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Fourier analysis of weekly soil moisture at Pune

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सार—केन्द्रीय कृषि वैधशाला, पुणे में 1958 से 1989 के दौरान एकत्रित किए गए साप्ताहिक मृदा आर्द्रता आंकड़ों का फोरियर विश्लेषण किया गया है। सामान्य आंकड़ों के प्रयोग से यह देखा गया कि पहले हार्मोनिक में जल की विभिन्न गहराईयों का आयाम (विस्तार) 0.76 से 1.12 सें० मी० तक होता है और उच्च हार्मोनिक में यह बहुत तीव्रता से घट जाता है। चौथे हार्मोनिक के मान, जल में केवल 0.03 से 0.07 सें० मी० के बीच होते हैं। तथापि, मृदा की गहराई पर निर्भर किए बिना 1984, 1985, 1987 और 1989 में उच्चतम विस्तार 0.45 से 1.35 सें० मी० तक पाया गया। सामान्य परिस्थितियों में मृदा सतह (7.5 सें० मी० की गहराई) के निकट अधिकतम मृदा आर्द्रता 30 सें० मी० की गहराई में 15 सितम्बर को पाई गई है और अधिकतम मृदा आर्द्रता 9 दिन के बाद अर्थात् 24 सितम्बर को पाई जाती है।

इस अध्ययन में विश्लेषित की गई चार मृदा गहराईयों में पहली, दूसरी तथा तीसरी हार्मोनिक में क्रमश: वर्ष 1984, 1985, 1987, 1989 के आंकड़ों एंव सामान्य आंकड़ों में कुल विसंगति के क्रमश: 9–46, 2–32 एवं 4–26 एवं 67–86, 6–14 और 1–6 प्रतिशत मान पाए गए। दक्षिण-पश्चिमी तथा उत्तर पूर्वी मानसून आने के कारण जून एंव अक्तूबर के बाद, मृदा आर्द्रता में परिवर्तन भी स्पष्ट रूप में प्रकट हुआ है।

ABSTRACT. The weekly soil moisture data collected at Central Agromet Observatory (CAgMO), Pune during 1958-1989 have been subjected to Fourier analysis. With normal data the amplitudes for various depths vary between 0.76 to 1.12 cm of water for first harmonic and decrease sharply with higher order harmonics. The values for fourth harmonic range from 0.03 to 0.07 cm of water only. However, the highest amplitude for the calendar years 1984, 1985, 1987 and 1989 is found to vary between 0.45 to 1.35 cm of water irrespective of soil depths. Under normal conditions the maximum soil moisture near the soil surface (7.5 cm depth) occurs on 15 September while at 30 cm depth, the soil moisture maximum occurs on 24 September, a delay of 9 days.

For the four soil depths considered in this study, the first, second and third harmonics represent respectively 9-46, 2-32 & 4-26 & 67-86, 6-14 and 1-6% of total variance respectively for each of the years 1984, 1985, 1987 and 1989 and normal data. The change in soil moisture pattern after June and October due to occurrence of southwest and northeast monsoon is well reflected.

Key words — Soil moisture, Fourier analysis, Soil-climate, Soil moisture cycles.

1. Introduction

The soil moisture under bare soil condition like the soil temperature, varies in a nearly regular pattern reflecting the annual cycle of rainfall. This cycle, however, is somewhat modified due to the intra-seasonal variation of rainfall caused by different synoptic situations. The observed soil moisture variation for various depths at the CAgMO, Pune indicates that the pattern is not a symmetrical one. Considerable variation is produced by the occasional thundershowers in the pre-monsoon season and the intra-seasonal variation of rainfall during southwest monsoon season.

Small but irregular fluctuations of meteorological and agrometeorological parameters are normally reduced by averaging the data series. However, the averaging (over periods ≥ 10 years) reduces, but does not eliminate, these deviations, particularly for soil related parameters down through the soil profile (Carson 1961). In such cases, the averaged data may be further analysed by means of Fourier analysis, reducing the soil moisture *versus* time curves to a series of Fourier coefficients. These coefficients give an objective description of the variation with depth of the amplitude of the soil moisture wave and the time location of soil moisture extremes. Several scientists (Pearce 1958, Pearce and Gold 1959, Carson 1963) have reported that the annual cycle of various meteorological and agrometeorological parameters is fairly well described by the first harmonic alone. On the other hand,others have also reported that first harmonic alone is not sufficient to describe the annual cycle (Lettau 1954, Krishnan and Kushwaha 1972) and that the effect of higher harmonics have to be considered. In this study, an attempt has been made to represent the actual observed variation of soil moisture from that of predicted one from first five harmonics for four selected years, *viz*., 1984, 1985, 1987 and 1989 which have guaged either excess, normal or deficient rainfall. The effect of higher harmonics have also been considered for representing the actual observed variation of normal soil moisture.

