

Vorticity of vector change of wind during 24 hours related to spatial distribution of rainfall during the next 24 hours

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सार - भारतीय क्षेत्र में मानसून के दौरान विभिन्न स्तरों पर पवन परिवर्तनों के अध्ययन से यह पता चलता है कि 24 घण्टों के दौरान पवन परिवर्तनों की पवन प्रवाही रेखाओं में चक्रवाती तथा प्रति चक्रवाती भ्रमिल होते हैं, जो पूर्वानुमान वाले दिन इन स्तरों पर वास्तविक पवन प्रवाह के पवन प्रवाही-विश्लेषण में नहीं दिखाई पड़ते। इन भ्रमिलों के क्षेत्र कभी-कभी उन क्षेत्रों में अगले 24 घण्टों में होने वाली वर्षा से संबंधित होते हैं। फिर भी, पवन परिवर्तनों का प्रति चक्रवाती तथा चक्रवाती चक्र जो स्पष्टतः बड़ी मात्रा में सकारात्मक या नकारात्मक भ्रमिलता परिवर्तनों की सूचना देता है, अक्सर नहीं होता। इस प्रकार संबंधित क्षेत्र में भ्रमिलता परिवर्तन मालूम करने के लिए परिवर्तन का वास्तविक मान अभिकलित किया जाता है।

सभी पायलेट बेलून एवं रेडियोसोन्ड स्टेशनों के लिए पूर्वानुमान वाले दिन 00 ग्री० मा० स० पर सदिश पवन परिवर्तन पिछले दिन के सदिश पवन परिवर्तन की तुलना में 850 मि० बार वायु-मण्डलीय दाब पर अभिकलित किया जाता है। क्षेत्र के विभिन्न स्टेशनों को जोड़कर कई त्रिभुजों में बांटा जाता है। ऐसा करते समय ध्यान में रखा जाता है कि इस प्रकार बनने वाले त्रिभुज समत्रिबाहु-त्रिभुज हों। इन त्रिभुजीय क्षेत्रों को भ्रमिलता वेलामी ग्रिड की सहायता से अभिकलित की जाती है। क्षेत्र में भ्रमिलता का ऊर्ध्वोपर घटक अगले 24 घण्टों में होने वाली वर्षा से संबंधित है। अगले 24 घण्टों की वर्षा के स्थानिक बंटन की प्रगुक्ति के लिए एक पूर्वानुमान योजना का सुझाव भी दिया गया है।

ABSTRACT. From the study of wind changes at various levels over Indian area during the monsoon period, it is seen that the stream-lines of 24-hour wind changes contain cyclonic and anti-cyclonic vortices, which are not indicated in the stream-line analysis of the actual wind flow at these levels on the day of forecast. The regions of these vortices are sometimes associated with rainfall over that area during the subsequent 24 hours. However, it is not too often that well defined cyclonic or anti-cyclonic circulation of wind changes which obviously indicate large positive or negative vorticity changes occur. Thus, in order to know the vorticity changes in the area concerned, the actual value of the change is calculated.

Vector wind changes at 00 GMT on the day of forecast with respect to its previous day are calculated at 850 mb for all PB/RW stations. The area is divided into a number of triangles by joining these stations suitably taking care that the triangles so formed are more or less equilateral. Vorticity of these triangular areas are calculated with the help of Bellamy's grid. The vertical component of the vorticity in the area is related to the rainfall which occurs during the next 24 hours. A forecast scheme is suggested for prediction of spatial distribution of rainfall during the next 24 hours.

1. Introduction

During the southwest monsoon accurate forecasting of spatial distribution of rainfall, especially in the absence of well defined systems is one of the most difficult and challenging problems for a forecaster. Even today with the transmission of surface and upper air prognostic charts on the facsimile circuits, the forecaster still relies on the conventional methods and his practical experience. The experience, of course, lies largely on the subjective methods which differ from forecaster to forecaster.

1.1. For the middle latitudes, a number of workers have tried to correlate the occurrence of precipitation

with the upper air circulation (Klein 1948, Starrett 1949). For the tropics also a number of Indian and other workers have tried to relate the rainfall with the upper air flow pattern. Das (1951) has explained the use of the convergence and divergence charts for precipitation forecasting. Venkataraman and Rao (1965) have related the rainfall distribution associated with a southwest monsoon depression with high level divergence. Buajitti (1963) has related the occurrence of rainfall with the vertical component of vorticity of 24 hours wind changes. In this paper efforts have been made to relate the vertical component of vorticity of the vector change of wind during the preceding 24 hours with the spatial distribution of rainfall during the next 24 hours over the Indian area.

