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Chaotic dynamics in Parkfield region, California and characteristics earthquakes

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सार — वर्ष 1969-1987 के दौरान पार्कफील्ड संजाल के माध्यम से पता लगाये गये केलिफोर्निया क्षेत्र में आये लगभग 75000 भूकम्पों के दौरान घटित हुए विप्लवों का परीक्षण दो विभिन्न पद्धतियों नामतः विलक्षण आकर्षक परिमाप और लयापूनव घातांक द्वारा किया गया। इस क्षेत्र में विलक्षण आकर्षक परिमाप 6.3 पाया गया जो भूकम्प प्रागुक्तिता के लिए कम से कम 7 प्राचलों का संकेत देते हैं। 0.045 के लघु सकारात्मक लयापूनव घातांक क्षेत्र में आगामी निर्धारणात्मक विप्लवों के लिए प्रमाण उपलब्ध कराते हैं जो आरम्भिक अवस्थाओं में प्रबल निर्भरता दर्शाते हैं। विशिष्ट पार्कफील्ड भूकम्पों की विप्लवकारी गतिकी की विवक्षाओं पर विचार-विमर्श किया गया। क्षेत्र का विलक्षण आकर्षक परिमाप प्लेट परिसीमा के रूपांतर किस्म का प्रतीक हो सकता है जो उत्तरीपश्चिमी हिमालय क्षेत्र के हिंदुकुश के निकट बताई गई महाद्वीपीय संघट्ट किस्म की प्लेट परिसीमा के विवरणों की तुलना में निम्न स्तर का है।

ABSTRACT. Based on about 75000 earthquakes in the California region detected through Parkfield network during the years 1969- 1987, the occurrence of chaos was examined by two different approaches, namely, strange attractor dimension and the Lyapunov exponent. The strange attractor dimension was found as 6.3 in this region suggesting atleast 7 parameters for earthquake predictability. Small positive Lyapunov exponent of 0.045 provided further evidence for deterministic chaos in the region which showed strong dependence on the initial conditions. Implications of chaotic dynamics on characteristic Parkfield earthquakes has been discussed. The strange attractor dimension in the region could be representative for the Transform type of plate boundary which is lower than that reported for continent collision type of plate boundary which is lower than that reported for continent collision type of plate boundary near Hindukush northwest Himalayan region.

Key words— Chaos, Strange attractor, California earthquake.

1. Introduction

Parkfield, California region is considered to be one of the most interesting zones in the world due to the recurrence of moderate size characteristic earthquakes (Bakun and Mc Evilly, 1984) repeating the same epicentre, magnitude, seismic moment, rupture area and southeast direction of rupture expansion suggesting a strictly periodic process. On the basis of this model and absence of any earthquake since 1966, the next characteristic Parkfield earthquake was predicted to occur between 1983 and 1993. This prediction did not materialise. Due to different inter-event intervals of 12 and 32 years between 1922, 1934 and 1966 Parkfield earthquakes, their occurrence appears to be more complex than originally envisaged.

Recent developments in chaos theory enable us to examine predictability of earthquakes in this region. Although Horowitz (1989) and Julian (1990) reported that a strange attractor exists in the Parkfield region of California, but Beltrami and Mareschal (1993) did not find any evidence of a chaotic process in the region through generation of a random series using seismic energy release of earthquake. They suggested that the occurrence of earthquakes in this region is random or had a strange attractor dimension larger than 12 implying inherent limitations in evolving a predictive model. In view of these contradictory results, the objective of this paper is to examine earthquake predictability using two different approaches namely Lyapunov exponent and strange attractor dimension. The results being representative of a transform type plate boundary near San Andreas fault, have been compared with the collision type boundary near Indian Eurasian plates.