TABLE 1

Harmonic components of soil moisture at different depths in respect of first three harmonics

Year	Soil depth (cm)	Mean soil moisture	Harmonic I			Harmonic II		Harmonic III	
			Amplitude	Variance	Date corresponding to the maximum degree	Amplitude	Variance	Amplitude	Variance
1984	7.5	1.54	0.55	17	10 Sep	0.65	24	0.68	26
	15	1.92	0.70	23	21 Sep	0.74	25	0.73	25
	30	2.67	0.94	31	4 Oct	0.74	19	0.76	20
	45	2.77	0.95	32	10 Oct	0.75	20	0.73	19
1985	7.5	1.43	1.14	38	8 Sep	0.32	3	0.68	14
	15	1.77	1.28	42	18 Sep	0.32	3	0.66	11
	30	3.00	1.35	33	8 Oct	0.43	3	0.57	5
	45	3.08	1.19	29	9 Oct	0.28	2	0.58	7
1987	7.5	1.71	0.37	9	11 Oct	0.70	32	0.31	6
	15	2.15	0.38	11	22 Oct	0·42	14	0.33	9
	30	3.07	0.45	14	25 Nov	0.17	2	0.14	11
	45	3.04	0.43	14	22 Nov	0.26	5	0.32	7
1989	7.5	1.34	0.90	45	13 Aug	0.36	7	0.32	6
	15	1.60	0.66	25	12 Aug	0.56	19	0.40	9
	30	2.60	0.86	25	30 Aug	0.55	10	0.54	10
	45	2.70	0.82	24	31 Aug	0.61	14	0.33	4
Normal (1958-80)	7.5	2.02	1.03	83	15 Sep	0.28	6	0.20	3
	15	2.59	1.12	86	17 Sep	0.35	8	0.11	1
	30	3.34	0.77	70	24 Sep	0.35	14	0.18	4
	45	3.43	0.76	67	22 Sep	0.33	12	0.23	6

2. Data

The weekly soil moisture observations are taken from 7.5, 15, 30 and 45 cm depths by gravimetric method at CAgMO, Pune. The soil at the experimental site is vertisol and extend upto 60 cm depth. The soil texture is more or less uniform with a nearly constant bulk density of 1.0-1.1 g/cm³. The experimental site is always kept free of vegetation. The soil moisture observations are taken at about 0800 IST. The data used in this study are those collected in the calendar years 1984, 1985, 1987 and 1989. The wide range of variation in soil moisture is primarily due to variation in rainfall.

Available weekly soil moisture data for the years 1958-80 are averaged to obtain a mean soil moisture pattern for the 52 standard weeks of the year. Data for some of the standard weeks for some of the years considered for this study are missing. But on an average mean weekly values are based on 10-18 years data except for the standard week No. 39 when it is based on 9 years average. Soil moisture (%) values are converted to cm of water by multiplying with bulk density and depth interval. This provides the root-zone water content in cm of water which is comparable to indicating the amount of rainfall/irrigation water in length unit.

3. Method of analysis

Since the computational procedures of Fourier analysis are well-known (Conrad and Pollak 1950, Panofsky and Brier 1958), only a brief summary is included here. Any time series can be represented by a series of sine and cosine functions. These series consisting of finite number of equally spaced data points can be accounted for by a finite number of sine and cosine terms in a Fourier analysis. For example, the weekly soil moisture (in cm of water) at various depths considered for the present study are periodic functions. The general form of the Fourier series is :

$$SM = \overline{SM} + \sum_{n=1}^{N/2} \left(A_n \cos \frac{360}{P} nx + B_n \sin \frac{360}{P} nx \right) \qquad (1)$$

where, $\overline{SM}$ is the mean of the 52 weekly averages for the year, N is the number of equally spaced data points which is 52 in this case, A_n and B_n are the amplitudes (half range) of the various terms, P is the period of the fundamental cycle, x is the time factor varying from 0 to 51. A more convenient representation of a Fourier analysis is :

$$SM = \overline{SM} + \sum_{n=1}^{N/2} C_n \sin \left(\frac{360}{P} nx + \phi_n \right) \qquad (2)$$

where, C_n is the amplitude of the nth harmonic and is given by $(A_n^2 + B_n^2)^{\frac{1}{2}}$ and ϕ_n is the phase angle given by $\tan^{-1} (A_n/B_n)$.

