MAUSAM, 51, 1 (January 2000), 81-84

551.501.74:530.1

Chaos analysis of pressure parameter over Ahmedabad

KAMALJIT RAY

Meteorological Centre, Ahmedabad – 380012, India

and

B.C. PANDA

Division of Agricultural Physics, IARI, New Delhi-110012,India

(*Received 9 April 1997, Modified 8 January 1998*)

सार - इस अध्ययन में अहमदाबाद स्टेशन के लिए दाब प्राचल के सहसंबंध विमा के ग्रासबर्गर और प्रोकेशिया के निदर्श का उपयोग करते हुए वायुमंडलीय आयामों का पता लगाने का प्रयास किया गया है। वायुमंडलीय गति की निदर्श तैयार करने के लिए आवश्यक सारभूत परिवर्तियों की संख्या के अनुसार न्यून परिसीमा की जानकारी के लिए एकल परिवर्ती काल श्रेणी पर आधारित दाब के खिंचाव के विमों का आकलन किया गया है। यदि मौसमी आँकडों का उपयोग करके अंतः वार्षिक और मौसमी परिवर्तिताओं का अलग कर दिया जाए तो दाब परिवर्ती के पाँच से सात के क्रम के न्यून आयामों का पता चलता है।

ABSTRACT. In the present study attempt has been made to obtain the dimensionality of atmosphere by using Grassberger and Proccacia's model of correlation dimension on pressure parameter for Ahmedabad station. Based on single variable time series, the dimension of pressure attractor is evaluated to obtain a lower bound on the number of essential variables necessary to model atmospheric dynamics. A low dimensionality of the order of five to seven for the pressure variable was obtained if interannual and seasonal variabilities are excluded by using seasonal data.

Key words — Chaos, Strange attractor, Embedding dimension.

1. Introduction

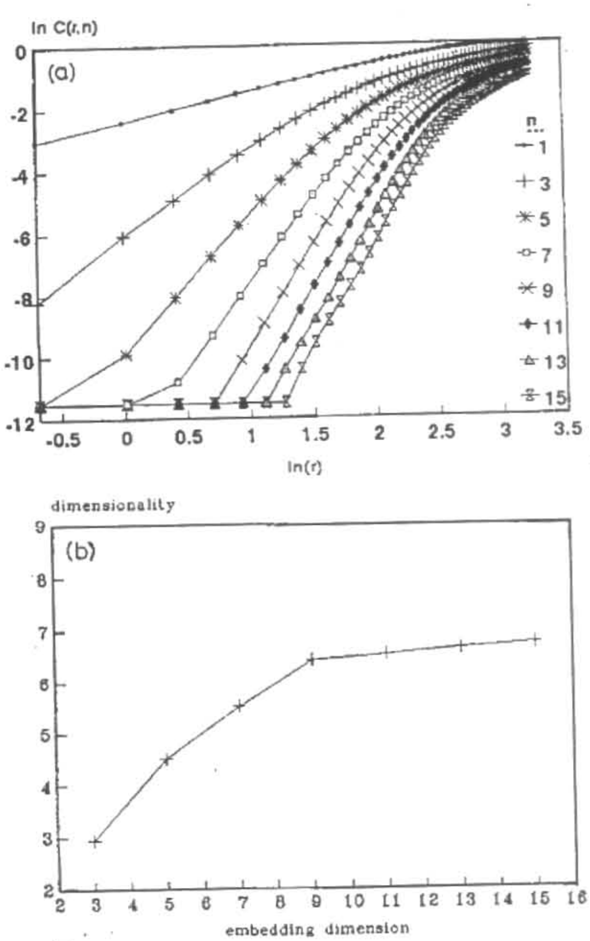
Lot of research is going on to get into the details of variability and predictability of atmospheric flows. Our study is based on phase space approach, which in turn is based on the works of Packard *et al.* (1980), Takens (1981), Grassberger and Procaccia (1983 a, b) and Broomhead and King (1986). According to Packard *et al.* (1980) and Ruelle (1981), if the mathematical formulation of any system is not available, then the information about the system can be deduced from studying the time series x (t) of a single variable, from that system. The theory behind such an approach is that a single variable observable from a dynamical system is the result of all interacting variables and thus any observable variable will contain the information of evolution of the dynamical system. Fraedrich (1986, 1987) used this approach to study the time series of daily pressure at Berlin and oxygen isotope record of platonic species. In this paper study is initiated to go into the details of evolution of any particular weather system over Gujarat region. A time series of sea level pressure for a period of 20 years has been treated to chaos analysis for the estimation of attractor dimension and corresponding embedding dimension (Packard *et al.* 1980, Grassberger and Procaccia 1983 a & b). Using the same method, Satyan (1988) has shown the dimension of annual rainfall around 5.1 and Kulkarni (1991) gave a lower value of around 3.8. Ray and Panda (1995) gave dimensionality value of 3 for temperature time series over short time scales.

