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A probabilistic study of monsoon daily rainfall at Calcutta by Markovian model and trend of rainfall during monsoon season

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सार - विभिन्न मानसून के महीनों के लिए कलकत्ता (अलीपुर) में विभिन्न वर्षा के अभिलक्षणों (जैसे वर्षा न होने वाले दिनों, हल्की वर्षा, मध्यम वर्षा, भारी वर्षा और बहुत भारी वर्षा) के लिए मार्कोवियन निदर्श द्वारा एक स्थानांतरीय अवस्था से दूसरी अवस्था तक एकल दूरी स्थानांतरीय संभावित सांचे का पता लगाया गया है। इस शोध-पत्र में शैनन के निदर्श द्वारा मानसून के वर्षा के अभिलक्षणों की स्थानांतरीय प्रणाली की अव्यवस्थता (अनिश्चितता) का अध्ययन किया गया है। कलकत्ता में मानसून की अवधि के दौरान पूर्णता परीक्षण द्वारा विभिन्न अवस्थाओं की अनुकूल अथवा प्रतिकूल स्थितियों का भी अध्ययन किया गया है। संभावित साँचे द्वारा इस प्रकार की वर्षा के अभिलक्षणों से प्राप्त हुए दीर्घकालीन संभावित सदिशों का पता लगाया गया है। इस शोध पत्र में मानसून अवधि के दौरान, जून से सितंबर तक के प्रत्येक मानसून महीने में हुई वर्षा की मात्रा की प्रवृत्ति का भी अध्ययन किया गया है।

ABSTRACT. One-step transitional probability matrices are obtained by Markovian model from one transitional state to other for different rainfall characteristics (such as, non-rainy days, light rains, moderate rains, heavy rains and very heavy rains) at Calcutta (Alipore) for different monsoon months. The disorderness (uncertainty) of the transitional system of the monsoon rainfall characteristics are studied by Shannon's model. The favourable or unfavourable condition of different states are also studied by redundancy test during monsoon period at Calcutta. A long-run probability vectors of such rainfall characteristics are found out from probability matrices. A trend in rainfall amounts during monsoon period at Calcutta for each of the monsoon month, June to September, has also been studied here.

Key words – Markov model, One-step transitional probability matrices, Entropy, Limiting probability vector, Redundancy, Trend.

1. Introduction

Several authors like, Gabriel and Neumann (1962) and Medhi (1976) have studied the probability of occurrences of dry and wet days through Markovian model. Basu (1988) in his earlier work has studied the probability of occurrences of different rainfall characteristics through one-step transitional probability matrices during monsoon period at Maithon by Markovian model, where, the rainfall characteristics are categorised into five different classes (amounts in ranges), according to definition, such as, non-rainy days, light rains, moderate rains, heavy rains and very heavy rains. Domenico (1972) has suggested the test of uncertainty of transitional probabilities of such occurrences for determining favourable or unfavourableness of the system through redundancy method.

The main objective of this study by Markovian model is to test whether the individual state of occurrence depends on previous states of occurrences and to know the probabilistic behaviour from one transitional state to another. Data of daily rainfall amounts at Calcutta (Alipore) during monsoon period for 30 years (1961-90) have been used here, to find out one-step transitional probability matrices for different states of rainfall characteristics during monsoon months (June to September) and favourable or unfavourableness of such transitional probabilities of different states are tested by redundancy of the system. A long-run probability vectors of different states are also found out by solving system of linear equations. An attempt has also been made to find out a trend in monsoon rainfall (1931-96) at Calcutta with the progress of monsoon from June to September through regression method.

2. Data used and processing

Daily rainfall amounts of Calcutta have been Categorised into five different classes, according to definition, as mentioned above, for each of the monsoon month. The frequency of occurrences of daily rainfall amounts at Calcutta from one transitional state to another for different categorised rainfall amounts for each of the monsoon month, June to September are found out and have been arranged in a 5×5 arrays.

One-step 5×5 transitional probability matrices from one transitional state to another for such frequency of occurrences have been computed for each monsoon month. The transitional probability matrix is given by

$$P_{ij} = \begin{bmatrix} P_{11} P_{12} \ldots\ldots\ldots\ldots P_{15} \\ P_{21} P_{22} \ldots\ldots\ldots\ldots P_{25} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ P_{51} P_{52} \ldots\ldots\ldots\ldots P_{55} \end{bmatrix}$$

where, P_{ij} $(i, j = 1, 2, \ldots\ldots, 5)$ is the transitional probability from the j th state of occurrence to the i th state and $\sum_{j=1}^{5} P_{ij} = 1$, for each i.