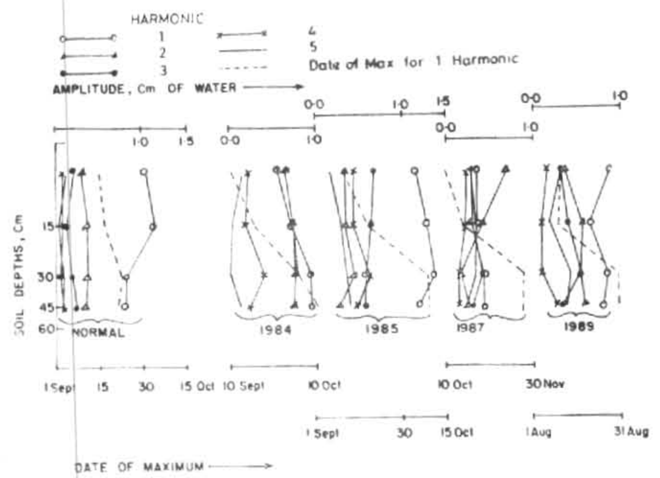

Fig. 1. Amplitude and date of maximum value of annual soil moisture cycle

4. Results and discussion

4.1. *Harmonic constants*

The values of amplitude and phase angle in respect of the first three harmonics fitted to weekly soil moistures at various depths are presented in Table 1 along with the mean soil moisture, for each of the years and the long-term mean. The largest amplitudes are found at deeper depths (*viz.*, 30 and 45 cm) in all the years except 1989, when it was maximum at 7.5 cm depth. However, with normal soil moisture data the largest amplitude is found at 15 cm depth for the first and second harmonics. In general, the annual wave has its largest amplitude at all soil depths. The amplitude remained around 0.5 cm of water and for most of the depths, except in the year 1987 when it was below 0.5 cm. The low amplitudes at all the depths during 1987 are associated with uneven distribution of rainfall experienced during the year. The heavy and even distribution of rainfall during 1984 is conspicuous with relatively large amplitude of about 1.0 cm of water. However, even with below normal rainfall (annual rainfall = 56.9 cm) during the year 1985, even distribution of rainfall has helped greater recharge as well as large amplitude of soil moisture (1.14-1.35 cm of water). On the contrary, during the year 1987 even with above normal rainfall (annual rainfall = 71.5 cm) at Pune, the amplitudes of the annual cycle of soil moisture at all the soil depths are small, much smaller than that observed during 1984, 1985, 1989 and with the normal data. The amplitude values in Table 1 shows that the years 1984 and 1987 are rather unusual because the amplitudes of the semi-annual cycle at 7.5 and 15 cm depths are larger than those of annual cycle. In the year 1985, high soil moisture has created soil moisture waves of considerable amplitude at all depths. The amplitudes of the annual, semi-annual and third harmonics were more or less same and ranged between 0.55 and 0.95. These variations may be due to the fact that soil moisture is not fully recharged in most of the years. With normal data, the values of the the first and fourth harmonics range from 1.12-0.76 and 0.03-0.07 cm of water respectively. Fig 1 is a plot of the amplitude variation with depth of the first five harmonics. The relative magnitude of the various harmonics falls more into line with the expected pattern at the deeper soil depths. The figure also shows that the moisture cycle in the soil follows the expected pattern of a damped, lagging wave.

