Mausam, (1985), **36**, 3, 259-266

551.515.1 : 551.511.2

A study of transformation of cyclones into waves and *vice versa* using kinematical determinant*

AIR VICE MARSHAL S. LAKSHMI NARAYANAN
Indian Air Force, New Delhi

सार — यह दिखाया गया है कि चक्रवातों तथा धारा रेखी तरंग प्रतिरूपों का एक सामान्य गुण बन्द शून्य आइसोप्लैथ है। धारा रेखी तरंग प्रतिरूपों में अविरल पवन क्षेत्र के अध्यारोपण से पारस्परिक रूपान्तरण हो जाता है। आइसोप्लैथ के भीतर/बाहर y_1 घनात्मक/ऋणात्मक है। यह L_1 शुद्ध गतिकीय निर्धारक है।

ABSTRACT. It is shown that a property common to cyclones and streamline wave patterns which undergo mutual transformation by superposition of a constant wind field is the closed zero L_1 isopleth inside/outside of which L_1 is positive/negative. Here L_1 is the kinematical determinant.

1. Introduction

It is often seen that atmospheric cyclones are born out of an initial streamline wave configuration and when cyclones disappear, a streamline wave configuration is seen in its place. From this, one can infer that there can be some property common to cyclones and waves. The object of this paper is to investigate the common property.

2. Wind field

In this paper, we consider a two dimensional wind field $\mathbf{V} = u\mathbf{i} + v\mathbf{j} = V\mathbf{t} = V(\cos\psi\mathbf{i} + \sin\psi\mathbf{j})$. $\mathbf{V}$ is finite, single-valued, continuous and differentiable. $V^2 = u^2 + v^2$ and $\tan\psi = \left(\frac{v}{u}\right)$. ψ is the angle $\mathbf{V}$ makes with x-axis and is reckoned positive in counterclockwise direction.

Kinematical properties

Divergence (Div) $= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = V\frac{\partial \psi}{\partial n} + \frac{\partial V}{\partial s}$

Vorticity (Vor) $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = V\frac{\partial \psi}{\partial s} - \frac{\partial V}{\partial n}$

Stretching Deformation (D_{st}) $= \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}$

Shearing Deformation (D_{sh}) $= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$

Kinematical Determinant (L_1) $= 4\mathbf{k}.\,[(\nabla u) \times (\nabla v)]$
$= (\text{Div})^2 + (\text{Vor})^2 - D^2_{sh} - D^2_{st}$

$$L_1 = 4\mathbf{k}.\left[\left(\nabla \frac{V^2}{2}\right) \times \nabla\psi\right] = 4\left|\nabla\psi\right|\frac{\partial}{\partial n_1}\left(\frac{V^2}{2}\right)$$

$$L_2 = L_1 - (\text{Div})^2 \,;\; L_3 = L_1 - (\text{Vor})^2$$

s and n are distances along and normal to streamlines.

$\frac{\partial \psi}{\partial s}$ is curvature. It is positive/negative when streamlines curve counterclockwise/clockwise.

$\frac{\partial \psi}{\partial n}$ is diffluence/confluence. It is positive/negative if streamlines are diffluent/confluent.

$(\nabla\psi \times \mathbf{k}).\,\nabla = \left|\nabla\psi\right|\frac{\partial}{\partial n_1}$ where $\frac{\partial}{\partial n_1}$ is differentiation along the isogon. n_1 is reckoned positive.

(*i*) from cyclone/spiral centre,

(*ii*) towards col point,

(*iii*) in clockwise direction of closed isogons enclosing a ψ minimum (maximum of ψ if ψ were reckoned as per meteorological practice) and

(*iv*) in counterclockwise direction of closed isogon enclosing a ψ maximum (minimum of ψ if ψ were reckoned as per meteorological practice).