Here, ' 1' indicates non-rainy days; ' 2 ' for light rains; ' 3 ' for moderate rain; ' 4 ' for heavy rain; ' 5 ' for very heavy rain.

3. Methodology

3.1. *Entropy of Markovian system and redundancy test*

A measure of uncertainty (entropy) of the transitional probability (P_{ij}) of the above system is given by Shannon's formula

$$H_i = -\sum_{j=1}^{5} P_{ij} \log P_{ij} \quad \text{, for each } i.$$

TABLE 1

One-step transitional probability matrices by Markovian model and limiting probability vectors for different categorised rainfall amounts during each of the monsoon month at Calcutta

(a) June

Markov probability matrix

+0.4300000	+0.1900000	+0.2000000	+0.1200000	+0.0600000
+0.4200000	+0.2200000	+0.1700000	+0.1300000	+0.0600000
+0.7100000	+0.1400000	+0.0700000	+0.0600000	+0.0100000
+0.4200000	+0.1900000	+0.1800000	+0.0700000	+0.1400000
+0.2100000	+0.3000000	+0.1500000	+0.0900000	+0.2500000

Limiting probability vector

+0.2009088
+0.2002212
+0.1985901
+0.2001954
+0.2000844

(b) JULY

Markov probability matrix

+0.5300000	+0.2000000	+0.1500000	+0.0700000	+0.0400000
+0.4000000	+0.2500000	+0.1800000	+0.1300000	+0.0400000
+0.3600000	+0.2800000	+0.1900000	+0.1000000	+0.0600000
+0.3400000	+0.2600000	+0.2100000	+0.1200000	+0.0700000
+0.4600000	+0.2300000	+0.1300000	+0.0800000	+0.1000000

Limiting probability vector

+0.2011523
+0.2002094
+0.1981559
+0.2000976
+0.2003848

(c) AUGUST

Markov probability matrix

+0.5500000	+0.2100000	+0.1300000	+0.0700000	+0.0400000
+0.3900000	+0.2400000	+0.2300000	+0.0800000	+0.0600000
+0.3800000	+0.3200000	+0.1800000	+0.0900000	+0.0300000
+0.3400000	+0.2300000	+0.2100000	+0.1000000	+0.1200000
+0.3000000	+0.2800000	+0.1200000	+0.1800000	+0.1200000

Limiting probability vector

+0.2000001
+0.2000000
+0.2000000
+0.2000000
+0.2000000

(d) SEPTEMBER

Markov probability matrix

+0.4600000	+0.2500000	+0.1100000	+0.1300000	+0.0500000
+0.4500000	+0.2400000	+0.1700000	+0.0900000	+0.0400000
+0.6500000	+0.1800000	+0.1100000	+0.0500000	+0.0100000
+0.4300000	+0.1800000	+0.2100000	+0.1000000	+0.0900000
+0.2900000	+0.2100000	+0.0900000	+0.1500000	+0.2600000

Limiting probability vector

+0.2003833
+0.1978091
+0.1999453
+0.2019461
+0.1999163

TABLE 2

Entropy and redundancy (in percent) during monsoon months at Calcutta

	Jun	Jul	Aug	Sep
H_1 (non- rainy day)	0.23	0.25	0.24	0.24
H_2 (light rain)	0.11	0.15	0.15	0.12
H_3 (moderate rain)	0.07	0.11	0.10	0.08
H_4 (heavy rain)	0.05	0.06	0.05	0.05
H_5 (very heavy rain)	0.03	0.03	0.03	0.03
H (weighted entropy)	0.50	0.59	0.58	0.52
R (redundancy)	28	15	17	26

where, H_i denotes the entropy of the i th state of occurrence ($i = 1,2, \ldots, 5$).

The entropy H of the Markovian system of the transitional probability matrix P_{ij} obtained from the probability of individual state of occurrence P_i and weighted H_i and is given by

$$H = \sum_{j=1}^{5} P_i H_i$$

where, P_i is the probability of the ith state of occurrence.

