

## Orographic effect of an elliptical meso-scale mountain on barotropic air-stream

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**सार** — इस शोध-पत्र में मैसोस्केल के त्रिविमीय दीर्घवृत्ताकार पर्वत पर और उसके आसपास स्थायी अवस्था वाले रुद्धोष्म, धर्षणहीन अनावर्ती बूसिने प्रवाह के लिए गणितीय निदर्श को विकसित करने का प्रयास किया गया है। मूल प्रवाह में क्षेत्रीय और साथ ही साथ याम्योत्तरी दोनों प्रकार के घटक पाए गए हैं। मूल प्रवाह को यह मानते हुए सुगम बनाया गया है कि मूल प्रवाह वेग के साथ ही साथ बर्न्ट-वैसला आवृत्ति उँचाई के साथ परिवर्तित नहीं होती है। यहाँ जेड — कोऑर्डिनेट को निर्धारित करने वाले समीकरणों का उपयोग किया गया है जिन्हें क्षोभ तकनीक का उपयोग करते हुए एकघातिक किया गया है।

एकघाती निर्धारित करने वाले समीकरण पुनः दोहरे फूरिए रूपान्तरण को प्रभावित करते हैं। बीजगणित के सरलीकरण के उपरांत क्षोभ उर्ध्वाधर वेग ( $w'$ ) और क्षोभ धारारेखा और विस्थापन ( $\eta'$ ) के फूरिए रूपान्तरण में द्वितीय क्रम में सामान्य अवकल समीकरण प्राप्त किए गए हैं।  $w'$  और  $\eta'$  दोनों को दोहरे समाकल के रूप में व्यक्त किया गया है। इन समाकलों का मूल्यांकन विश्लेषणात्मक रूप से करना कठिन है। अतः पर्वत के निचले भाग में पवन के सुसंगत स्थान निर्धारण से इन समाकलों के उपगामी मान प्राप्त किए गए हैं। उपर्युक्त समाकलों के उपगामी विस्तार का मूल्यांकन करते हुए तरंग संख्या डोमेन में पल्याण बिंदुओं से बचने का ध्यान रखा गया है, जहाँ पर तरंग संख्या वेक्टर ( $k, l$ )  $90^\circ$  पर या उससे अधिक के कोण पर मूल प्रवाह वेक्टर ( $U, V$ ) के साथ झुका हुआ है।

उपगामी समाधान के परिणामों से यह पता चलता है कि  $Uy - Vx = 0$  रेखा पर मैदानी भागों के मध्य में  $w'$  और  $\eta'$  दोनों की रोधिका के अनुवात में कमी आती है जबकि उपगामी समाधान के मामले में  $X_1^{-1}$  के रूप में इसमें कमी आती है जहाँ  $X_1, Uy - Vx = 0$  रेखा पर मापी गई दूरी है। उपगामी समाधान से यह ज्ञात होता है कि क्षैतिज समतल पर नोडल रेखा अतिपरवलय (हाइपरबोला) है जो  $Uy - Vx = 0$  और  $Ux + Vy = 0$  रेखाओं पर अक्ष है। उपगामी समाधान से  $Uy - Vx = 0$  रेखा पर उर्ध्वगामी पवन के झुकने और लगभग इसी रेखा पर उँचाई पार्श्विक प्रसार का भी पता चलता है।

**ABSTRACT.** An attempt has been made to develop a mathematical model for a steady state adiabatic, frictionless, non-rotating and Boussinesq flow over and around a meso-scale three-dimensional elliptical mountain. Basic flow is assumed to have both zonal as well as meridional components. Basic flow is simplified by assuming that basic flow velocity, as well as the Burnt -Vaisala frequency do not change with height. Here the governing equations in  $z$ -coordinate are used, which are linearized using perturbation technique.

The linearized governing equations are again subjected to a double Fourier transformation. After some algebraic simplification the second order ordinary differential equations (ODE) in the Fourier transform of perturbation vertical velocity ( $w'$ ) and that of perturbation streamline displacement ( $\eta'$ ) are obtained. Both the  $w'$  and  $\eta'$  are expressed as double integrals. It is difficult to evaluate these integrals analytically. So the asymptotic values of those integrals, which are valid at a far down wind location of the mountain, have been obtained. While evaluating the asymptotic expansion of the above integrals, care has been taken to avoid those saddle points in the wave number domain, where the wave number vector ( $k, l$ ) is inclined with the basic flow vector ( $U, V$ ), at an angle greater than or equal to  $90^\circ$ .

Results of the asymptotic solution shows that in the central plane along the line  $Uy - Vx = 0$  both  $w'$  and  $\eta'$  falls off down wind of the barrier, more over in case of asymptotic solution they fall off as  $X_1^{-1}$ , where  $X_1$  is the distance measured along the line  $Uy - Vx = 0$ . Asymptotic solution shows that nodal lines on the horizontal plane are hyperbola the axes of

which are along the lines  $Uy - Vx = 0$  and  $Ux + Vy = 0$ . Asymptotic solution also shows upwind tilting along the line  $Uy - Vx = 0$  and lateral spreading about the same line with height.