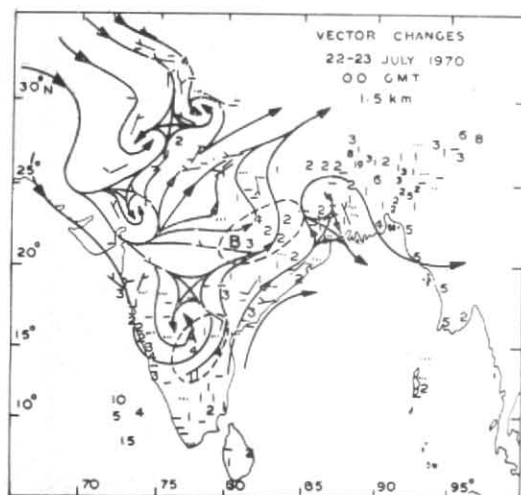


Fig. 1. Stream line analysis for 24-hr vector changes of wind

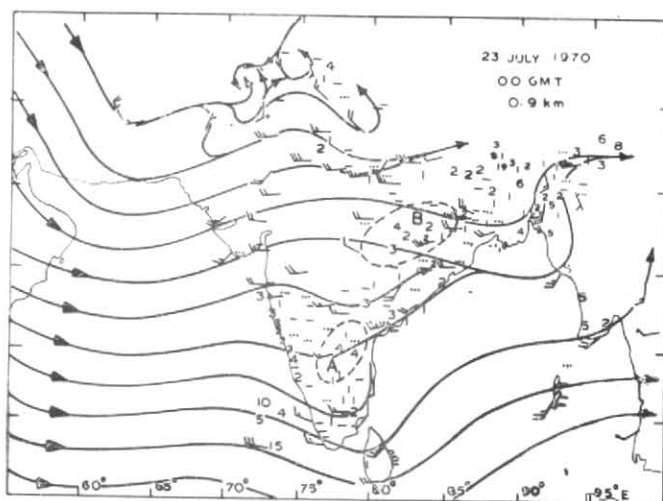


Fig. 2. Stream line analysis and rainfall distribution

2. Method of analysis

The method adopted here is more or less on the same lines as was originally developed by Buajitti (1963) for Thailand, which is described below :

For horizontal motion, the equations of motion after ignoring the friction and small terms may be written as :

$$\frac{du}{dt} - fv = -\frac{1}{\rho} \frac{\partial p}{\partial x} \quad (1)$$

$$\frac{dv}{dt} + fu = -\frac{1}{\rho} \frac{\partial p}{\partial y} \quad (2)$$

where u and v are the horizontal components of wind, f is the coriolis parameter and ρ is the density of the air. These equations may be written as :

$$v = \frac{1}{f\rho} \frac{\partial p}{\partial x} + \frac{1}{f} \frac{du}{dt} \quad (3)$$

$$\text{and } u = -\frac{1}{f\rho} \frac{\partial p}{\partial y} - \frac{1}{f} \frac{dv}{dt} \quad (4)$$

Differentiating (3) and (4) w.r.t. time and assuming ρ as constant, the following expressions are obtained :

$$\frac{\partial v}{\partial t} = \frac{1}{f\rho} \frac{\partial}{\partial x} \left(\frac{\partial p}{\partial t} \right) + \frac{1}{f} \frac{\partial}{\partial t} \left(\frac{du}{dt} \right) \quad (5)$$

$$\frac{\partial u}{\partial t} = -\frac{1}{f\rho} \frac{\partial}{\partial y} \left(\frac{\partial p}{\partial t} \right) - \frac{1}{f} \frac{\partial}{\partial t} \left(\frac{dv}{dt} \right) \quad (6)$$

if t is taken as 24 hours, $\partial u/\partial t$ and $\partial v/\partial t$ are the u and v components of the vector change of the wind in 24 hours. Representing these components of the vector

change of wind in 24 hours by \bar{u} and \bar{v} we can write (5) and (6) as :

$$\bar{v} = \frac{1}{f\rho} \frac{\partial}{\partial x} \left(\frac{\partial p}{\partial t} \right) + \frac{1}{f} \frac{\partial}{\partial t} \left(\frac{du}{dt} \right) \quad (7)$$

$$\bar{u} = -\frac{1}{f\rho} \frac{\partial}{\partial y} \left(\frac{\partial p}{\partial t} \right) - \frac{1}{f} \frac{\partial}{\partial t} \left(\frac{dv}{dt} \right) \quad (8)$$