The favourable and unfavourableness of the system is tested by redundancy R and is given by

$$R = 1 - H / H_{\max}$$

where, $H_{\max}$ is the maximum possible entropy and the value is equal to log 5. When the redundancy value R tends to 1, the system tends to maximum favourable condition, *i.e.*, almost certain.

3.2. *Limiting probability vector*

Clarke and Disney (1970) has discussed the behaviour of 'long-run' probability of states for aperiodic system and for large value of 'n', the probability of state $p_j^{(n)}$ tends to a limiting probability vector V and is given by

$$V = \lim_{n\to\infty} p_j^{(n)} = \lim_{n\to\infty} p^{(n)}$$

The quantity $V = (v_1, v_2, v_3, v_4, v_5)$ is a steady state probability of the system.

To determine the limiting probabilities in finite Markov - chain model, the vector V satisfy a system of linear equations and is given by

$$V_j = \sum_{k=1}^{5} V_k P_k$$

and $v_j = 1$ for $j = 1, 2, 3, 4, 5$.

The limiting probability vector V is determined by solving the above system of linear equations by Gauss - Jordon method through computer simulation. The vector V is expressed as an unique stationary probability vector in Markov - chain process.

3.3. *Trend of monsoon rainfall*

The trend values of monsoon rainfall (Y_c) are fitted by a regression line, as discussed by Croxton and Cowden (1956) and is given by

$$Y_c = a + b X_i$$

where, X_i corresponds to the ith year of which trend values Y_c for each year for each of the monsoon month is to be determined; 'a' and 'b' are constants to be found out by the 'Least square method'.

$$a = \sum Y / N$$

$$b = \sum XY / \sum X^2$$

An increase or decrease in trend values of rainfall are determined by the value of 'b', positive or negative respectively.

4. Discussion and results

4.1. *Markov probability matrices and limiting probability vector*

One- step transitional probability P_{ij} are obtained by dividing the frequencies of one state of occurrence for the day followed from another state of occurrence of the previous day f_{ij} by the frequencies of the state f_i *i.e.* $P_{ij} = f_{ij} / f_i$. These are arranged in 5×5 matrices for each of the monsoon month from June to September at Calcutta for different classes of the categorised rainfall amounts (in ranges) and are shown in Table1.

A long-run limiting probability vectors for each of the state of occurrences of the different classes of the categorised rainfall amounts for each monsoon month at

TABLE 3

Rainfall total and trend (in mm) for each of the monsoon month at Calcutta

Year	June		July		August		September	
	Total	Trend	Total	Trend	Total	Trend	Total	Trend
1961	254.7	256.9	108.6	273.7	426.1	372.2	313.8	176.7
1962	259.9	259.0	257.2	280.1	172.6	370.6	228.1	184.2
1963	297.6	261.1	291.5	286.4	214.1	368.9	394.8	191.8
1964	96.5	263.2	626.1	292.7	289.7	367.3	195.1	199.3
1965	309.9	265.3	355.6	299.0	411.8	365.6	346.7	206.9
1966	403.2	267.5	131.6	305.3	275.3	364.0	202.8	214.4
1967	102.6	269.6	217.5	311.7	372.2	362.3	316.6	222.0
1968	598.3	271.7	494.6	318.0	597.7	360.7	111.4	229.5
1969	118.7	273.8	290.7	324.3	429.7	359.0	339.9	237.1
1970	537.1	275.9	277.0	330.6	280.5	357.4	524.3	244.6
1971	358.7	278.0	540.8	336.9	475.0	355.7	232.2	252.2
1972	82.6	280.1	292.3	343.3	556.1	354.1	238.6	259.7
1973	198.8	282.2	221.3	349.6	362.1	352.4	387.6	267.3
1974	132.5	284.3	369.6	355.9	363.9	350.8	279.7	274.8
1975	207.9	286.4	385.6	362.2	340.5	349.1	215.7	282.4
1976	139.7	288.5	449.6	368.5	355.6	347.5	232.9	289.9
1977	456.0	290.6	469.3	374.9	334.0	345.8	222.9	297.5
1978	224.2	292.7	439.2	381.2	312.4	344.2	944.7	305.0
1979	179.9	294.8	396.2	387.5	270.2	342.5	166.8	312.6
1980	382.2	296.9	340.5	393.8	361.0	340.9	123.9	320.1
1981	172.0	299.0	447.3	400.1	249.4	339.2	312.4	327.7
1982	130.4	301.1	185.1	406.5	484.1	337.6	99.3	335.2
1983	258.7	303.2	282.0	412.8	340.1	335.9	138.2	342.8
1984	971.3	305.3	368.1	419.1	462.7	334.3	107.4	350.3
1985	254.5	307.4	498.8	425.4	261.9	332.6	204.4	357.9
1986	238.3	309.5	484.3	431.7	137.1	331.0	832.4	365.4
1987	248.3	311.6	489.8	438.1	509.9	329.3	230.7	373.0
1988	625.1	313.7	332.8	444.4	416.4	327.7	250.5	380.5
1989	218.6	315.8	309.7	450.7	206.8	326.0	302.5	388.1
1990	281.7	317.9	878.9	457.0	157.3	324.4	366.7	395.6
1991	322.0	320.0	345.3	463.3	405.4	322.7	383.9	403.2
1992	480.5	322.1	491.8	469.7	179.4	321.1	218.0	410.7
1993	308.7	324.2	472.9	476.0	409.9	319.4	445.1	418.3
1994	248.6	326.3	412.7	482.3	355.3	317.8	175.1	425.8
1995	145.7	328.4	384.8	488.6	270.5	316.1	591.3	433.4
1996	410.5	330.5	203.1	494.9	539.4	314.5	157.3	440.9