**Key words** – Meso-scale elliptic mountain, Lee wave, Asymptotic expansion.

## 1. Introduction

The study of the perturbation in a stably stratified air stream by an obstacle may be broadly divided into two categories. In one category the obstacle is assumed to have an infinite extension in the crosswind direction, so that the flow essentially becomes two-dimensional (2-D). In the other category the obstacle is assumed to have finite extension in the crosswind direction and the flow becomes essentially three-dimensional (3-D). The study on 2-D mountain wave problem was first addressed by Lyra (1943). He considered a 2-D model with uniform air-stream of constant static stability and obtained solutions using Green's functions. He obtained lee waves, which decreased downstream and increased upward. But this upward increase of wave amplitude was contrary to the observation. Queney (1947, 1948) proposed a complete theory of adiabatic perturbations in a stratified and rotating atmosphere, and applied this theory to the flow of air-stream over a 2-D bell shaped mountain with half width 'a'. Like Lyra (1943) Queney also took uniform basic flow and constant static stability.

The studies on three-dimensional mountain wave problem were first addressed by Scorer and Wilkinson (1956). They synthesized one isolated three-dimensional hill by superposition of infinite ridges inclined at different angles but intersecting at a point, which was expressed mathematically by an integral. They computed interference pattern of the lee wave systems of the component ridges. They found lee wave pattern very similar to that produced by a ship moving across the surface deep water. In their result, lee waves were confined within a wedge-shaped region, the corner of which being vertical and through the hill top, where the half angle of the wedge was dependent on the air stream character. Wurtele (1957) represented the 3-D orographic barrier in the form of semi-infinite plateau of height 'h' with narrow width '2b' in the cross wind direction. He considered the incoming wind ( $U$ ) and buoyancy frequency ( $N$ ) to be independent of height. His theory predicted the region of updraft, which had a horseshoe shape and was located some distance downstream of the barrier. Crapper (1959) presented a 3-D small perturbation approach of waves produced in a stably stratified air stream flowing over a mountain. He obtained the fundamental solution for a doublet disturbance in an air stream in which Scorer's parameter remains constant and then it was extended to that for a disturbance caused by a circular mountain in the same air stream. He showed that

circular mountain can give rise to waves which have greater amplitude than those produced by an infinite ridge in the same air stream. Crapper (1962) considered the airflow across a 3-D barrier with elliptical contour for two types of air stream. In one case the Scorer parameter (Scorer, 1949)  $l$  was constant with height, in other case it was assumed to fall off exponentially with height. In each

of the above cases  $\frac{l}{U} \frac{d^2 U}{dz^2} = q^2$  was kept constant. The

result showed that when  $l$  is constant, then the form of the waves was determined by the value of  $q$ . They also showed that when  $l$  falls off exponentially, the waves closely resembled ship waves for any value of  $q$ . Sawyer (1962) studied gravity waves in the atmosphere as a 3-D problem. He derived an equation, for the vertical variation of the amplitude of the standing waves, when the wind varied with height and the wave was periodic in the horizontal. He solved the equation numerically for specified two or three layer atmosphere to determine possible wavelengths in the horizontal directions for lee waves. He obtained results for the cases when wind direction changed with height as well as for the cases when wind direction remained same in the vertical. He showed interestingly that Scorer's (1949) condition for the occurrence of lee wave was no longer applied for wave motion in 3-D. He showed that in 3-D, lee waves are always possible in a two-layer atmosphere. Das (1964) studied 3-D lee waves associated with a large circular mountain with some ideal atmospheric condition. Since the dimension of the mountain, he took, was large (1000 km), hence he had to consider the effect of Coriolis force. The nodal lines in his solution were systems of concentric circles, whereas those in the Wurtele's (1957) were system of hyperbola. Das attributed this difference to the geostrophic assumption taken by him. Smith (1980) examined the stratified hydrostatic flow over a bell shaped 3-D isolated mountain using linear theory. Solutions for various parts of the flow field were obtained using analytical method and numerical Fourier analysis. The flow aloft was found to be composed of vertically propagating mountain waves. The maximum amplitude of these waves occurred directly over the mountain, but there was considerable wave energy trailing downstream along the parabolas  $y^2 = \frac{Nzax}{U}$ ; where  $U$ ,  $N$  are respectively the constant basic zonal wind and buoyancy frequency.

Somieski (1981) studied the stratified hydrostatic flow over a three dimensional circular mountain. He

derived a 2<sup>nd</sup> order wave equation from the primitive equation including constant rotation and vertical wind shear of the mean flow. He solved the equation numerically. He showed that in case of no shear and constant static stability, the nodal lines are parabolic for a circular mountain of diameter 50 km. Olafssen and Bougeault (1996) explored the hydrostatic flow over an elliptical mountain barrier of aspect ratio 5. They took upstream profiles of wind ( $U$ ) and stability ( $N$ ) constant ignored the effect of Coriolis force. Under such conditions their result showed the flow characteristics to be dependent mainly on the non-dimensional mountain height  $\frac{Nh}{U}$ . They found that for all values of  $\frac{Nh}{U}$ , a substantial part of the flow was diverted vertically above the mountain. They found generation of potential vorticity in the wake of the mountain, leading to the creation of lee vortices.

Dutta *et al.* (2002) have made a theoretical study on the problem of 3-D lee waves across a meso-scale elliptical mountain. In this study the basic flow has been assumed to be solely normal to the major ridge of the elliptical mountain.