*Paper was presented in the symposium "Indo-French School on recent advances in Computer Techniques in Meteorology, Biomechanics and Applied Systems" held at I.I.T., New Delhi, 4-13 February 1980

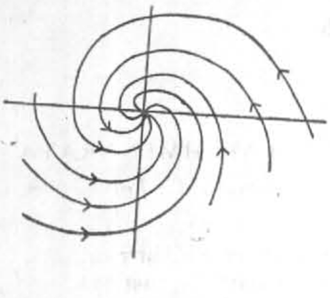


Fig. 1. Streamline pattern

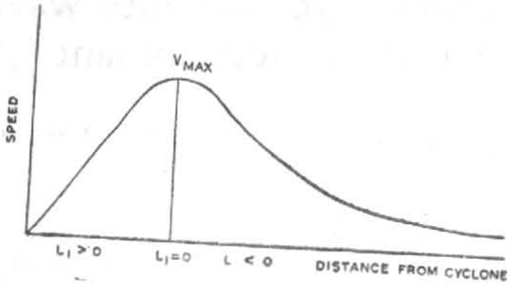


Fig. 2. Speed profile

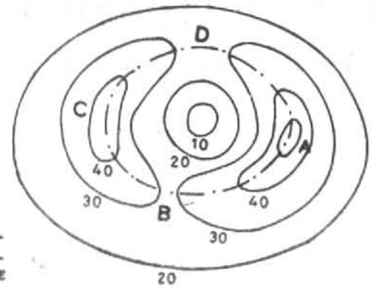


Fig. 3. Speed field

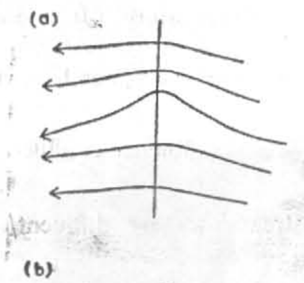


Fig. 4(a & b)

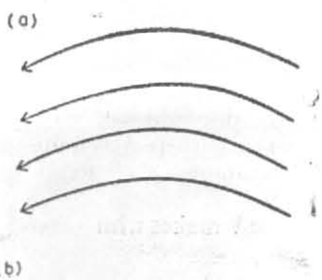
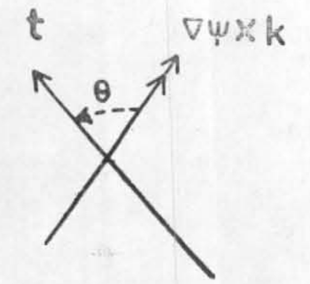


Fig. 5(a & b)

Meteorological reckoning of ψ is just the opposite. In figures, isogons are labelled with values following meteorological usage and the direction along which n_1 is reckoned positive is indicated by double barbs.

$$\tan\theta = \frac{[\{(\nabla\psi)\times\mathbf{k}\}\times\mathbf{t}].\mathbf{k}}{[(\nabla\psi\times\mathbf{k}).\mathbf{t}]} = \frac{\partial\psi/\partial s}{\partial\psi/\partial n}$$



θ is the angle between the isogon and streamlines. Three theorems (Lakshminarayanan 1978) proposed to be used in the present study are listed for easy reference.

1(a). ($\nabla\psi\times\mathbf{k}$) vector lines which are the same as isogons originate from a point where $\mathbf{V}=0$ and L_1 is positive. ψ will be found to increase continuously and change by $+2\pi$ on making one complete counterclockwise circuit around the point.

1(b). ($\nabla\psi\times\mathbf{k}$) vector lines which are the same as isogons terminate at a point where $\mathbf{V}=0$ and L_1 is negative. ψ will be found to decrease continuously and change by -2π on making one complete counterclockwise circuit around the point.

2(a). Streamlines are spiral shaped in an area where L_1, L_2 and L_3 are positive, $L_1\neq L_2$, $L_1\neq L_3$ and where an isolated point $\mathbf{V}=0$ exists.

2(b). Streamlines are closed lines in an area where L_1, L_2 are positive, $L_1=L_2$ and L_3 is zero or negative and where an isolated point $\mathbf{V}=0$ exists.

2(c). Streamlines are parabolic shaped or radial lines in an area where L_1, L_3 are positive, $L_1=L_3$ and L_2 is either zero or negative and where an isolated point $\mathbf{V}=0$ exists.

2(d). Streamlines are hyperbolically shaped as at a col point in an area where L_1, L_2, L_3 are negative and where an isolated point $\mathbf{V}=0$ exists.

3. The ring of maximum wind enclosing the spiral centre is the zero L_1 isopleth inside/outside which L_1 is positive/negative.