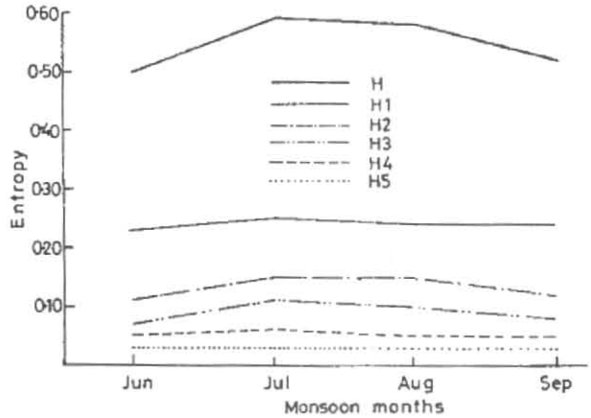

Fig. A. Graph showing entropy of different categorised rainfall for monsoon months alongwith their weighted values at Calcutta

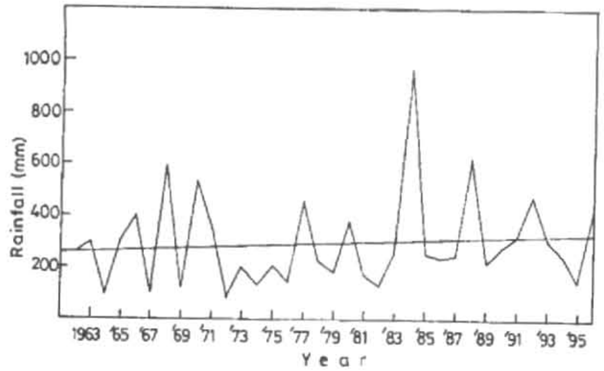

Fig. 1. June rainfall & trend at Calcutta

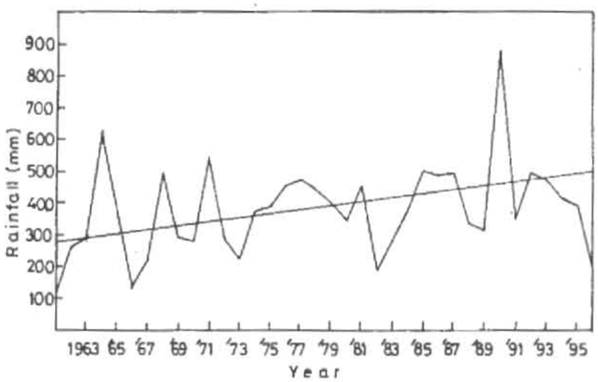

Fig. 2. July rainfall & trend at Calcutta

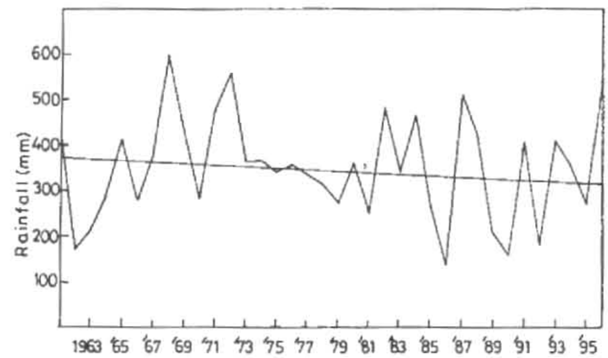

Fig. 3. August rainfall & trend at Calcutta

Calcutta have been found out from the above probability matrices by solving the system of linear equations. In order to solve Markov probability vectors, a object-oriented computer programme has been developed in C ++ language. The programme follows Gauss- Jordon method of matrix inversion to solve the probability vectors. The programme is based on DOS - environment compatible to Window- operating system also. The special feature of the programme is that it generates the matrices dynamically at run- time. The results for each of the monsoon month are also shown in Table 1.

It is found from Table 1, that the limiting probability vectors for non- rainy days in Calcutta is maximum in all the monsoon months, whereas, minimum values for the same vary for different classes of the categorised states of rainfall in different monsoon months. In this study, it reveals that the limiting probability vector for the month of August in Calcutta are equally probable for all states of occurrences of the categorised rainfall.

