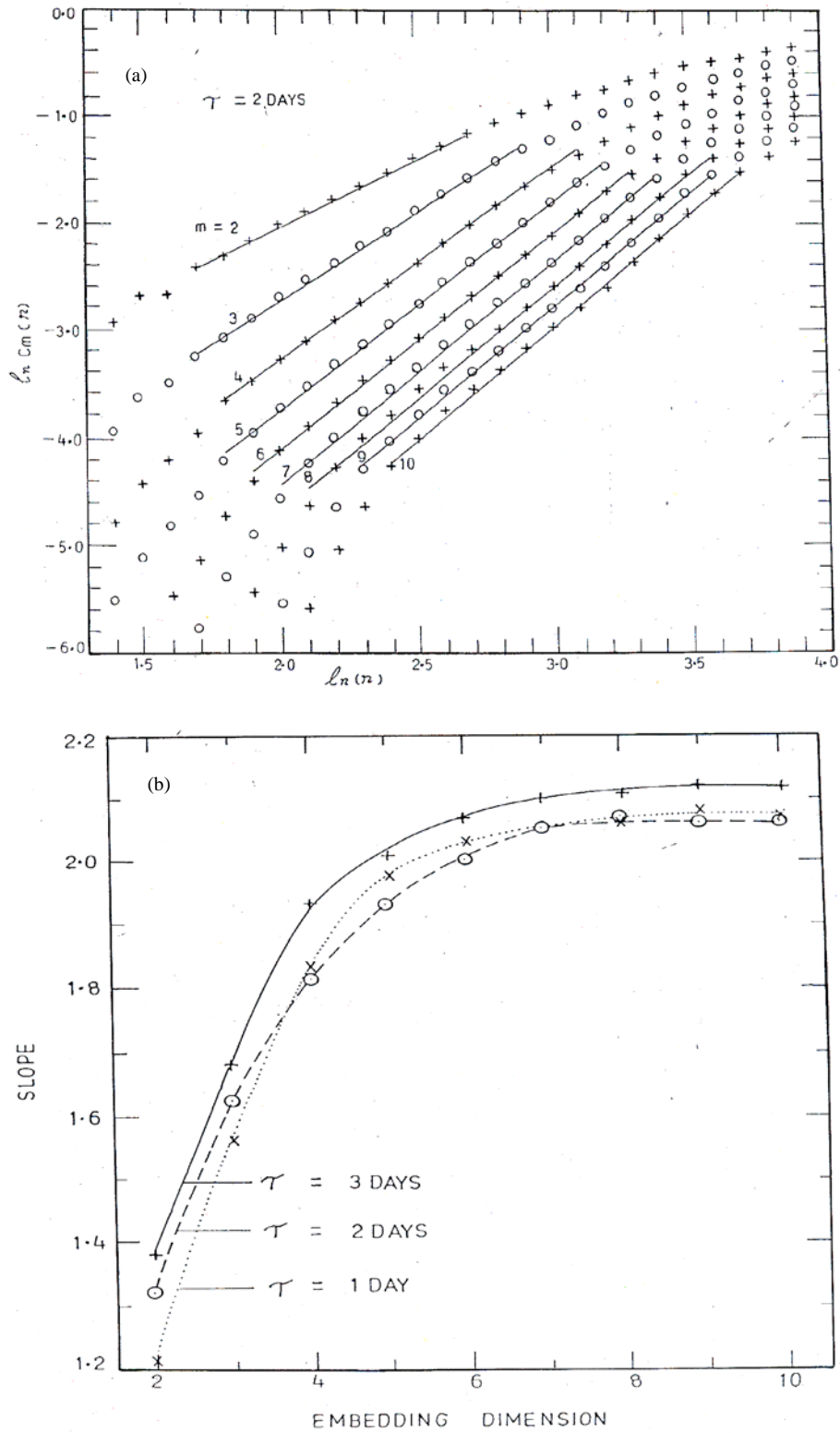


Fig. 3. Number of earthquakes per day recorded at Anklachh Seismological Observatory for the period 1 March 1986 to June 1988

It may be mentioned that care is taken to avoid spurious results being obtained by keeping the number N of earthquakes such that the criterion, $2 \log(N) > D$ is satisfied (Ruelle, 1990). It may be noted from the next

section that the data set available for this region fully to meet this criterion in view of low strange attractor dimension obtained from the time series of earthquakes in Valsad region.



Figs. 4(a&b). (a) Plot of $\log C_m(r)$ versus $\log r$ based on daily frequency of earthquakes for various embedding dimension for a delay parameter of 2 days and (b) Slope versus embedding dimension in Valsad region using daily frequency of earthquakes for time delay of 1, 2 and 3 days

(ii) *Lyapunov exponent*

The solution of equations (1) conceptually follows solutions that start within a hypersphere of radius r . As the solution evolves, the hypersphere is deformed into a hyper ellipsoid with principal axes $\varepsilon_i(t)$.