From the foregoing discussions it appears that in most of the studies on 3-D mountain wave problem, the basic flow is assumed to consist of only that component ( $U$ ), which is normal to the major ridge of the mountain. Those studies did not consider other component of basic flow ( $V$ ), which is parallel to the major ridge of the mountain. But in the real atmosphere at any level horizontal wind may have both components, *viz.*, the component normal to the major ridge as well as the component parallel to the major ridge. So, it is necessary to investigate, atleast qualitatively, the effect of 'V' component on the pattern of perturbation vertical velocity ( $w'$ ) and stream line displacement ( $\eta'$ ) associated with 3-D lee wave.

The objective of the present study is to develop a 3-D lee wave model across a meso-scale elliptical 3-D mountain, with a basic flow having both the components 'U' and 'V' and thereby to study the effect of 'V' component.

## 2. Methodology

To develop the model following assumptions are made:

- (i) Steady state flow
- (ii) Friction less flow

- (iii) Adiabatic flow
- (iv) Boussinesq flow
- (v) Non-rotating flow

The basic flow is assumed to have both the components  $U$  and  $V$ , normal and parallel to the major ridge of the mountain respectively. It is again simplified by assuming  $U$ ,  $V$  and the Brunt-Vaisala frequency ( $N$ ), to be invariant with height. The smoothed profile of the 3-D elliptical mountain is expressed analytically as

$$h(x, y) = \frac{H}{1 + \frac{x^2}{a^2} + \frac{y^2}{b^2}} \quad (1)$$

Where,  $H$  is the maximum height of the mountain at the center (0,0) and  $a$ ,  $b$  are half widths of the mountain along the two components of basic flow.

Under above assumptions (i)-(v), the linearized governing equations may be written as

$$U \frac{\partial u'}{\partial x} + V \frac{\partial u'}{\partial y} = -\frac{1}{\rho_0} \frac{\partial p'}{\partial x} \quad (2)$$

$$U \frac{\partial v'}{\partial x} + V \frac{\partial v'}{\partial y} = -\frac{1}{\rho_0} \frac{\partial p'}{\partial y} \quad (3)$$

$$U \frac{\partial w'}{\partial x} + V \frac{\partial w'}{\partial y} = -\frac{1}{\rho_0} \frac{\partial p'}{\partial z} + \frac{g\theta'}{\theta_0} \quad (4)$$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0 \quad (5)$$

$$U \frac{\partial \theta'}{\partial x} + V \frac{\partial \theta'}{\partial y} + w' \frac{d\theta_0}{dz} = 0 \quad (6)$$

Where  $u', v', w', p', \theta'$  are perturbation zonal, meridional, vertical components of wind and perturbation pressure and potential temperature respectively and  $\theta_0 = \theta_0(z)$ ,  $\rho_0 = \rho_0(z)$  are respectively the potential temperature and density of the basic state flow at the height  $z$ .

Now equations (2)-(6) are subjected to double Fourier transform given by

$$\hat{f}(k, l, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y, z) e^{-i(kx+ly)} dx dy \quad (7)$$

and

$$f(x, y, z) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{f}(k, l, z) e^{i(kx+ly)} dk dl \quad (8)$$

where  $\hat{f}$  is the double Fourier transform of  $f$ .

Then equations (2)-(6) transformed to

$$i(kU + lV)\hat{u} = -ik \frac{\hat{p}}{\rho_0} \quad (9)$$

$$i(kU + lV)\hat{v} = -il \frac{\hat{p}}{\rho_0} \quad (10)$$

$$i(kU + lV)\hat{w} = -\frac{1}{\rho_0} \frac{\partial \hat{p}}{\partial z} + g \frac{\hat{\theta}}{\theta_0} \quad (11)$$

$$i(k\hat{u} + l\hat{v}) + \frac{\partial \hat{w}}{\partial z} = 0 \quad (12)$$

$$i(kU + lV)\hat{\theta} + \hat{w} \frac{d\theta_0}{dz} = 0 \quad (13)$$

Where,  $\hat{u}, \hat{v}, \hat{w}, \hat{p}, \hat{\theta}$  are double Fourier transforms of  $u', v', w', p', \theta'$  respectively. Now eliminating  $\hat{u}, \hat{v}, \hat{p}, \hat{\theta}$  from equations (9)-(13), we obtain

$$\frac{\partial^2 \hat{w}}{\partial z^2} + \frac{1}{\rho_0} \frac{d\rho_0}{dz} \frac{\partial \hat{w}}{\partial z} + (k^2 + l^2) \left[ \frac{N^2}{(Uk + Vl)^2} - 1 \right] \hat{w} = 0 \quad (14)$$

where  $N = \sqrt{\frac{g}{\theta_0} \frac{d\theta_0}{dz}}$  is the Brunt-Vaisala

frequency, which has been assumed to be invariant with height for the present study. Now by the substitution

$$\hat{w}(k, l, z) = \left( \frac{\rho_0(0)}{\rho_0(z)} \right)^{1/2} \hat{w}_1(k, l, z) \quad (15)$$

the equation (14) further simplified to

$$\frac{\partial^2 \hat{w}_1}{\partial z^2} + (k^2 + l^2) \left[ \frac{N^2}{(Uk + Vl)^2} - 1 \right] \hat{w}_1 = 0 \quad (16)$$

Now, the earlier workers have shown that

$$\left( \frac{\rho_0(0)}{\rho_0(z)} \right)^{1/2} \approx e^{\frac{(g-R^*\gamma)z}{2R^*T}}, \text{ where } Y, \bar{T}(z), \text{ are basic state}$$

lapse rate and temperature at level  $z$ .