4.2. *Disorderness of the Markovian system and redundancy values*

The entropies of the different categorised states of rainfall amounts and also for the weighted entropies for each of the monsoon month, June to September at Calcutta are given in Table 2. It is found that the maximum values of entropies are lying between 0.23 and 0.25 during the monsoon period from June to September at Calcutta for non-rainy days; while, the minimum values for the same are 0.03 during the monsoon period for very heavy rains, as shown in the Table 2. The weighted entropy of the system varies from maximum values 0.59 in July to minimum value 0.50 in June during the monsoon period at Calcutta.

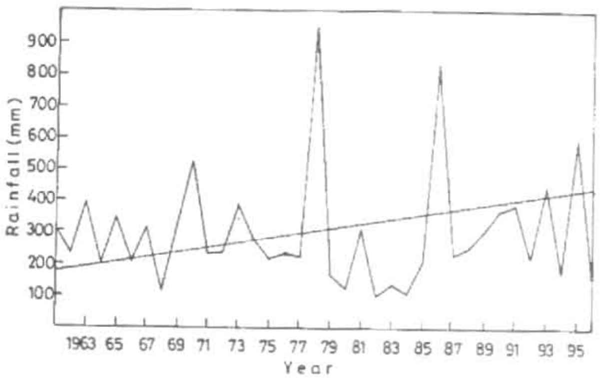

Fig. 4. September rainfall & trend at Calcutta

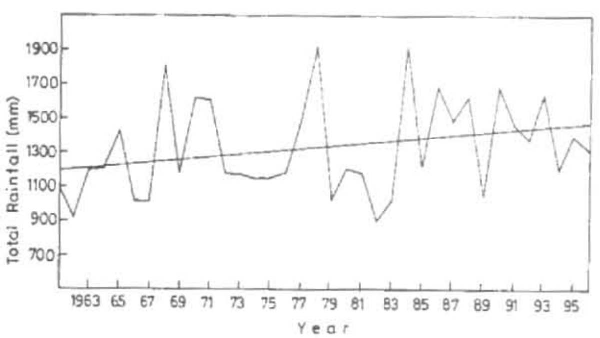

Fig. 5. Total monsoon rainfall & trend at Calcutta

The redundancy values of the system during monsoon period at Calcutta are obtained as shown in Table 2. The maximum value is 28 % during the month of June and minimum value is 15 % during July at Calcutta. This indicates that the favourableness of the Markovian system is maximum in June and minimum in July at Calcutta during the monsoon period.