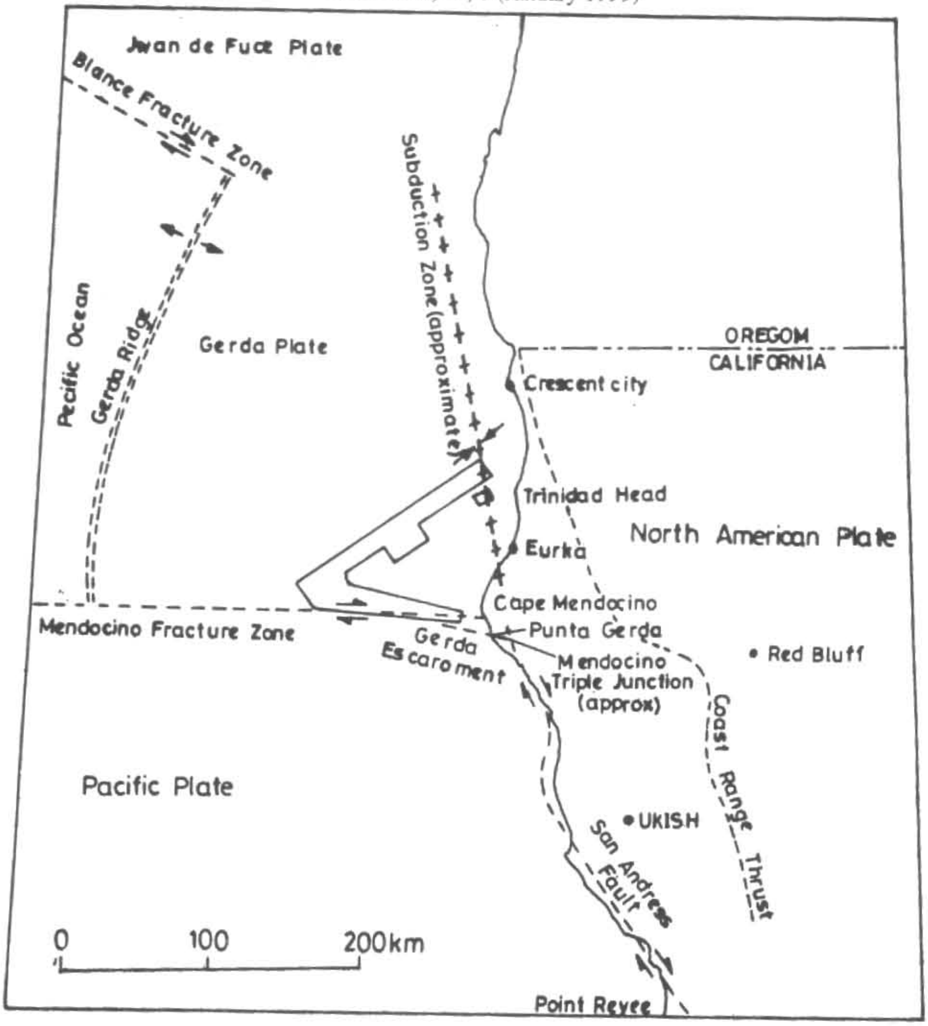


Fig. 1. Plate boundaries near San Andreas fault system, California (After Eaton, 1988)

2. Geotectonics and seismicity

Plate tectonics model suggested relative motion of the north American, Pacific and Juan de Fuca plates along their common boundaries in northern California namely the San Andreas fault (transform type), the Mendocino fracture zone and the coastal subduction zone north of Cape Mendocino. Contemporary seismicity has brought to light many interesting features pertaining to the plate boundary and the faults in the region, namely the branching and spacing of major strike slip faults and relatively aseismic zone between the actively creeping San Andreas fault and the reverse fault earthquakes along both the east and west flank of the ranges (Fig. 1). The section of San Andreas fault near Parkfield is tectonically more interesting because the great 1857 earthquake in southern California was preceded by foreshocks with their epicentres near Parkfield. Correlation of seismicity and discontinuities or bends in the mapped fault trace provided the bash for an extension and refinement of the "stuck" and "creeping" patch model of the San Andreas fault in central California.

Keeping in view that Parkfield area has been subjected to six earthquakes between 1857 and 1966 with an average repetition rate of 22 years, and other studies, and official earthquake prediction made by the U.S. Geological Survey (according to which a moderate earthquake (M 6) was likely to occur before 1993 in this region), an extensive network of monitoring programme was, therefore, taken up in this segment as a support to the prediction of earthquakes of Central California (Fig. 2). Such a programme could have sufficient justification if predictability of earthquakes could be examined using deterministic chaos.

3. Methodology and data

(i) Strange attractor dimension

We adopt Grassberger and Procaccia (1983) method to estimate from a statistical analysis of the distances between pairs of points on the trajectory and then the dimension of the attractor. Details, are given by Srivastava *et al.* (1994) and Bhattacharya *et al.* (1995).