3. Cyclone

To focus attention, we choose northern hemispheric cyclones characterised by counterclockwise curved streamlines spiralling inward and terminating at a point called the cyclone centre. $\mathbf{V}=0$ at the cyclone centre. Since the cyclone centre is the point of intersection of zero isopleth of u separating positive from negative values of u, and zero isopleth of v, separating positive from negative values of v, $L_1\neq 0$ at the centre. Further, L_1 cannot be negative by Theorem 2(a) and hence must be positive at the cyclone centre.

Proceeding from the centre in any direction, the *speed increases*, reaches a maximum whereafter it decreases. The locus of such points of maximum is the Ring of Maximum Wind (RMW). RMW encloses the centre. The sign of L_1 can be determined from $\frac{\partial}{\partial n_1}\left(\frac{V^2}{2}\right)$ using $L_1=4\left|\nabla\psi\right|\frac{\partial}{\partial n_1}\left(\frac{V^2}{2}\right)$. The isogons originate from cyclone centre [Theorem 1(b]. Proceeding along any isogon, we note $\frac{\partial}{\partial n_1}\left(\frac{V^2}{2}\right)$ is positive, zero/negative inside / on / outside the RMW. Hence L_1 is positive/zero/negative at all points inside/outside the RMW. The significant characteristic of the cyclone which we will use is the closed zero L_1 isopleth inside/outside which L_1 is positive/negative.

Fig. 1 illustrates the streamline field, Fig. 2 speed profile in any direction from the cyclone centre and Fig. 3 the speed field of a cyclone.

4. Stream wave configuration

Figs. 4 (a) & 5(a) represent two streamline wave configuration and their corresponding isogon fields are in Figs. 4(b) & 5(b). Fig. 4(a) represents a laterally damped wave (*see* p. 69 Palmer *et al.* 1955) and is characterised by closed isogon configuration. The closed isogons enclosing a ψ maximum are separated by an isogon from closed isogons enclosing ψ minimum. Fig. 5(a) gives an undamped wave with no closed isogon. We will not consider these types of waves. A unique isogon configuration save for an additive constant to ψ, is always obtainable from a given streamline field by evaluating ψ at every point. But to obtain a streamline field from a given isogon field, it is necessary that the unit vector is defined at least at one point. Depending on the definition of unit vector, the streamline pattern changes. If we define the unit vector as normal to $P_1 P_2$ (*see* illustration Fig. 6) we get back the streamline wave pattern (Fig. 4a) from isogon field (Fig. 4b). But if we define unit vector as parallel to $P_1 P_2$ as illustrated in Fig. 7, we get the streamline pattern as in Fig. 8. The angle between the isogon and the streamline at the point of definition in the first case is 90° and in the second case is zero.

We note that (*i*) different streamline patterns result depending on the definition of unit vector. (*ii*) in the specific case of an isogon field characterised by closed isogons enclosing a ψ maximum separated by an isogon from closed isogons enclosing a ψ minimum a streamline wave pattern results if the angle between the separating isogon and the streamline is near about a right angle, *i.e.*,

$$\left|\frac{\partial\psi}{\partial s}\right| \gg \left|\frac{\partial\psi}{\partial n}\right|$$

and a confluenting/diffluenting streamline pattern results if

$$\left|\frac{\partial\psi}{\partial n}\right| \gg \left|\frac{\partial\psi}{\partial s}\right|$$

5. Closed zero isopleth of kinematical determinant

L_1 vanishes at points where (*i*) speed and/or direction is maximum/minimum/minimaximum and (*ii*) at points where isopleth of speed touches isogon. In the specific context of cyclone, isopleth of speed in general touches isogons at all points on RMW excepting points of maxima/minimaxima where $\nabla(V^2)$ is zero and isogons pass through them. It may be noted that at cyclone centre, $\nabla(V^2)$ is zero, $\nabla\psi$ is infinite and L_1 is finite positive and at col point, $\nabla(V^2)$ is zero, $\nabla\psi$ is infinite and L_1 is finite negative.