The maximum value of any harmonic occurs when $[(360/P)\,nx + \phi_n] = 90°$. Thus for $n = 1$, various values of $(360/P)\,nx$ corresponding to the maximum values for varying depths from 7.5 to 45 cm have been computed. These angles are appropriately converted to obtain the dates of the first maxima which are presented in Table 1. During the calendar years 1984, 1985, 1987 and 1989 and with normal data, the annual wave attains maximum amplitudes between the months August and November. The dates of occurrence of maximum soil moisture near the soil surface, *i.e.*, at 7.5 cm depth are 37th, 36th, 41st and 33rd standard week respectively in 1984, 1985, 1987 and 1989 while at a depth of 45 cm the soil moisture maximum is delayed by about 4 weeks except during the year 1987 and 1989 when it was delayed by 6 and 3 weeks respectively. With normal data the maximum soil moisture near the soil surface is that recorded on 37th standard week while at depths of 30 cm and 45 cm the soil moisture maximum is delayed by about one week.

Fig. 1 shows the date on which the first harmonic reached its maximum value. This figure shows that the soil moisture cycle under Pune condition follows the expected pattern of a damped, lagging wave. In general, the time of maximum soil moisture at 7.5 cm depth is in the first fortnight of September, however with some exception in the years 1987 and 1989 when it is in the first fortnight of October and August respectively. At 15 cm depth the soil moisture maximum is delayed by about one week. At 45 cm depth the soil moisture maximum is delayed by approximately 4 weeks for the calendar years 1984 and 1985. It is interesting to note that for the calendar year 1987, not only the maximum reached much later than the normal date of occurrence of maximum but also at 45 cm depth the maximum occurred a few days earlier than that at 30 cm depth. This difference may be attributed due to less diffusion of soil moisture between 30 and 45 cm depths under less water potential gradient which becomes common feature under normal rainfall condition. The high water holding capacity of vertisol which does not reach saturation particularly at deeper levels is also reflected in this process. The inability of southwest monsoon rainfall to saturate the soil moisture profile in the month of June-July indicates that early-sown rabi crop or late-sown kharif crop would be more suitable for rainfed farming in this semi-arid tract. It is observed in the Deccan Plateau that the soil moisture storage capacity is the limiting factor for the success of rabi crops which are grown under receding soil moisture conditions. Limited soil depths put limitations on soil moisture storage capacities which ultimately affect the production. Under such a condition, it has been observed that by advancing the sowing date of rabi crops to early September, the accumulating and receding, soil moisture conditions are utilized which increases the production by about 100 per cent probably due to efficient utilization of fertilizer and native nutrients from the soil (Patil *et al.* 1981). Working with 5 districts in drought prone region of Maharashtra, Samui and Jog (1983) also observed that primary peak of water availability occurs from 37th to 39th standard weeks at Pune. This confirm