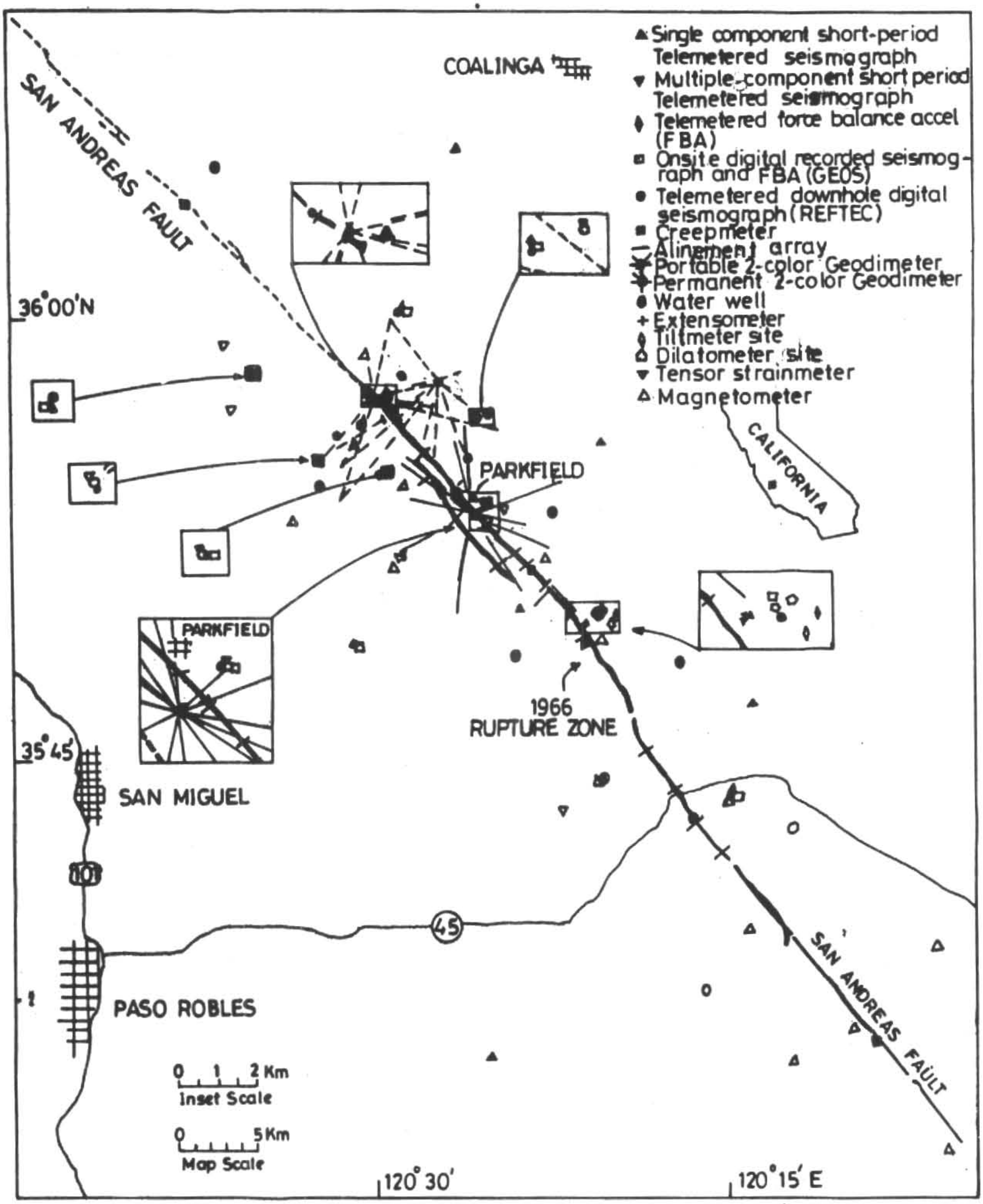


Fig. 2. Monitoring network in central California

(ii) Lyapunov exponent

We consider dynamics of earthquakes in the (n=I) dimensional phase

$$\frac{dx}{dt} = f(x) \tag{1}$$

Small deviations δx from the non-linear time evolution follow the related differential equation

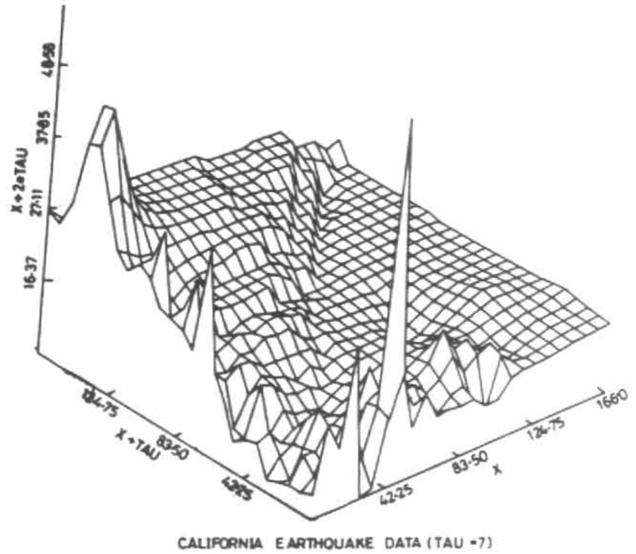


Fig. 3. Phase space diagram (3-D) for the earthquake occurrence in California region

$$\frac{d}{dt}(\delta x) = \frac{df}{dx}\Big|_{x_0}(\delta x) \qquad (2)$$

Which is linearised about the state $x_0 = x(t_0)$ at time t_0. These deviations grow or shrink exponentially with time $t = t_0 + mr$ increasing by m steps of duration T.