2. Materials and methods

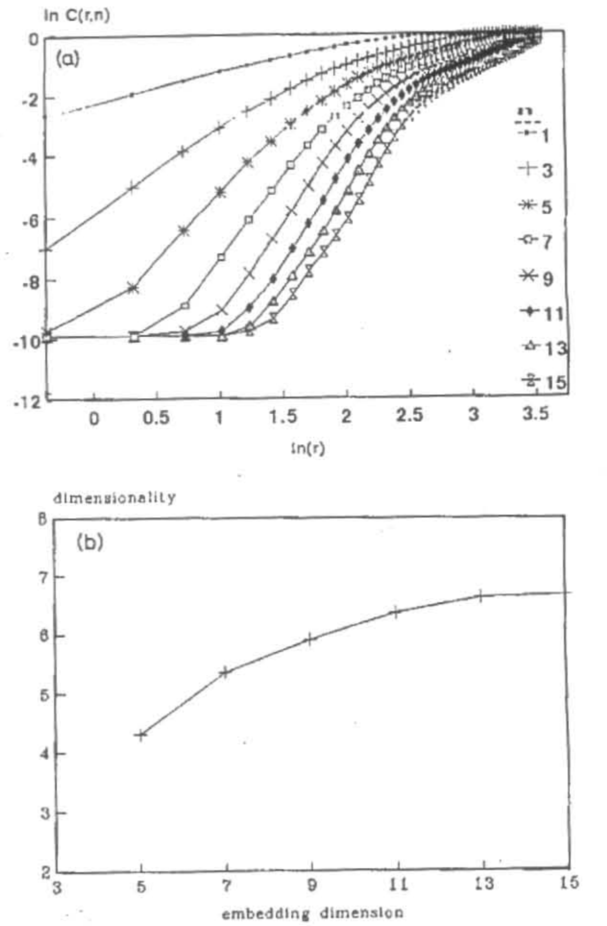
Synoptic data of mean sea level pressure height in geopotential metres (gpm) from 1970 to 1990, collected from IMD, Pune was used for the analysis.

2.1. *Evaluation of correlation function*

The pressure attractor is embedded into an n-dimensional phase space spanned by a time series and its



Figs. 1 (a&b). (a) Dependence of ln C (r, n) on ln (r) for annual pressure time series with increasing embedding dimension (n) and (b) Dimensionality (d) as function of embedding dimension (n) for annual pressure time series



Figs 2. (a&b). (a) Dependence of ln C (r, n) on ln (r) for summer pressure time series with increasing embedding dimension (n) and (b) Dimensionality (d) as function of embedding dimension (n) for summer pressure time series

time shifted values, so that a point in the space is described by :

$$X(t) = f\{ X(t), X(t+1), ...X[t+(n-1)] \} \quad (1)$$

The correlation function was calculated using following equation of Grassberger and Proccacia

$$C(r,n) = \lim\left[\frac{1}{N^2}\sum H\left(r - \left|X_i - X_j\right|\right)\right] \quad (2)$$

where N – Number of points of long time series

H – Heaviside function

$H(y) = 1$ if $y \geq 0$,

$= 0$ if $y < 0$

X_i and X_j – points on attractor

X_i-X_j – Euclidean distance in n-dimensional space

2.2. *Evaluation of scaling exponent*

C (r, n) behaves as a power of r for sufficiently small r, $C(r, n) \alpha\, r^d$

