551.515.4 : 551.553.21 : 519.217

PROBABILISTIC ESTIMATION OF THUNDERSTORM ACTIVITY AT A TROPICAL INLAND STATION, PUNE DURING POST-MONSOON SEASON USING MARKOV CHAIN MODELS

1. Daily thunderstorm day data occurred during post-monsoon (September-November) season for 11 year (1970-80) period at a tropical inland station, Pune (18° 32' N, 73° 51' E, 559 m asl) are analyzed by using the principle of two state Markov-chain probability model to investigate the distribution of occurrence and non-occurrence of thunderstorms. The data have also been tested by using Akaike's Information Criterion (AIC). The results of the study revealed that the first order Markov Chains is found to be the best fit model for thunderstorm forecast which has described the appropriate period (7 days) for the occurrence of thunderstorms over Pune.

2. In India, the interest in the study of thunderstorm climatology has been evinced for a long time. Thunderstorms are the main channels of energy exchange in the atmosphere. They are also important weather phenomena related to weather and atmospheric electricity. Many scientists (Moid, 1995; Manohar *et al.*, 1991 and Paul, 2001) have studied this particular phenomena through different angles and the results were

discussed. The motto of these studies is to examine the thunderstorm occurrences by using different methods such as numerical, synoptic, statistical etc. Time series analysis is one of the statistical methods.

Though the study of this phenomenon is important to scientists of atmospheric electricity, it is also one of the important aviation hazards. Statistical knowledge of its occurrence at different time and months in a year is helpful to a forecaster. Hence, in the present study the authors have made an attempt to examine the occurrence and non-occurrence of thunderstorm activity over the Pune region during post-monsoon season by using the well known Markov Chain Models.

The Markov chain has found wide applications. Many scientists (Liakatas and Charantonis, 1990; Thiyagarajan *et al.*, 1995; Pant and Shivare, 1998; Dahale and Puranik, 2000; Dasgupta and De, 2001; Kulkarni *et al.*, 2002) have used this model to forecast the occurrence of precipitation, threshold temperature, frequency of wet and dry spells, thunderstorms etc. From the studies referred above, it seems that most of these were carried out at a station which receives large amount of precipitation or a station where thunderstorms are more frequent. In the present study, the authors have used a tropical inland station namely, Pune (India) which receives the total of 20 to 25 thunderstorms per year (Manohar *et al.*, 1991).

Order	Transition probabilities		n step probabilities for I st order		
	Probabilities		n step	Probal	oilities
I.	$P_{00} = 0.91$	$P_{10} = 0.57$	1.	0.91	0.57
	$P_{01} = 0.09$	$P_{11} = 0.43$	2.	0.09 0.88 0.12	0.43
II	$P_{000} = 0.89$	$P_{001} = 0.11$	3.	0.12 0.87	0.24 0.83 0.17
	$P_{010} \!=\! 0.50$	$P_{011} = 0.50$	4.	0.13	0.85
	$P_{110} = 0.21$	$P_{111} = 0.78$	5.	0.14 0.86	0.15
	$P_{100} = 0.63$	$P_{101} = 0.36$	6.	0.15	0.14
			7.	0.14 0.86 0.14	0.14 0.86 0.14

Estimates of transition probabilities of two state Markov Chains for first and second order along with *n*th step probabilities for the first order Markov chain

TABLE 1

3. Daily data on thunderstorms that occurred during post-monsoon season (September-November) for the 11-year period (1970-80) at a tropical inland station Pune (18°32'N, 73°51'E, 559 m asl) forms the data set for the present study. The data have been obtained from India Meteorological Department (IMD), Pune. The original data were in the form of precised dates of thunderstorm occurrence. These dates have been transformed into a sequence of time series of the daily observation of the occurrence and non-occurrence of the thunderstorm labeled as binary (1, 0) events beginning from 1st September to 30th November.