ABCDA in Fig. 3 is the zero isopleth of L_1. A and C are points of maxima of speed : B and D are points of

Fig. 6

Fig. 7

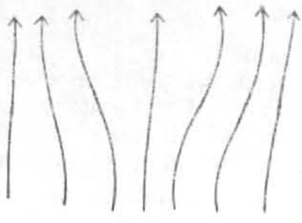


Fig. 8

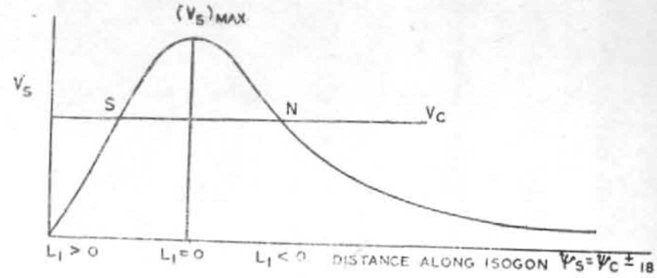


Fig. 9

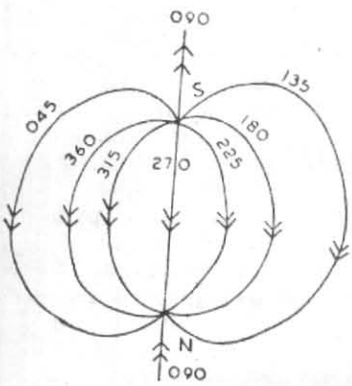


Fig. 10

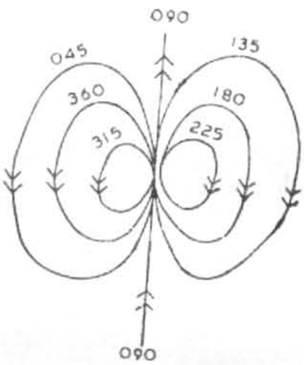


Fig. 11

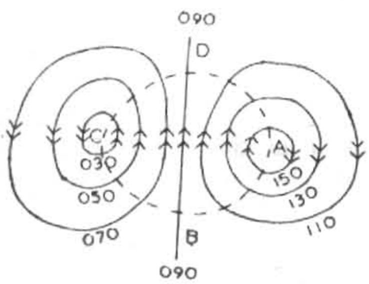


Fig. 12

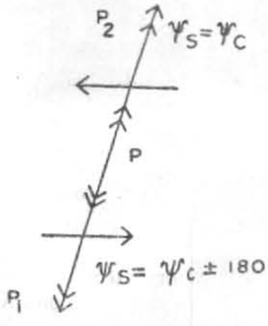


Fig. 13

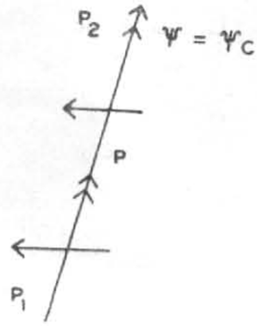


Fig. 14

minimaxima of speed : at all other points, isogons and isopleths of speed touch.

In the context of streamline wave configuration, L_1 is zero at points of ψ maximum and ψ minimum. Hence if there exists a closed L_1 zero isopleth passing through the points of maximum and minimum of ψ, and inside/outside of the closed zero L_1 isopleth, L_1 is positive/negative, we can evolve a simple rule on the sequence of occurrence of maximum and minimum of speed and direction as follows :

Going round the zero isopleth of L_1 the sequence of occurrence of a point of maximum and a point of minimum of direction and speed will be maximum of speed, maximum of direction (minimum of direction if reckoned as per meteorological usage) minimum of speed and minimum of direction (maximum of direction if reckoned as per meteorological usage).

6. Transformation

We study the transformation of a cyclone $\mathbf{V}_s$ into a wave by superposing a constant wind field $\mathbf{V}_c$. The field to be studied is $\mathbf{V}=\mathbf{V}_s+\mathbf{V}_c$. We note that L_1 field for $\mathbf{V}$ and $\mathbf{V}_s$ are the same.