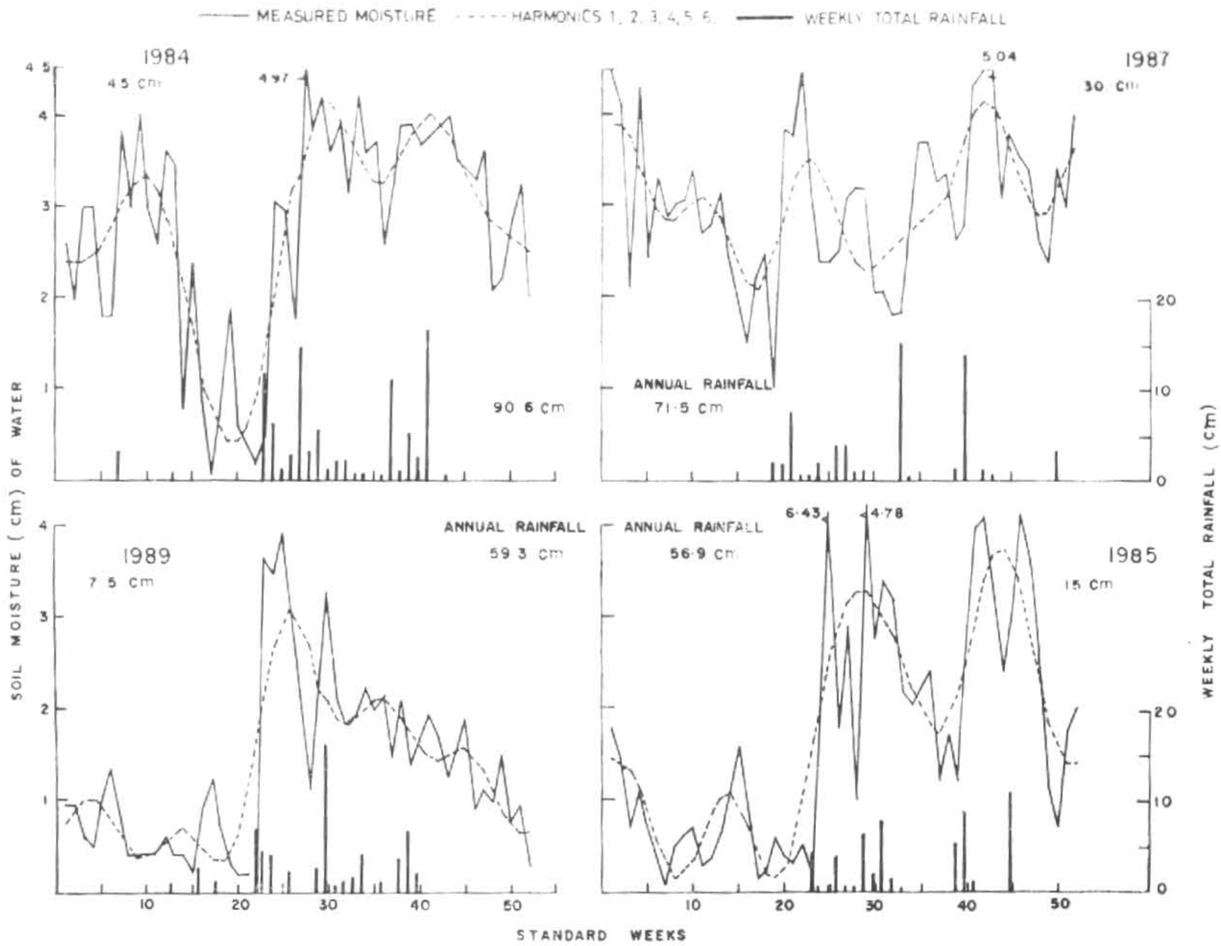

Fig. 2. Weekly soil moisture at different depths and that predicted by first five harmonics alongwith weekly total rainfall

that under normal conditions early sown rabi crops would be in a position to utilize the accumulating and receding soil moisture at Pune as soil moisture builds up gradually.

4.2. *Variance explained by different harmonics*

It may be seen from Table 1 that the first harmonic explains 17-32, 29-42, 9-14 and 24-45% of the total variance at different depths for the calendar years 1984, 1985, 1987 and 1989 respectively. This clearly indicates that unlike soil temperature, the annual wave of soil moisture is not able to give a fairly good representation of the soil moisture regime. The percentage variance contributed by the annual wave is minimum during 1987 presumably due to uneven distribution of rainfall. The variance contributed by the second harmonic is lowest in the year 1985 and it accounted for only 2-3% at different depths. Except for the year 1985, higher harmonics generally accounted for larger percentage of variance. It may also be seen from Table 1 that the first harmonic of normal data explains 67-86% of the total variance at different depths, while the second and third harmonics represent 6-14% and 1.0-6% of the total variance respectively. The total variance explained by the first three harmonics amounts to 93 and 96% at 7.5 and 15 cm depths respectively, whereas at 30 and 45 cm depths it explains 85 to 88% variance.

It is interesting to note that in all the years and soil depths considered in this study only about 50% of the variance is accounted for by the first five harmonics except in 1984 when it accounted for more than 70% and in 1987 when it accounted for as low as 32-56% of the variance at different depths. Such variation is attributed to variation in the soil moisture values caused by partial recharge of soil profile. However, with normal data more than 80% of the variance of soil moisture at 7.5 and 15 cm depths is accounted for by the first harmonic, showing that the annual wave alone is able to give a fairly good representation of the soil moisture regime. At deeper levels, the percentage variance contributed by the annual wave gradually decreases and reaches 67-70%. This may be attributed due to the fact that on many occasions soil moisture does not penetrate the soil profile at lower depth especially when rainfall is not sufficient to recharge the soil profile. On account of these, the higher harmonics assume greater importance to account for the observed soil moisture distribution at deeper depths.