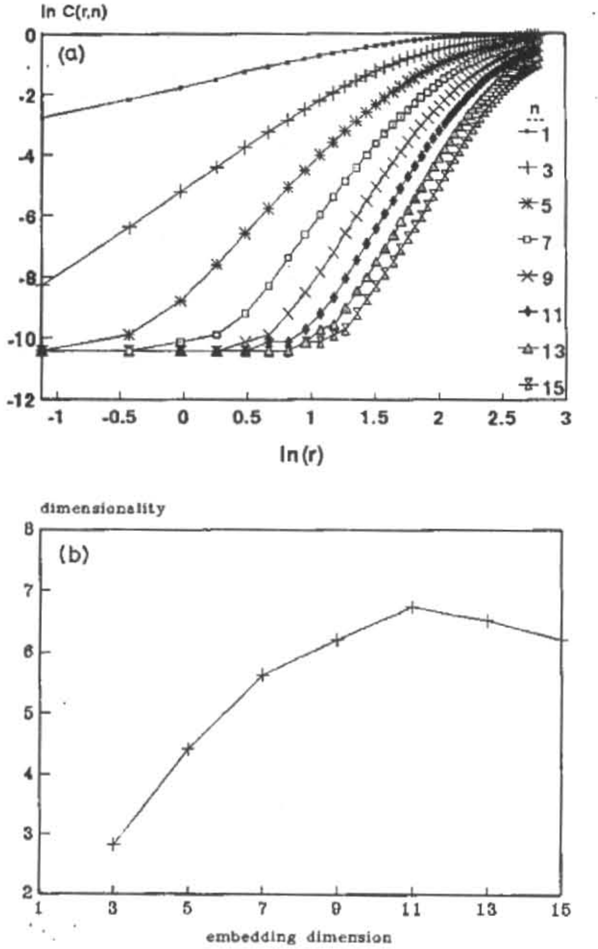
For small values of r, ln (r, n) *vs* ln r curve will have a linear portion. The slope of the linear portion in the ln C (r, n) *vs* ln (r) curve is calculated.

2.3. *Evaluation of dimensionality of attractor D*

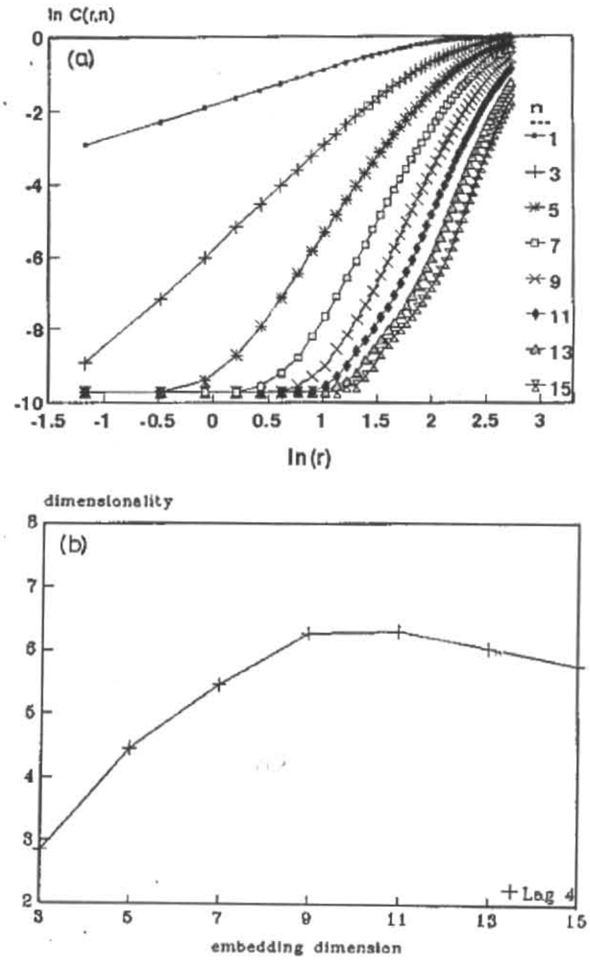
By repeating the above process for increasing n (n = 2, 4, 6,--) we obtain successive estimates of the attractor dimension. If the slope converges to a limiting value $D = d(n) = d(n+1)$, then D is the true correlation dimension of the attractor and the corresponding embedding dimension n is the measure of the no. of variables sufficient to model the dynamics (Fraedrich 1986). A non-integer value of D indicates the presence of a strange attractor (Ruelle and Takens 1971) signifying deterministic but chaotic dynamics. The time series is termed purely random if there is no saturation of slope value.