Cyclone centre and col point

Since $\mathbf{V}$ must vanish at the cyclone centre, we investigate by graphical method the point/points, if any, where $\mathbf{V}$ vanishes. At the point where $\mathbf{V}=0$, the difference between the directions of $\mathbf{V}_s$ and $\mathbf{V}_c$ must be 180°, *i.e.*, $\psi_s=\psi_c\pm180$. To focus attention, if we superpose an easterly constant wind field, the cyclone centre must be on the 270° isogon. Further at the new centre, V_s must be equal to V_c. Hence we construct the speed profile of V_s on the isogon $\psi_s=\psi_c\pm180$ and superpose on it V_c speed as in Fig. 9.

Case (i). $V_c<(V_s)_{max}$. At S and N, $\mathbf{V}$ vanishes. At S, $\mathbf{V}=0$, $L_1>0$ and at N, $\mathbf{V}=0$ and $L_1<0$. The cyclone centre is at S and a col point is created at N.

Case (ii). Keeping the direction of $\mathbf{V}_c$ to be the same, we can choose higher values of speed. As V_c increases, S and N approach each other as well as RMW and merge with one another when $V_c=(V_s)_{max}$.

Case (iii). For values of $V_c>(V_s)_{max}$ there is no point where $\mathbf{V}$ can vanish and hence there can be no cyclone centre and col point.

Isogon field of $\mathbf{V}$

Case (i). $V_c<(V_s)_{max}$. The isogons emanating from cyclone centre S terminate at the col point N as illustrated (Fig. 10).

Case (ii). $V_c=(V_s)_{max}$.

S and N coincide at a point on the RMW. The isogon $\psi_s=\psi_c\pm180$ between S and N vanishes in the limit. All other isogons are closed as illustrated (Fig. 11).

Case (iii). $V_c>(V_s)_{max}$

The cyclone centre and col point have disappeared. The isogons are closed lines enclosing a ψ maximum and a ψ minimum. The isogon $\psi_s=\psi_c$ is the separating isogon. The point of maximum A and point of minimum C of ψ are on the zero isopleth of L_1 as illustrated in Fig. 12 by ABCDA.

Speed field

$$V^2=V_s^2+V_c^2+2V_s\,V_c\cos(\psi_s-\psi_c)$$

Case (i) $V_c<(V_i)_{max}$. The speed is a minimum at the cyclone centre and col point. The RMW preserves its character of being the locus of points of maximum speed. Even though the positions of maxima and minimaxima are altered, still they continue to be on the zero isopleth of L_1.

Case (ii) $V_c=(V_s)_{max}$. S and N coincide at a point on the zero isopleth of L_1. The character of RMW is changed with the appearance of a minimum speed at the point of coincidence.

Case (iii) $V_c>(V_s)_{max}$. The zero isopleth of L_1 is no longer the locus of points of maximum speed. It is a locus of points of maximum as well as minimum speed. Proceeding along the isogons from outside to inside of the closed zero isopleth of L_1, we note that $\partial V^2/\partial n_1$ must be negative, zero and positive at points outside, on and inside the zero isopleth. Hence on these points where $\partial V^2/\partial n_1$ vanishes while proceeding from outside to inside, the speed must be minimum. In Fig. 12, points on ABC have minimum speed. By a very similar reasoning, we note points on CDA must be maximum speed. Therefore, for $V_c>(V_s)_{max}$ the zero isopleth of L_1 is made up of two portions :

(*i*) where speeds have maximum values,

(*ii*) where speeds have minimum values while proceeding along the isogons.

Angle between separating isogon and streamline

For values of $V_c>(V_s)_{max}$, the isogon $\psi_s=\psi_c$ and $\psi_s=\psi_c\pm180$ together constitute isogon $\psi=\psi_c$ as in Fig. 14. The angle θ between the isogon $\psi=\psi_c$ and the streamline of $\mathbf{V}$ is the same as the angle between $\psi_s=\psi_c$ and $\psi_s=\psi_c\pm180$ and $\mathbf{V}_s$ streamlines. In the atmospheric cyclones, curvature is predominant, *i.e.*, $\left|\frac{\partial\psi}{\partial s}\right|\gg\left|\frac{\partial\psi}{\partial n}\right|$ and hence as in Figs. 13 and 14, the angle between the isogon and the streamline is near about a right angle. Hence we conclude that the cyclone is transformed into a damped streamline wave configuration. If there is a spiralling pattern in which $\left|\frac{\partial\psi}{\partial n}\right|\gg\left|\frac{\partial\psi}{\partial s}\right|$, the transformation will result in a confluent/diffluent pattern.