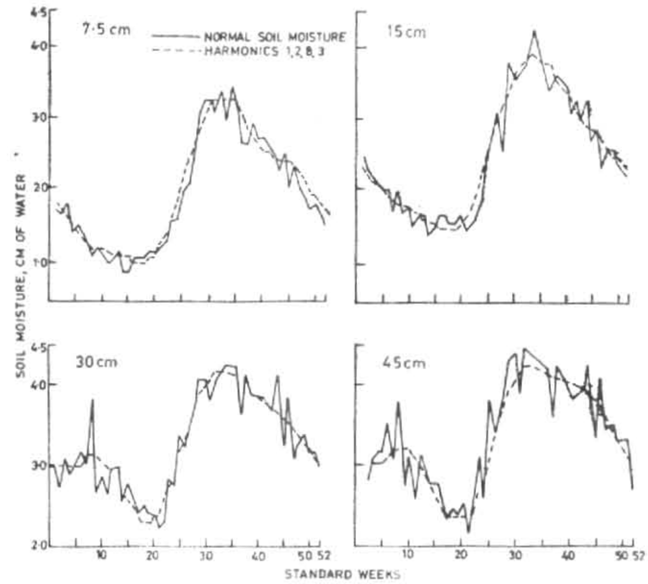

Fig. 3. Weekly normal soil moisture at 7.5, 15, 30 and 45 cm depths and that predicted by first three harmonics

4.3. *Weekly soil moisture and its synthesis*

Depthwise variation of soil moisture for some of the representative years comprising deficient, normal and excess rainfall and for the mean soil moisture are presented in Figs. 2&3 respectively. It may be seen that the highest and the lowest peaks correspond well with the period of rainfall and a little or no rainfall in southwest monsoon and pre-monsoon seasons respectively. However, there are some variations in the moisture content in the soil profile due to variation of rainfall. In the year 1984 when 90.6 cm of rainfall was received the soil moisture ranged between 3 and 4.5 cm for a longer period (between 27th and 46th weeks) at an average depth of 45 cm. Whereas soil moisture at 30 cm average depth varied considerably with 5 peaks in the year 1987 when rainfall received was 71.5 cm. In the year 1985 and 1989 when rainfall were 56.9 and 59.3 cm respectively, the variation of soil moisture at different depths were comparatively less than that of 1984 and 1987. Fig. 2 also confirms that distribution of rainfall played the important role in soil moisture variation. Some soil moisture peaks during winter and pre-monsoon seasons, when there was no rainfall, may be due to observational error in the gravimetric method under relatively dry conditions. Sources of error may also be due to spatial variation of soil moisture, due to formation of cracks and mixing of top soil with the deeper depths soil as soil does not stick to the auger under dry conditions during these seasons.

The original data are reconstructed from the Fourier coefficients by using all $N/2$ harmonics, where $N=P=52$ in this case. The weekly soil moisture values were reconstructed using first five harmonics only. Fig. 2 shows the variation of weekly soil moisture along with estimated soil moisture by the first five harmonics for some selected soil depths. Only one soil depth for each of the years is presented as others will show more or less the similar patterns. The fourth harmonic of mean soil moisture data explains only 0.11-0.52% of the total variance. Thus with reasonably good accuracy the original data of weekly soil moisture for the individual years and for normal data at various depths are represented in terms of the first five and first three harmonics respectively. By substituting the values of mean moisture ($\overline{SM}$), amplitude (C_n) and phase angle (ϕ_n) in Eqn. (2) in respect of the first five and first three harmonics, the weekly soil moisture data for the years 1984, 1985, 1987 and 1989 and for normal data were reconstructed. These data alongwith the observed weekly soil moisture for average depths 7.5, 15, 30 and 45 cm are plotted in Figs. 2&3. It may be seen from these figures that the annual soil moisture cycle in vertisol situated in the semi-arid tract of India can be fairly well described by the combination of first three or first five harmonics. It would also be seen from Figs. 2&3 that the combination of the first three and first five harmonics fit the observed values fairly well during the southwest monsoon and reasonably well during pre and post-monsoon seasons except for the fact that they fail to represent some of the fluctuations of the soil moisture.

5. Conclusions

(*i*) Soil moistures and their maxima/minima at different depths at any standard week can be estimated for Pune on the basis of their annual periodicity with the help of harmonics computed by Fourier technique.

(*ii*) For mean data the annual soil moisture cycle, in vertisol at Pune, can be fairly well described by the combination of first three harmonics.

(*iii*) Annual soil moisture cycle depicts that maximum recharge of soil profile would be in or around the month of September.

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