$$\delta x = \delta x_0 \exp(Lmv) \qquad (3)$$

The characteristic exponent (or Eigen value) L is defined by the Jacobian

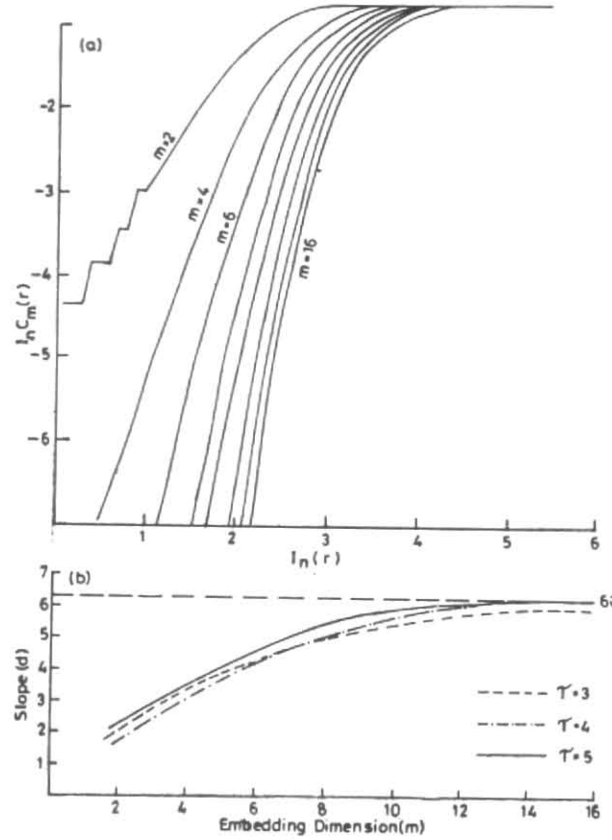
$$L = \frac{df}{dx} \quad \text{or} \quad d\ ln\ \frac{dx}{dt} \qquad (4)$$

If $L < 0$ (or >0), the system (2) is stable (unstable), $D=(\delta x)$ being small or grows exponentially.

We can generalise the concept for trajectories in one dimensional phase space to the mean exponential rate of divergence of two initially close trajectories in n dimensional phase through

$$L(x_0, \delta x_0) = \lim_{m \to \infty} \frac{1}{m\tau}\ ln|\delta x| \qquad (5)$$

Here, L takes one of n values $L_1, L_2 ......Ln$ which in general would be largest. We can relate n Eigen vectors of the Jacobian matrix $\frac{\delta fi}{\delta xi}$ giving several values of Lyapunov exponents. The rate of the exponential growth of an infinitesimal vector $\delta x(t)$ in the n-dimensional phase space is given by the largest of the Lyapunov characteristic exponents L. Positive value of Lyapunov exponent describes a direction in which the two nearby trajectories diverge showing strong sensitiveness to initial conditions (Wolf *et al.* 1985).



Figs.4(a&b). (a) Distance dependence of the correlation function for a sequence of embedding dimensions in California region (b) Dimensionality D of the attractor as a function of embedding dimensions

The dense network of seismographic stations operated by the U.S. Geological Survey enables us to detect microearthquake activity along the San Andreas fault system in central California. We have used 75000 earthquakes monitored by the Parkfield network California during the years 1969 to 1987. Fig. 3 shows the phase space diagram for this region which suggests complexity of earthquake occurrence in a qualitative manner. We used number of earthquake every two days.

It may be mentioned that care was taken to avoid spurious results being obtained because the number of earthquakes N, fulfilled the criterion $2 \log_{10} N \geq D$ where, D is the fractal dimension (Ruelle, 1990). Also, the delay time in the chaotic time series was kept below the upper bound as suggested by Lai *et al.* (1966). The basic idea in this criterion is to choose two delay so that the coordinates Xn and $Xn+1$ are independent of each other but not completely uncorrelated.