Differentiating (7) and (8) partially w.r.t. x and y respectively and subtracting (8) from (7) we get,

$$\begin{aligned} \frac{\partial \bar{v}}{\partial x} - \frac{\partial \bar{u}}{\partial y} &= \frac{1}{f\rho} \left[\frac{\partial^2}{\partial x^2} \left(\frac{\partial p}{\partial t} \right) + \frac{\partial^2}{\partial y^2} \left(\frac{\partial p}{\partial t} \right) \right] + \\ &+ \frac{\partial}{\partial x} \frac{\partial}{\partial t} \left[\frac{1}{f} \left(\frac{du}{dt} \right) \right] + \frac{\partial}{\partial y} \frac{\partial}{\partial t} \left[\frac{1}{f} \left(\frac{dv}{dt} \right) \right] \\ \text{or } \frac{\partial \bar{v}}{\partial x} - \frac{\partial \bar{u}}{\partial y} &= \frac{1}{f\rho} \nabla^2 \left(\frac{\partial p}{\partial t} \right) + \dots \dots \dots \quad (9) \end{aligned}$$

From (9) it is seen that the vertical component of vorticity of the vector change of wind is related to the Laplacian of the pressure tendency represented by the first term on the r.h.s. Vorticity will be positive with the positive value of the Laplacian and negative with the negative value of the Laplacian of the pressure tendency. The second term on the r.h.s. will also have its own contribution to the vorticity value. The Laplacian of the pressure tendency measures the difference between the local and average values of pressure tendency. This indicates the movement of a pressure wave in the area under consideration.

2.1. Fig. 1 shows the stream line analysis for 24-hours vector changes of wind of 22 and 23 July 1970 (0000 GMT) at 1.5 km. Fig. 2 shows the stream lines drawn for the actual wind at 0.9 km on 23 July 1970 (0000 GMT). Numbers plotted in Figs. 1 and 2 show the rainfall distribution on 24 July 1970 at 0300 GMT. It can be seen from Fig. 2 that the

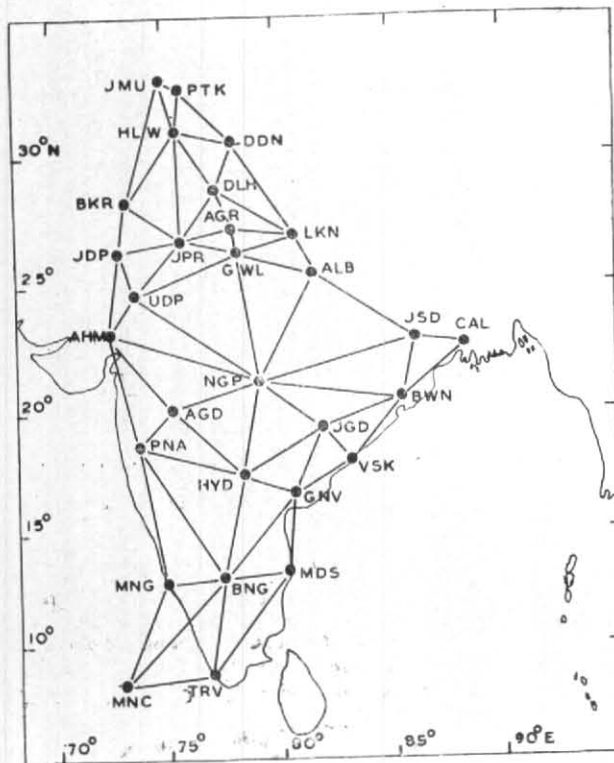


Fig. 3. Area divided into a number of more or less equilateral triangles

stream line analysis at 0.9 km which is an important level for forecasting precipitation during this period of the year does not show any sign of rainfall over the areas marked A and B in the figures. The study of the wind changes at 1.5 km (Fig. 1) provides a quite satisfactory clue for prediction of rainfall in these areas during the next 24 hours.

2.2. It is thus seen that the stream lines drawn for 24 hours vector changes of wind at different levels sometimes contain cyclonic and anticyclonic vortices which are not indicated in the stream line analysis of the actual wind flow at these levels on the day of forecast. The regions of these vortices are sometimes associated with rainfall over that area during the subsequent 24 hours. However, it is not too often that well defined cyclonic or anticyclonic circulations of wind changes which obviously indicate large positive or negative vorticity changes occur. Thus in order to know the vorticity changes in the area concerned, the actual value of the change is calculated.