7. Mathematical models

We illustrate the transformation of cyclone into wave with the help of a model, the author (Lakshminrayanan 1975) has developed :

$$\mathbf{V}_s=\mathbf{V}_i+\mathbf{V}_R\,;\ \ \mathbf{V}_i=A\,\nabla\phi\,;\ \ \mathbf{V}_R=B(\nabla\phi\times\mathbf{k})$$

$$\phi=\exp\left\{-\frac{(x^2+y^2)}{2\sigma^2}\right\}$$

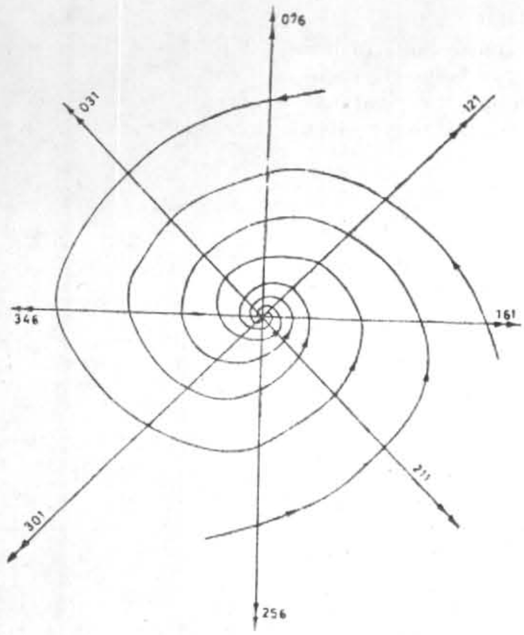


Fig. 15

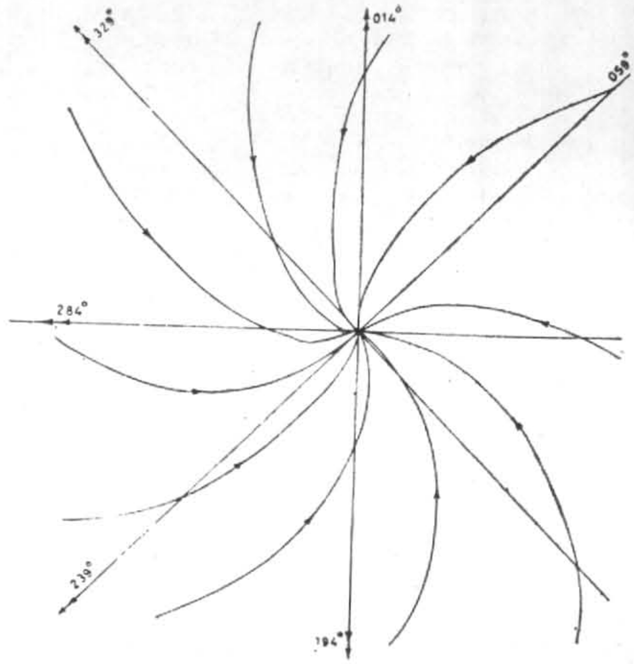


Fig. 16

Fig. 17

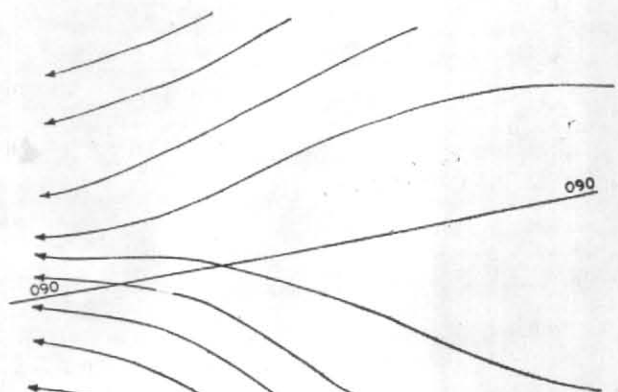


Fig. 18

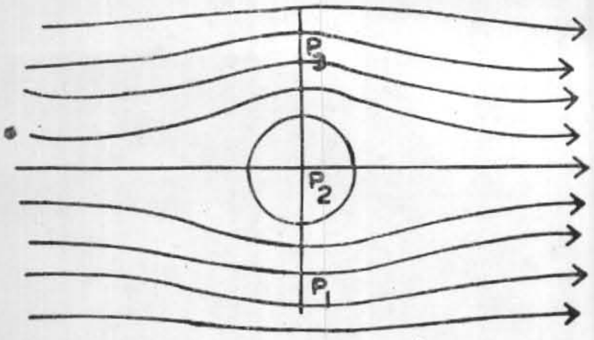


Fig. 19

σ is the radius of RMW

$$\frac{A}{\sigma} = (V_i)_{\max} e^{\frac{1}{2}} \; ; \quad \frac{B}{\sigma} = (V_R)_{\max} e^{\frac{1}{2}}$$

$$-\frac{A}{\sigma^2}\phi = V\frac{\partial\psi}{\partial n} \; ; \; \frac{B}{\sigma^2}\phi = V\frac{\partial\psi}{\partial s} \; ;$$

$$\tan\theta = \frac{B}{-A} = \frac{(V_R)_{\max}}{-(V_i)_{\max}}$$

The spiralling model is obtained by superposing inflow $\mathbf{V}_i$ field on a counterclockwise closed streamline field $\mathbf{V}_R$ with $(V_R)_{\max} = 20$ kt and $(V_i)_{\max} = 5$ kt,

$$\left|\frac{\partial\psi}{\partial s}\right| \gg \left|\frac{\partial\psi}{\partial n}\right| \quad (\theta \text{ is approximately } 104)$$

Fig. 15 gives the streamline of $\mathbf{V}_s$ where σ is taken as 100 km.

Fig. 16 gives the wave pattern after superposing an easterly wind of 30 kt.

With $(V_R)_{\max}=5$ kt and $(V_i)_{\max}= 20$ kt.

$$\left|\frac{\partial\psi}{\partial n}\right| \gg \left|\frac{\partial\psi}{\partial s}\right| \quad (\theta \text{ is approximately } 166^\circ)$$