4.3. *Monsoon rainfall trends*

The trend values (Y_c) of rainfall for each year (1961-96) for the monsoon period at Calcutta for each of the monsoon month are calculated by fitting regression lines, as discussed by Basu (1982) in his earlier work. These trend values of rainfall for each of the monsoon month for each year during the monsoon period at Calcutta are shown in Table 3. Regression lines for each of the monsoon month at Calcutta are shown in Figs. 1 to 4 and are given by the following equations

$$Y_c = 292.73 + 2.11\ X_i \quad ; \text{ for June}$$

TABLE 4

Total monsoon rainfall (in mm) during June to September and its trend (in percent) at Calcutta

Year	Rainfall	Trend	% Trend
1961	1103.2	1189.2	93
1962	917.8	1197.1	77
1963	1198.0	1204.9	99
1964	1207.4	1212.8	100
1965	1424.0	1220.7	117
1966	1012.9	1228.6	82
1967	1008.9	1236.5	82
1968	1802.0	1244.3	145
1969	1179.0	1252.2	94
1970	1618.9	1260.1	128
1971	1606.7	1268.0	127
1972	1169.6	1275.9	92
1973	1169.8	1283.7	91
1974	1145.7	1291.6	89
1975	1149.7	1299.5	88
1976	1177.8	1307.4	90
1977	1482.2	1315.3	113
1978	1920.5	1323.1	145
1979	1015.1	1331.0	76
1980	1207.6	1338.9	90
1981	1181.1	1346.8	88
1982	898.9	1354.7	66
1983	1019.0	1362.5	75
1984	1909.5	1370.4	139
1985	1219.6	1378.3	88
1986	1692.1	1386.2	122
1987	1478.7	1394.1	106
1988	1624.8	1401.9	116
1989	1037.6	1409.8	74
1990	1684.6	1417.7	119
1991	1456.6	1425.6	102
1992	1369.7	1433.5	96
1993	1636.6	1441.3	114
1994	1191.7	1449.2	82
1995	1392.3	1457.1	96
1996	1310.3	1465.0	89

$Y_c = 381.17 + 6.32\ X_i$; for July

$Y_c = 344.19 - 1.65\ X_i$; for August

$Y_c = 305.04 + 7.55\ X_i$; for September

Origin is 1978 and X_i units, one year each .

The rainfall amounts and trend values for each of the monsoon month are given in Table 3. The positive slopes indicate the upward tendency of rainfall, while, the negative slope indicates the downward tendency for the same. It is seen from above equations, the highest increasing tendency of rainfall at Calcutta during September month in comparison to June and July months, while, decreasing tendency of rainfall are noticed during the month of August.

Similarly, the trend values of rainfall during the monsoon period (June- September) as a whole, are shown in Fig. 5 and is given by a equation

$$Y_c = 1323.13 + 7.88\ X_i$$

Origin is 1978 and X_i units, one year each.

The monsoon rainfall amounts, trends and percent of trends are given in Table 4. It is seen that the tendency of monsoon rainfall at Calcutta during 36 years of study is increasing in nature for the second half periods after the year 1978 in comparison to first half, prior to 1978

5. Conclusive remarks

The following salient features have been revealed by the qualitative study of daily monsoon rainfall during monsoon period at Calcutta.

(*i*) The total probabilistic behaviour for each of the monsoon month at Calcutta are obtained from one- step transitional matrices by Markovian model and a long- run limiting probability vectors for each of the monsoon month are found out, as discussed above and shown in Table 1. The limiting probability vectors for all states of occurrence of the categorised rainfall for peak monsoon month August at Calcutta are equally probable whereas the same vary for other monsoon months.

(*ii*) The disorderness (entropy) of such probability for each of the categorised classes of rainfall amounts at Calcutta during monsoon period with their weighted entropy values have also been discussed alongwith favourable or unfavourableness of occurrences of each state have been tested by redundancy method, as shown in Table 2. The entropy of the probability is maximum 0.25 for the category rainfall of non-rainy day in July at Calcutta during monsoon period whereas the same is minimum 0.03 for the category rainfall of very heavy rain at Calcutta for all the monsoon months, as shown in Fig.A.

(*iii*) The trend values, increasing or decreasing of rainfall for each of the monsoon month , June to September at Calcutta have been found out by regression method, as discussed above and shown in Figs. 1 to 5. The percent trend of monsoon rainfall during the monsoon period for 36 years at Calcutta have also been found out, which vary year to year from 66% to 145% without any periodicity , as shown in Table 4.

The model, as discussed above, may be applied for probabilistic behaviour study and the trend of monsoon rainfall during monsoon period, as a whole, over any place, irrespective of geomorphic character of the place.

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