4. Results and discussion

Fig. 4(a) shows the plot of Cm(r) versus log (r) for embedding dimensions from 2 to 16 for time lag of 3 days. Slope is computed from the straight line portion marked (xx) for various embedding dimensions. The plot of slope versus corresponding dimension is shown in Fig. 4(b) for time lags of 3,4 and 5 days. It may be noted that saturation takes place for consecutive time delays of 4 and 5 days, giving strange attractor dimension of 6.3. This suggests that atleast 7 parameters are needed for the predictability of earthquakes in the region. However, Horowitz (1989) found that the underlying structure can be represented with only six degrees of freedom based on lesser data of 15196 earthquakes. Further evidence of chaotic process in the region was provided through this study which found a small positive value of the largest Lyapunov exponent (095).

Based on slider block models coupled to each other, Huang and Turcotte (1990) brought out some features of Parkfield seismicity. Ryapov and Ito (1996) studied the dynamic behaviour of a system composed of two elastically coupled blocks on a moving rough plate and surmised that some of the characteristics in the fault model originate in the non-linearity of a dynamical system with a few degrees of freedom. Such developments, though based on much simple models vis-a-vis complex fault system nevertheless support chaotic dynamics.

It is surmised that a strange attractor dimension of 6.3 could be representative of a transform type of plate boundary. However, higher strange attractor dimensions of 6.9 and 9.8 have been reported near Hindukush (Bhattacharya and Srivastava, 1992) and northwest Himalayas (Bhattacharya, 1990). On the other hand, Pavlos *et al.* (1994) reported a strange attractor dimension of 2.4 in the sub-duction zone of a Eurasian Pacific plate boundary near Japan. The results being representative of limited regions could be considered preliminary to distinguish different type of plate boundaries on the basis of strange attractor dimensions.

It may be mentioned that non-chaotic earthquake occurrence was found near Oroville reservoir (Srivastava *et al.* 1996). This was in contrast to the results reported for Koyna (Srivastava *et al.* 1994), Aswan and Nurek reservoirs (Srivastava *et al.* 1995) where evidence of deterministic chaos in the occurrence of earthquakes was reported.

It is interesting to note that while describing recurring moderate size characteristic earthquakes in Parkfield region, Bakun and Mc Evilly (1998) noted that the seismic moment for 1966 earthquake was 20% greater than that for the earlier events and an unexplained 10 year advance of the 1934 could be considered as an exception. Occurrence of chaotic dynamics in the region would make the hypothesis of "Characteristic earthquakes" or strict periodicity less acceptable. This is corroborated by Mc Closkey and Bean (1994) who reported that models in which fractal sealing is broken at high magnitudes predict that characteristic events and recurrence behaviour will be unstable in time. Application of their model also suggests that a small failure on one block in this region may trigger a characteristic earthquake on the other. Thus sensitivity of the double block model to changes in the initial conditions as found by the positive value of the largest Lyapunov exponent would render the "Predicted" time of occurrence of large earthquakes less reliable. In other words, while the chaotic dynamics may enable us short-term predictability of characteristic earthquakes of magnitude 6, the prediction of a great earthquake in southern California due to trigering effect of Parkfield earthquakes caused by local fluctuations of strain accumulation would remain uncertain. In other words, strong sensitivity to the initial conditions in the chaotic regime would be governed by the temporal fluctuations of seismicity in Parkfield region.

Recent developments in chaotic dynamics have raised some questions about premonitory seismicity pattern (Shaw *et al.* 1992) for this region. Srivastava *et al.* (1996) have shown that the strange attractor dimension remains constant within 2 and 4 around Shillong in northeast India justifying to use earthquakes upto epicentral distance of 450 km. for delineation of seismicity pattern in northeast India. However, Singh *et al.* (1996) have found that if a chotic system can be compounded through another chaotic system, higher strange attractor dimensions could result vis-a-vis individual fault system. This concept may remain valid even if we consider characteristic Parkfield earthquakes as a periodic system because by compounding through another chaotic system, the overall dynamics is manifested as chaotic. Future studies may resolve these aspects for California region.

5. Conclusions

The present study has brought out the following results:

(*i*) The largest Lyapunov exponent being a small positive value and the strange attractor dimensions of 6.3 in California region unequivocally suggests the existence of deterministic chaos in the region. At least 7 parameters are needed for earthquake predictability in this region.

(*ii*) Strange attractor dimension is relatively lower in Parkfield, California region near transform type of plate boundary as compared to northwest India close to Indian Eurasian plate boundary. It is, however, much larger as compared to sub-duction type of plate boundary near Pacific. The results being preliminary need validation from different type of plate boundaries in other regions.

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