Lyapunov exponent,

$$\lambda_i = \lim(t \rightarrow \infty) \lim(r \rightarrow 0) [\varepsilon_i(t)/(r.t)] \quad (6)$$

If $\lambda_i < 0$, all solutions that start with initial conditions close to each other will converge, *i.e.*, there is no sensitivity to initial conditions. But if just one λ_i is positive, the nearby solutions will diverge, *i.e.*, there will be extreme sensitivity to the initial conditions. The growth of uncertainty in time t is given by

$$N = N_0 e^{\lambda t} \quad (7)$$

where N_0 is the initial condition and λ is related to the concept of entropy in information theory and also related to another concept, *i.e.*, the Lyapunov exponent, which measures the rate at which nearby trajectories of a system in phase space diverge. Its unit is reciprocal of time e.g. second⁻¹. The largest Lyapunov exponent λ , of the time series is found using the computer programme given by Wolf *et al.* (1985). A positive Lyapunov Exponent suggests chaotic nature of earthquake while a non-chaotic system is characterized by negative Lyapunov Exponents.

5. Data analysis

We have considered 21,830 earthquakes in the epicentral area given in Fig. 2 for the period March 1986 to June 1988 for Valsad region. The observable variable $x(t)$ has been considered as the number of earthquakes every day recorded at Anklachh observatory and this data is shown in Fig. 3. The total number $N = 851$. The delay parameter τ was chosen as 1, 2 and 3 days.

In order to obtain the strange attractor dimension of the earthquake sequence, we compute $C_m(r)$ for different values of r using the equation (4) for various embedding dimensions m . We note that $\log C_m(r)$ saturates at large values of r due to finite size of the attractor and at small values of r due to the finite number of data points. In this plot we choose a scaling region, where $\log C_m(r)$ is linear to $\log r$. For each embedding dimension the slope of the straight line passing through the points in the scaling region is obtained and these slopes against embedding dimensions are plotted in Fig. 4(b). The value of slope for delay parameters of one and two days, converge to 2.1

which gives the dimension of the strange attractor. In addition, the largest value of the Lyapunov exponent for the same data sets gave positive value of 0.13804, suggesting strong dependence upon initial conditions.

During June 1988 to December 1988, swarm activity declined significantly. To see its effect, we have extended the analysis including the earthquakes recorded till the end of 1988 and found that the largest Lyapunov exponent remained positive (0.11340) supporting again chaotic dynamics for the occurrence of earthquakes in the region.

6. Results and discussion

Figs. 4(a&b) show that the value of strange attractor dimension is 2.1 implying that atleast 3 parameters are needed for modeling earthquake system in Valsad region, of Gujarat. In the neighbouring Koyna region, Srivastava *et al.* (1994) found strange attractor dimension of 4.4. A low value of a strange attractor dimension of 3.4 was also reported in Aswan region of Egypt which is characteristic of shield region similar to Peninsular India (Srivastava *et al.* 1995). It may be mentioned that the sequence of volcanic eruptions have been found to be deterministically chaotic with a low dimension of about 2 in La Reunion and 4 in Hawaii regions (Sornete *et al.*, 1991). Such volcanic regions show a predominance of earthquake swarms. However, their strange attractor characteristics have not been reported so far.

Comparing the low value of strange attractor dimension in the Peninsular India with that in the tectonically active Himalaya-Northeast India region, we find that the lower value of strange attractor dimension in Valsad and Kyona regions may be a characteristics of intraplate seismicity noting that higher dimensions ranging from 6.1 to 8.5 have been found in Northwest Himalaya and Northeast India which lie in a complex tectonic region close to Indian-Eurasian plate boundary (Bhattacharya, 1990; Bhattacharya and Srivastava, 1992; Srivastava *et al.*, 1995, 1996). It is also interesting to note that higher strange attractor dimension of 7.2 was also reported in Nurek region in Tadjikistan (Bhattacharya *et al.*, 1995). On the other hand, a relatively lower value of 6.1 with positive Lyapunov exponent was found in California region (Srivastava and Sinha Ray, 1999). Much lower value of 3.2 near Japan can be attributed to difference in tectonics (Pavlos *et al.*, 1994).

It may be mentioned that the statistical characteristics of earthquake swarms have been studied in several regions of the world with Matsushiro swarms in Japan as an exceptional example (Drakopoulos *et al.* 1972). Further insight into the dynamics of earthquake swarms can be provided when adequate data in other tectonic regions become available.

7. Conclusions

The above study has brought to light that the earthquake swarm near Valsad, Gujarat in the Peninsular India are characterised by a strange attractor dimension of 2.1 and a positive Lyapunov exponent. The results provide further support to the difference in the chaotic dynamics between intraplate and interplate tectonics of the Indian plate region.

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