3. Results and discussion

The correlation function was estimated for t = 4 days for annual and seasonal pressure time series. Fig. 1(a) shows the curve of ln C (r, n) on ln r for increasing embedding dimensions n for a time lag of 4 days t = 4 for daily pressure values for a period of 20 years. In the figure



Figs. 3 (a & b). (a) Dependence of ln C (r, n) on ln (r) for winter pressure time series with increasing embedding dimension (n) and (b) Dimensionality (d) as function of embedding dimension (n) for winter pressure time series



Figs. 4 (a & b). (a) Dependence of ln C (r, n) on ln (r) for monsoon pressure time series with increasing embedding dimension (n) and(b) Dimensionality (d) as function of embedding dimension (n) for monsoon pressure time series

the scaling region is seen to be sandwiched between depopulation and saturation, where depopulation is referred to the lower part of the saturation curve, below which no pair of points exist for the r separation and at the other extremes when the value of r exceeds the set diameter, the correlation function increases no further and hence it is saturated.

The dimension of the attractor increases with each further coordinate added to the state space *i.e.* with increasing embedding dimension n [Fig. 1 (b)]. Even at embedding dimension n = 15 to 20, there was no indication that the pressure attractor is sufficiently embedded and has reached its limiting dimension indicating the randomness of time series. This can also be explained in terms that annual time series includes different weather phenomena in different seasons and thus a number of factors could be said to be guiding the pressure variations on annual basis.

In order to remove inter annual variability and season to season variation, pressure parameter records were reduced to separate seasonal samples of 19 winters, 20 summers and 20 monsoons, each season lasting for four months commencing on 1 November, 1 March and 1 June respectively. The correlation functions were individually evaluated for each season so as to discard the long range processes and inter annual variability. Scaling the weather attractor described by the trajectories of the seasonal time series led to different results. Fig 2(a) shows the dependence of ln C (r, n) on ln r for increasing embedding dimensions n for a summer pressure time series at a time lag t=4. In this figure, again, the scaling region is sandwiched between depopulation and saturation, but for higher value of embedding dimension, saturation was seen to be approached. The situation becomes clear from Fig. 2(b), which shows the scaling exponent (d) of the summer pressure attractors as a function of number n of the state space coordinates. The value of d increases with the increase in embedding dimension (n) and for n=12, value of slope saturates to an attractor dimension d equals to 6.75. The observed fractal dimensionality indicates the deterministic chaos of the weather system with its supposed sensitivity on initial conditions.

For winter season pressure time series, in the plot of $\ln C(r, n)$ *Vs* $\ln r$ for increasing embedding dimensions n [Fig. 3 (a)], the scaling region showed a shift towards higher value with the increase in the state space coordinates. An attractor dimension of $D \approx 6.75$, was obtained for an embedding dimension (n) of 11 [Fig. 3(b)].