2.3. Data used

Situations of the southwest monsoon of 1968, 1970 and 1972 in which no well defined systems were apparent on the synoptic charts, but the resulting rainfall was well distributed in certain areas were selected for the study. Situations were selected in such a way that the rainfall areas have a good coverage over the country barring the hilly terrain. Wind recorded at the Pibal/Rawin stations in those areas at 850 mb level on the day of forecast and on the previous day

TABLE 1
Vorticity value ($\times 10^{-5}$)

	<-1.0	Between -1.0 & +2.0	>+2.0	Total
Rainfall over more than half the area	12	40	4	56
Rainfall over half or less area	20	62	4	86
No rainfall	15	22	0	37
Total	47	124	8	179

TABLE 2
Normalised contingency ratio vorticity value ($\times 10^{-5}$)

	<-1.0	Between -1.0 & +2.0	>+2.0
Rainfall over more than half the area	0.8420	1.0436	1.2128
Rainfall over half or less area	0.8782	1.0706	1.0183
No rainfall	1.3808	0.8392	0.7119

were used to work out the 24 hours vector change of wind at these places. Rainfall recorded at 0300 GMT of the next day of the forecast at all the rain gauge stations, departmental and non-departmental including State rain gauge stations, wherever possible were used to determine the spatial distribution of the rainfall. While determining the spatial distribution of the rainfall, amounts less than 2.5 mm were not taken into consideration in accordance with the practice of the India Met. Dep.

2.4. In brief, vector wind changes at 0000 GMT on the day of forecast with respect to its previous day were calculated at 850 mb for all the PB/RW stations in the areas and situations which were selected for study. These were plotted on the charts. The area was divided into a number of triangles as shown in Fig. 3. By joining these stations suitably, taking care that the triangles so formed are more or less equilateral and not elongated, Vorticity of these triangular areas was calculated with the help of Bellamy's grid. Rainfall recorded at 0300 GMT on the next day of the forecast at all the rain gauge stations (departmental and non-departmental including State rain gauge stations) situated in the area were used to determine the spatial distribution of rainfall in these areas. While determining the spatial distribution, rainfall amounts less than 2.5 mm were not taken into consideration. A contingency table was prepared to evolve a forecast scheme by relating the spatial distribution of rainfall to the vorticity value in the area under study.

TABLE 3
Observed class

Class using the scheme	Area of rainfall more than 50%	Forecast Area of rainfall 50% or less	No rainfall	Total
Area of rainfall more than 50%	3	2	0	5
Area of rainfall 50% or less	5	12	1	18
No rainfall	1	4	4	9
Total	9	18	5	32

2.5. Contingency table

The total number of cases studied (179) were divided into three categories in this case, viz., (1) having rainfall over more than half the area, (2) having rainfall over half and less area and (3) having no rainfall at all. According to departmental terminology category 1 covers widespread and fairly widespread rainfall and category 2 scattered and isolated. The contingency table containing the number of cases under various ranges of vorticity values determined in this study is given in Table 1.

Since, the sample size in the various categories of rainfall distribution is not the same in the Table 1, 'Normalised contingency ratio' was calculated with the help of the following relation:

$$R'_{ij} = 1 + (R_{ij} - 1) \left[f_{ij}^o \times \frac{kI}{N} \right]^{1/2}$$

$$\text{and } R_{ij} = \frac{f_{ij}}{f_{ij}^o}$$

where,

f_{ij} is the observed frequency in the (i, j) cell,

f_{ij}^o is the frequency expected on the hypothesis of independence, and thus calculated as $f_{ij}^o = S_j \times S_i / N$ where S_j and S_i are the marginal totals.

N is the total number of cases taken for study.

The normalised contingency ratio is given in Table 2.

3. Discussion

For the three ranges of vorticity values, i.e., (1) < -1.0 , (2) between -1.0 and $+2.0$ and (3) $> +2.0$, the maximum value of the normalised contingency ratio is obtained for the blocks of (i) No rainfall, (ii) rainfall covering half or less area and (iii) rainfall covering more than half the area, respectively. These ranges can give a guidance to forecast the spatial distribution of rainfall. As the area considered for study has a good coverage all over the country, the above ranges can be made use of in all the regions. However, it was seen that some areas are always having larger values of the vorticity compared to others for the same type of distribution of rainfall. The reason is obvious that topography of the area plays a part in contributing to the vorticity.

3.1. The spatial distribution of the rainfall has been taken in this study on a broader scale as the number of cases for distribution in the finer limits were very few. However this study is being extended to make a finer division of the spatial distribution at a range of 25% each and considering smaller areas. The results will be published in a separate paper later on.

4. Verification of the forecast scheme and skill score

The scheme of forecast was tested for working out the skill score on two years data, i.e., for 1973 and 1974. Here again those cases were tested where the forecast issued by Regional Centres for spatial distribution were out by two or more than two stages, if each stage is considered as comprising of 25 per cent of the area. The results are shown in Table 3.

If T is the total number of forecasts and C is the number of correct forecasts according to the scheme, the skill score is given by:

$$S = (C - E) / (T - E)$$

where E is the number of correct forecasts by conventional methods. Since the cases considered are only those where the forecasts by conventional methods were out by two stages or more, the value of E is 0 and as such the skill score is .59.

Acknowledgement

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