While obtaining equation (16) the terms  $\left( -\frac{1}{2\rho_0} \frac{d^2\rho_0}{dz^2} \right)$  and  $\frac{1}{4\rho_0^2} \left( \frac{d\rho_0}{dz} \right)^2$  have been neglected because they are less, by at least one order of magnitude, than the other terms in the square bracket.

Now if  $\eta'(x, y, z)$  be the perturbation streamline displacement, then we have

$$w'(x, y, z) = U \frac{\partial \eta'}{\partial x} + V \frac{\partial \eta'}{\partial y} \quad (17)$$

Hence,  $\hat{w}(k, l, z) = i(kU + lV)\hat{\eta}$ . Then it is readily seen that  $\hat{\eta}$  also satisfies equation (14). Now, by the substitution

$$\hat{\eta}(k, l, z) \approx e^{\frac{(g-R_y)z}{2RT}} \hat{\eta}_1(k, l, z) \quad (18)$$

we obtain

$$\frac{\partial^2 \hat{\eta}_1}{\partial z^2} + (k^2 + l^2) \left[ \frac{N^2}{(Uk + Vl)^2} - 1 \right] \hat{\eta}_1 = 0 \quad (19)$$

Equations (16) and (19) are solved subject to the following boundary conditions:

(i) At the lower boundary stream line pattern follow the contour of the mountain,

(ii) At the upper boundary radiative boundary condition is imposed i.e., mountain wave is allowed to propagate vertically.

Now using the upper boundary condition (ii), the general solution of equation (16) and (19) can be taken as

$$\hat{w}_1(k, l, z) = Ae^{imz} \quad (20)$$

and

$$\hat{\eta}_1(k, l, z) = Be^{imz} \quad (21)$$

where A, B are constants to be determined using lower boundary condition and  $m$  is given by,  $m^2 = \left[ \frac{N^2}{(kU + lV)^2} - 1 \right] (k^2 + l^2)$ . Clearly  $m$  may be recognized as the vertical wave number of the vertically propagating mountain wave.

At the lower boundary we have  $\eta'(x, y, 0) = h(x, y)$ .

Hence,  $\hat{\eta}(k, l, 0) = \hat{h}(k, l)$ .

Now  $\hat{h}(k, l) = 2\pi abHK_0 \left( \sqrt{a^2k^2 + b^2l^2} \right)$ . Detail of this derivation has been explained in Dutta *et al.* (2002). Here  $K_0(x)$  is the Bessel function of second kind of order zero. Hence,  $B = 2\pi abHK_0 \left( \sqrt{a^2k^2 + b^2l^2} \right)$ . Again the linearized lower boundary condition for  $w'$  may be given by

$$w'(x, y, 0) = U \frac{\partial \eta'(x, y, 0)}{\partial x} + V \frac{\partial \eta'(x, y, 0)}{\partial y}.$$

Hence,

$$\hat{w}(k, l, 0) = i(kU + lV)\hat{\eta}(k, l, 0).$$

Hence  $A = 2i\pi(kU + lV)abHK_0 \left( \sqrt{a^2k^2 + b^2l^2} \right)$ .

Thus solution of (16) and (19) are given by:

$$\hat{w}_1(k, l, z) = 2i\pi(kU + lV)abHK_0 \left( \sqrt{a^2k^2 + b^2l^2} \right) e^{imz} \tag{22}$$

and

$$\hat{\eta}_1(k, l, z) = 2\pi abHK_0 \left( \sqrt{a^2k^2 + b^2l^2} \right) e^{imz} \tag{23}$$

Therefore,

$$w'(x, y, z) = c \times \text{Real part of } I_1 \text{ and}$$

$$\eta'(x, y, z) = c \times \text{Real part of } I_2, \text{ where,}$$

$$c = \frac{abH}{2\pi} e^{\frac{(g-R^* \gamma) z}{2R^* T}} \text{ and}$$

$$I_1 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} i(kU + lV)K_0 \left( \sqrt{a^2k^2 + b^2l^2} \right) e^{i(kx+ly+mz)} dkdl \tag{24}$$

$$I_2 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} K_0 \left( \sqrt{a^2k^2 + b^2l^2} \right) e^{i(kx+ly+mz)} dkdl \tag{25}$$

The double integrals  $I_1$  and  $I_2$  are difficult to evaluate analytically. So they are amenable to the method of stationary phase. According to this method, first those points in the wave number( $k, l$ ) domain are found out, where the phase ( $kx + ly + mz$ ) is stationary. Those points are termed as saddle points. Then the entire integrand is expanded in Taylor's series about the saddle point and the first term of the expansion is retained as the asymptotic approximation of the integrals, which is valid at far down wind location of the mountain.