The month of June was overlapping for both monsoon and summer seasons and hence it was included for the analysis of both seasons. Although the onset time of monsoon for Ahmedabad is in the later part of June, still the complete season was defined from June to September. The monsoon pressure attractor had a dimensionality of $D \approx 6.25$ for an embedding dimension $n \approx 11$ (Fig. 4). Similar results were obtained by Fraedrich (1987) for a 15 year time series of daily surface pressure. They explained this by saying that annual time series includes weather phenomena in winter and summer, the long range processes from season to season and the inter annual variability. A dynamical system which included all these aspects, was dependent on a large number of independent variables and inherent noise and thus saturation was not obtained even for increase in coordinates upto twenty.

The results of this analysis that summer, winter and monsoon separately provided small dimensions but the annual data did not, seemed to be valid, considering that there were two to three distinct regimes of flow for which the value of a parameter was changing between seasons.

4. Conclusions

Based on single variable time series the dimension of pressure is evaluated using Grassberger and Proccacia (1983a,b) developed from theory of chaos to obtain a lower bound on the number of essential variables needed to model the dynamics. Furthermore, an upper bound for the number of variables sufficient to model the dynamics of the attractor are also estimated. The procedure is based on the cumulative distribution of the distances of all data points (time trajectories) as they occur in phase spaces. They are spanned by an increasing number of independent coordinates to obtain a sufficient embedding of the attractor of the dynamical system; the coordinates are defined by the single variable time series and its time lags.

We observed a low dimensionality (between five and seven) of the weather attractor, if inter annual variability and seasonal variations are excluded by using seasonal data sets. The observed fractal dimensionality accounts for a chaotic dynamical system and its strong dependence on initial conditions and that pressure as a parameter evolves on a low dimensional attractor after inter-annual variability and season to season variations have been excluded, by using only seasonal data sets. An embedding dimension between 11 and 13 can be interpreted as the upper limit for the number of variables guiding the pressure parameter. Indirectly it can be inferred that forecast of atmospheric conditions for more than 10 to 13 days are very sensitively controlled by initial conditions and difficult to assess.

References

Broomhead, D.S. and King, G. P., 1986, " Extracting qualitative dynamics from experimental data", *Physica* , 217-236.

Fraedrich, K., 1986, " Estimating the dimension of weather and climate attractors", *J. Atmos. Sci.*, **43**, 419-432.

Frae drich, K., 1987, " Estimating weather and climate predictability on attractors", *J.Atmos. Sci.*, **44**, 722-728.

Grassberger, P. and Procaccia, I., 1983a, " Characterization of strange attractors", *Phys. Rev. Lett.*, **50**, 346-349.

Grassberger, P. and Procaccia, I., 1983b, " Measuring the strangeness of strange attractors", *Physica*, **90**, 189-208.

Kulkarni,T.R., 1991, "Monsoon subdivisional rainfall references: Dimensionality and predictability", *Adv. Atmos. Sci.*, **8(3)**, 351-356.

Packard, N.N., Crutchfield, J.P., Farmir, J.D. and Shaw, R.S., 1980, "Geometry from a time series", *Phys. Rev. Lett.* **45**, 712-716.

Ray, Kamaljit and Panda, B.C., 1995, "Estimation of attractor dimension of a temperature time series over very short time scales", Proc. TROPMET.-95, NRSA, Hyderabad.

Ruelle, D., 1981, "Chemical kinetics and differentiable dynamical systems", In Non-Linear phenomena in chemical dynamics (Pacault, A. and Vidal, C.Eds.), Springer-Verlag, 30-37.

Ruelle, D. and Takens, F., 1971, "On the nature of turbulence", *Commun. Math. Phys.*, **20**, 167-192.

Satyan, V., 1988, "Is there an attractor for the Indian Summer monsoon", *Proc. Indian Acad. Sci. (Earth Planet Sci.)*, **97**, 49-52.

Takens, F., 1981, "Detecting strange attractors in turbulence", In Dynamical System and Turbulence, Warwick, Springer-Verlag, 366-381.