4. The most common class or model of stochastic process used to represent time series of discrete variables is known as the Markov Chain (Wilks, 1995). A Markov Chain can be imagined as being based on the collection of a "state" of model system. Here, the word 'state' represent binary event -1 (occurrence) and event - 0 (non-occurrence) of a thunderstorm. The length of each state is equal to the time separation between two consecutive binary events in the above time series observations. The Markov Chain can either remain in the same state or change to the other state. Generally, there are three orders of Markov Chains (Kulkarni, *et al.* 2002).

The data used in the present study have been tested by these 3 orders. Instead of conventional Chi-square test, a decision procedure based on the extension of the 'maximum likelihood principle' has been used (Chin, 1977). The proper order of the Markov Chain for modeling the time series of thunderstorm occurrence has been assessed by using Akaike's Information Criterion (AIC).

TABLE 2

Akaike's information criterion for the Markov Chains of first and second order

Order (m)	Log likelihood	Akaike's Information Criterion (AICm)
Ι	-20.30	44.64
П	-21.69	51.37

Table 1 shows the estimated transition 5. probabilities for the state 1 and 0 for 1st and 2nd order Markov chains and n^{th} step probabilities only for the 1st order Markov Chain. From this table, it is seen that for 1st order Markov Chain the probability of (0 state) followed by (0 state) is observed to be the highest ($P_{00} = 0.91$), while the probabilities of state (1) and (0) followed by state (1) are more or less equal ($P_{10} = 0.57$, $P_{11} = 0.43$). These features can also be seen to some extent with higher order of Markov Chain where the corresponding probabilities are $P_{000} = 0.89$, $P_{100} = 0.63$, $P_{001} = 0.11$, $P_{101} = 0.36$ respectively. Thus, these probabilities imply that during the post-monsoon season, the chances of nonoccurrence of thunderstorm days are rather more as compared to occurrence of thunderstorm days.

Table 2 shows the AIC values for the 1st and 2nd order Markov Chains along with the log likelihood (L_m). From this table, it is seen that the for the 1st and 2nd order Markov Chains L_m values differ by ~ 1.4 units and the corresponding AIC values differ by ~ 6.73 units. This difference of 6 units in AIC is significant at 95%

TABLE 3

Observed and theoretical values of mean recurrence time for first and second order Markov Chain

	Mean recurrence time (days)					
Order	Thunderstorm days		Non-thunderstorm days			
	Observed	Theoretical	Observed	Theoretical		
I.	7.00	7.14	1.29	1.16		
II.	7.00	7.44	1.29	1.29		

statistically. These values suggest that 2-state 1^{st} order Markov Chain minimize the AIC functions (Chin, 1977). Thus, the 1^{st} order Markov Chain model is observed to be more suitable to describe the data series under investigation. As such the *n*-step probabilities have been obtained only for the first order and given in Table 1.

These *n*-step probabilities are the elements of a matrix of P^n with P is the 1-step transition matrix. This model has been executed upto 7th step and found that the values of P^n becomes constant after 5th step onwards (0.86 and 0.14) which implies that the probabilities of occurrence and non-occurrence of thunderstorm are independent from its initial state. The mean recurrence time for thunderstorm and non-thunderstorm days have been calculated using the reciprocal of the steady state probabilities and it is shown in Table 3.

From Table 3, it is seen that the mean recurrence time period for the state (1) over Pune region during post monsoon season is 7.14 days and that for state (0) is 1.16 days. These recurrence periods are compared with the observed recurrence periods and found to be more or less equal.

6. Time series analysis of the post-monsoon season thunderstorm occurred over the Pune region during 11 year period revealed that the data series under investigation can be best explained by the 1^{st} order Markov Chain Model. During the post-monsoon season the probability of thunderstorm occurrence is observed to be minimum. Computation of *n*-step transition probabilities suggested that after the 5^{th} step these probabilities become constant and hence become independent of its initial state.

7. The authors are thankful to the Director General of Meteorology, IMD, New Delhi for supplying

the data used in this study. They are also thankful to the Director, Indian Institute of Tropical Meteorology, Pune and Head, P.M.&A. Division for his constant support and encouragement during the research activity.

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