### 3. Discussion

Following Dutta *et al.* (2002), the asymptotic approximation of  $w'(X_1, Y_1, Z)$  and  $\eta'(X_1, Y_1, Z)$ , obtained from (24) and (25) are given by:

Thus,

$$w'(X_1, Y_1, Z) = -e^{\left( \frac{g-R^* \gamma}{2R^* T} z \right)} \frac{abHN^3 AX_1 Z \sin \left( \frac{ZR}{\rho} \right)}{\rho R(U^2 + V^2)} \tag{26}$$

and,

$$\eta'(X_1, Y_1, Z) = e^{\left( \frac{g-R^* \gamma}{2R^* T} z \right)} \frac{abHN^2 A \cos \left( \frac{ZR}{\rho} \right)}{(U^2 + V^2)} \tag{27}$$

where,

$$A = \frac{X_1 Z \sqrt{\rho^4 + (X_1 Y_1)^2}}{2\rho^3 R^2 \sqrt{1 + 4 \left( \frac{X_1 Y_1 Z R}{\rho^4 + X_1^2 Y_1^2} \right)^2}} K_0(\text{Arg}_0),$$

$$\text{Arg}_0 = \frac{(NX_1 Z) \sqrt{a^2 (U\rho^2 - VX_1 Y_1)^2 + b^2 (V\rho^2 + UX_1 Y_1)^2}}{\rho^3 R(U^2 + V^2)}$$

$$\rho^2 = Y_1^2 + Z^2, R^2 = X_1^2 + \rho^2,$$

$$X_1 = \frac{N(Ux + Vy)}{(U^2 + V^2)}, Y_1 = \frac{N(Uy - Vx)}{(U^2 + V^2)} \text{ and } Z = \frac{Nz}{\sqrt{U^2 + V^2}}$$

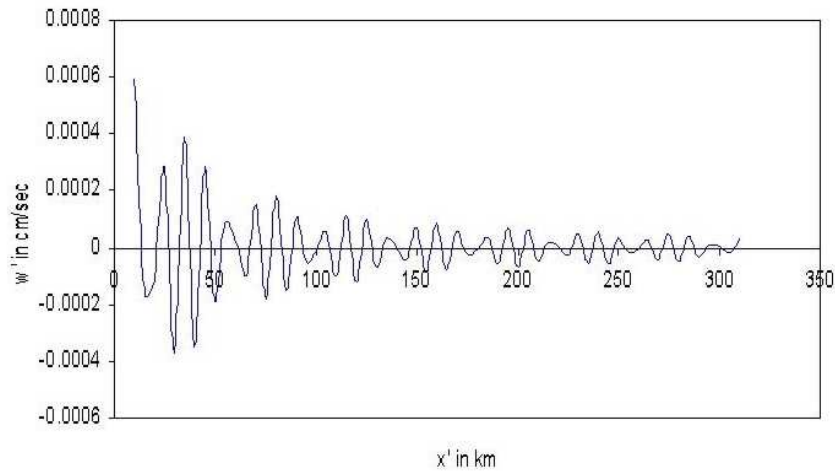


Fig. 1(a). Down wind variation of  $w'$  computed asymptotically, along the line  $Uy-Vx=0$

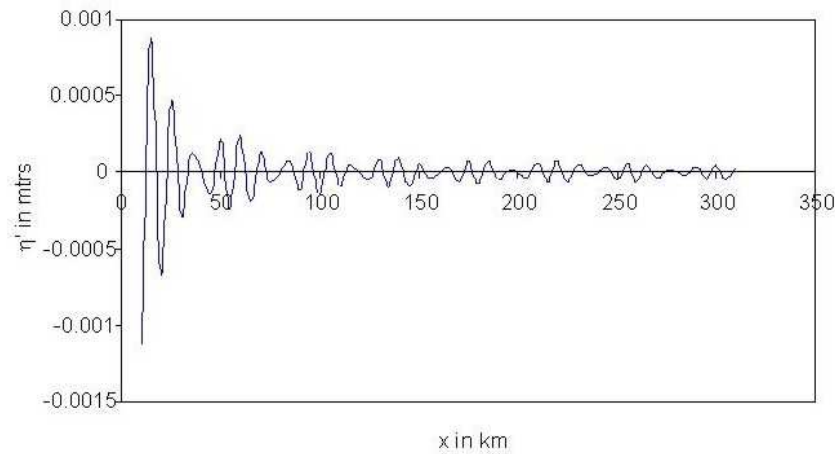


Fig. 1(b). Down wind variation of  $\eta'$  computed asymptotically, along the line  $Uy-Vx=0$

While obtaining asymptotic expression for  $w'$  and  $\eta'$  care has been taken to avoid all those saddle points, in the wave number domain, from the contour, which are inclined with the basic flow vector  $(U, V)$  at an angle of  $90^\circ$  or more.

From the asymptotic solution (26) it is clear that  $w'=0$  for  $z=0$ . Thus it represents, the lee wave (Wurtele,1957). Now, for  $Y_1 = 0, \rho = Z, R^2 = X_1^2 + Z^2,$

$$Arg_0 = \frac{(NX_1)\sqrt{(a^2U^2 + b^2V^2)}}{R(U^2 + V^2)} \text{ and}$$

$$A = \frac{X_1}{2R^2} K_0(Arg_0).$$

So at any level  $Z(\neq 0),$  for  $Y_1 = 0, w'$  and  $\eta'$  can be expressed as

$$w'(X_1, 0, Z) = -e^{\left(\frac{g-R^* \gamma}{2R^* T} z\right)} \frac{abHN^3 X_1^2 \sin\left(\sqrt{X_1^2 + Z^2}\right) K_0\left(\frac{NX_1 \sqrt{a^2U^2 + b^2V^2}}{(U^2 + V^2)\sqrt{X_1^2 + Z^2}}\right)}{2(X_1^2 + Z^2)^{3/2} (U^2 + V^2)} \tag{28}$$

$$\eta'(x, 0, z) = e^{\left(\frac{g-R^* \gamma}{2R^* T} z\right)} \frac{abHN^2 X_1 \cos\left(\sqrt{X_1^2 + Z^2}\right) K_0\left(\frac{NX_1 \sqrt{a^2U^2 + b^2V^2}}{(U^2 + V^2)\sqrt{X_1^2 + Z^2}}\right)}{2(X_1^2 + Z^2)(U^2 + V^2)} \tag{29}$$

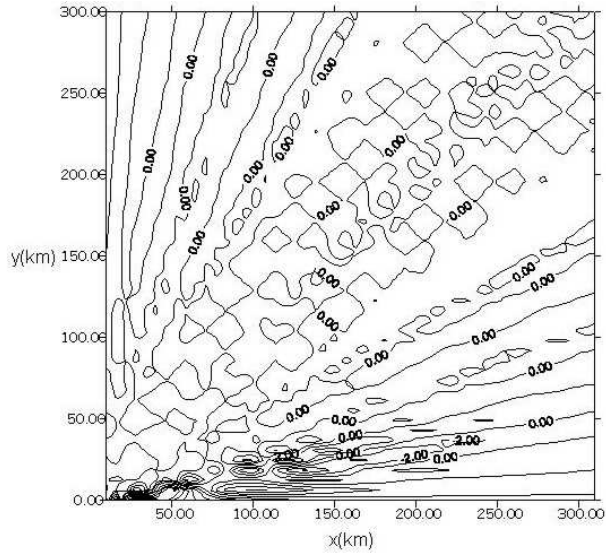


Fig. 2(a). Contour of  $w'$  at 4 km level taking 'V' component

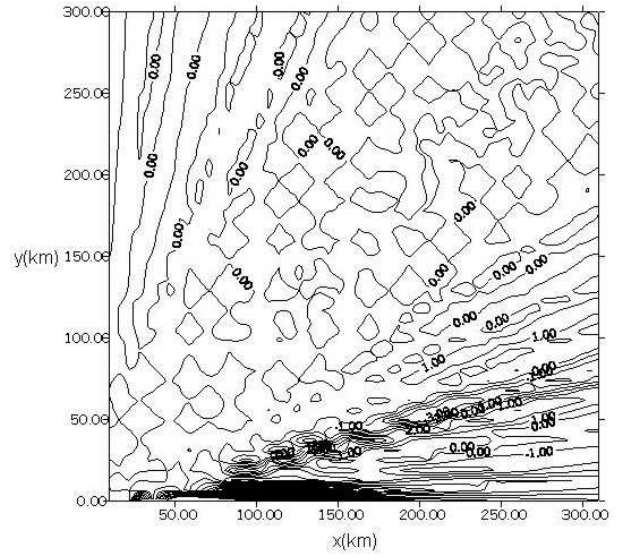


Fig. 2(b). Contour of  $w'$  at 8 km taking 'V' component

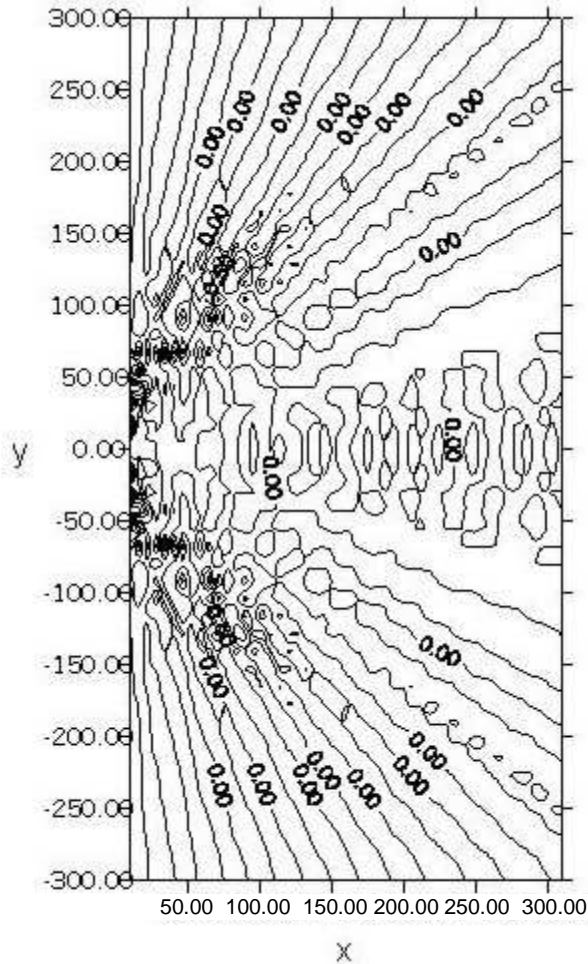


Fig. 2(c). Contour of  $w'$  at 4 km without taking 'V' component

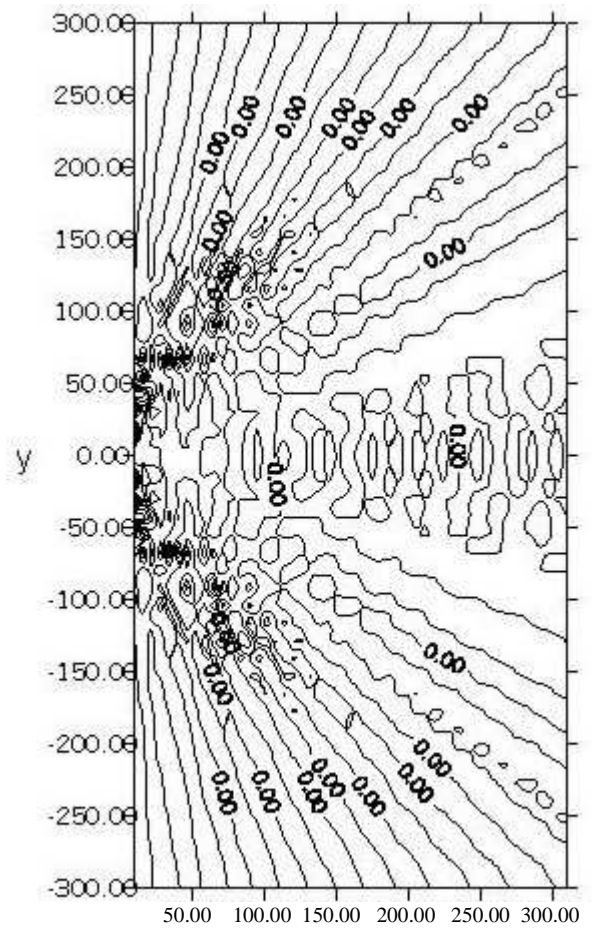


Fig. 2(d). Contour of  $w'$  at 8 km without taking 'V' component

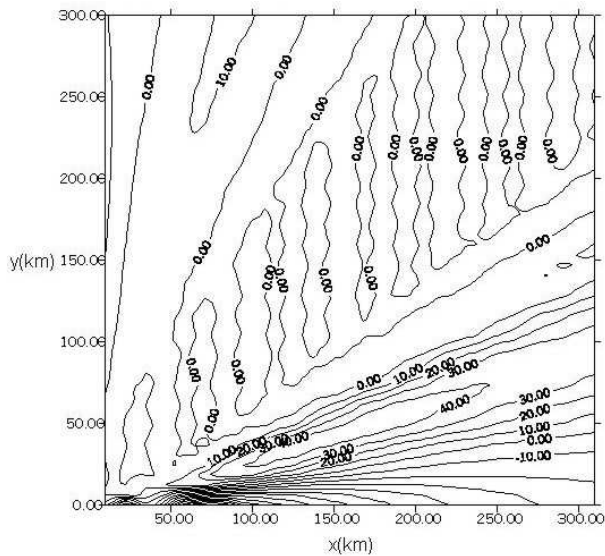


Fig. 3(a). Contour of  $\eta'$  at 4 km level taking 'V' component

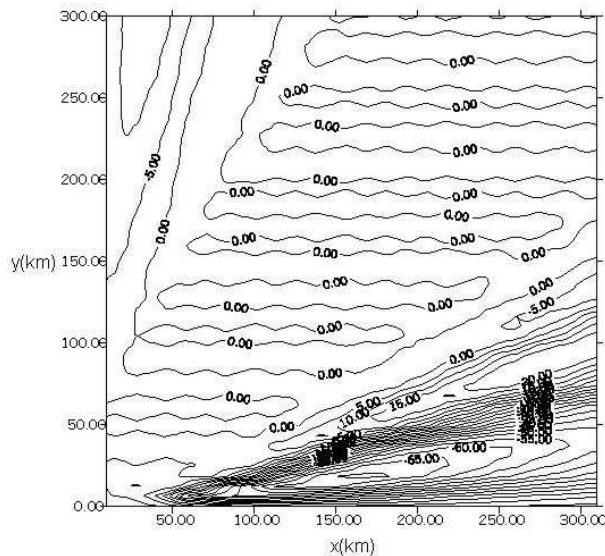


Fig. 3(b). Contour of  $\eta'$  at 8 km level taking 'V' component

From the above expressions for  $w'$  and  $\eta'$  it is clear that, along the line  $Uy - Vx = 0$ , both of them decay down wind of the barrier at a rate proportional to  $X_1^{-1}$ , i.e., inversely proportional to the distance along the line  $Uy - Vx = 0$ , owing to the presence of the terms  $\frac{X_1^2}{(X_1^2 + Z^2)^{3/2}}$  and  $\frac{X_1}{(X_1^2 + Z^2)}$  respectively. This is clearly reflected in Figs. 1 (a&b), which show the

downwind variation of  $w'$  and  $\eta'$  in the central plane along the line  $Uy - Vx = 0$ . In these figures the fluctuations may be attributed to the product of the damping factors  $\frac{X_1^2}{(X_1^2 + Z^2)^{3/2}}$ ,  $\frac{X_1}{(X_1^2 + Z^2)}$  with the Bessel function.

Following the analysis, made in Dutta *et al.* (2002), it can be shown that the nodal lines (where  $w' = 0$ ) on any horizontal plane ( $Z = Z_0$ ) are system of hyperbolas, the axes of which are along the lines  $Uy - Vx = 0$  and  $Ux + Vy = 0$  respectively and the latus rectum of which increases with increase in height. Due to this the updraft regions are crescent shaped, symmetrical about the line  $Uy - Vx = 0$ , tilting upwind with height along the line  $Uy - Vx = 0$  and spreading laterally with height about the same line. These are clearly reflected in Figs. 2(a&b) and in Figs. 3(a&b), which show the contours of  $w'$  and  $\eta'$  at 4 km and 8 km incorporating the effect of 'V' component. These figures show that the contours are approximately crescent shaped with axis of symmetry being inclined with the E-W direction by some angle. Figs. 2(c&d) show the contour of  $w'$  at 4 km and 8 km level, without taking the 'V' component, which are crescent shaped, symmetrical about E-W direction. Comparing these two figures with Figs. 2(a&b) it is clear that the result of incorporation of the 'V' component is, to rotate the axis of symmetry of crescent shaped updraft region by an angle of  $\tan^{-1}\left(\frac{V}{U}\right)$ . Lateral spreading of the wave, as shown in Figs. 2(a&b) and in Figs. 3(a&b), is due to the presence of divergent part in the lee wave, a typical characteristic of 3-D lee wave. The upwind tilting and lateral spreading may physically be interpreted as the upwind trailing of wave energy along the line  $Uy - Vx = 0$  and lateral spreading of wave energy about the same line.

Gjevic and Marthinson (1978) had also found diverging type as well as transverse type lee wave pattern analyzing satellite photograph to study the lee wave patterns generated by isolated islands in the Norwegian Sea and the Barents Sea. In the former case the crests were observed to be oriented outwards from the centre of the wakewhere and in later case the crests were nearly perpendicular to the wave direction. In the Figs. 2(a&b) and 3(a&b) crescent shaped updraft regions are found symmetric about the line  $Uy - Vx = 0$ . Wurtele (1957), obtained crescent shaped updraft region, symmetric about x-axis (i.e. about the line  $y = 0$ ), taking constant basic flow with only U-component. Now due to the presence of V-component, there is a meridional forcing acting at all



level, causing the symmetric (about the line  $y = 0$ ) crescent shaped updraft region to rotate. This may be the possible cause for the orientation of the crescent shaped updraft region in the present study.

Nodal lines in the study of Das (1964) were concentric circles which may be attributed to the geostrophic approximation made by him and the larger scale of the barrier taken by him. Smith (1980), taking hydrostatic approximation, obtained parabolic shaped nodal lines. Someiski (1981) taking hydrostatic approximation has also shown that if the diameter of the circular obstacle is 50 km, then flow becomes non-geostrophic and the nodal lines in that case become parabolic shaped. In the present study neither geostrophic nor hydrostatic approximations are made. Hence the nodal lines are system of Hyperbola in conformity with the earlier findings of Wurtele (1957), only difference is that in the present study the axes of the Hyperbola have been rotated through an angle  $\tan^{-1}\left(\frac{V}{U}\right)$  due to the presence of  $V$ -component in the basic flow.

It is known that the correct prediction, atleast qualitatively, of the region of upward motion associated with mountain wave is very important for aviation. In most of the studies, cited above, the basic flow has been assumed to have only ' $U$ ' component for a 3-D mountain, with major ridge being N-S oriented. And those studies have predicted more or less crescent shaped region of updraft motion, (except Das, 1964) symmetric about E-W direction. These results do seem to differ more from reality, because at every level the basic wind may have both components ' $U$ ' and ' $V$ ' instead of having only ' $U$ ' component. But the present study has considered the effect of both these components for a 3-D mountain, with major ridge being N-S oriented. The result of the present study shows that although the region of upward motion is crescent shaped but it is not symmetrical about E-W direction, rather it is symmetrical about a line inclined at an angle of  $\tan^{-1}\left(\frac{V}{U}\right)$  with E-W direction. So, the results of present study are capable to predict, atleast qualitatively, more accurately the region of updraft associated with 3-D meso-scale lee wave.

Even the mountain wave cloud to the lee of the Mount Fujiyama, as shown in Wurtele (1957), was of crescent shaped, but it was not symmetrical about the central axis, rather the axis of crescent shaped cloud region had some inclination with the central axis which was not predicted by Wurtele (1957). The inclination can be predicted if the effect of  $V$  is incorporated in the model. So the result of this present study can be taken as the

generalization of the results of Wurtele (1957), Dutta *et al.* (2002).

#### 4. Conclusions

From the above study following conclusions may be made:

(i) The asymptotic solutions for  $w'$  and  $\eta'$  show that, in the central plane along the line  $Uy - Vx = 0$  both decrease down wind of the barrier at a rate proportional to  $X_1^{-1}$ .

(ii) Nodal lines on the horizontal planes are Hyperbola, the axes of which are along the lines  $Uy - Vx = 0$  and  $Ux - Vy = 0$ .

(iii) In the horizontal plane contour of  $w'$ , as obtained by asymptotic method, shows that the regions of updraft are crescent shaped which are symmetrical about a line inclined at an angle of  $\tan^{-1}\left(\frac{V}{U}\right)$  with central axis.

(iv) Asymptotic solution for both  $w'$  and  $\eta'$  shows upwind tilting along the line  $Uy - Vx = 0$  and lateral spreading about the same line with height.

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