Fig. 18 gives the streamline pattern after superposition with an easterly current of 30 kt. Fig. 17 gives the streamline pattern of the $\mathbf{V}_s$ field from which Fig. 18 was obtained. It is easily seen that a cyclone (*i*) in which |curvature| $\gg$ confluence, transforms into a streamline wave and (*ii*) in which |confluence| $\gg$ |curvature|, transforms into a confluenting streamline pattern. Conversely, we can choose the streamline wave pattern or the confluenting pattern as in Fig. 16 or Fig. 18 and obtain the spiral patterns as in Fig. 15 or Fig. 17 by superposition of westerly wind of 30 kt.

8. Discussion

An excellent treatment of transformation of streamline wave into cyclone using isogon field is given by Palmer *et al.* 1955. In this, the role of speed field is ignored. The theorems developed by the author (Lakshminarayanan 1978) advocate use of kinematical determinant to define col point, spiral centre, isogon fields associated with these two and the RMW. Present study uses L_1 in the context of transformation as well.

Meteorologists will easily concede birth of cyclones from waves from experimental evidence but the spiral formation from a confluenting/diffluenting pattern may not find easy acceptance. Noting that the concepts developed in this paper are applicable to fluid motion also, we will give an easy example of fluid flow round a right circular cylinder (Fig. 19). The diffluent/confluent patterns are seen to the left/ right of $P_1P_2P_3$. Experimental evidence shows that vortices are formed with such patterns.

Taking a limited area, we can write the wind field $\mathbf{V}$ as $\mathbf{V}=\mathbf{V}_m+\mathbf{V}'$

where, $$\frac{\iint \mathbf{V}\,dxdy}{\iint dxdy} = \mathbf{V}_m = \text{Space mean wind vector}$$

and $\mathbf{V}'$ is the deviation from the mean wind vector.

Further $\iint \mathbf{V}'dxdy=0$

If the spiralling circulation were modelled as $\mathbf{V}_s= A\nabla\phi+ B\nabla\phi\times\mathbf{k}$, $\mathbf{V} \iint \mathbf{V}_s\,dxdy$ vanishes for the area of integration enclosed by ϕ isopleth. Hence, choosing a suitable area in which $\mathbf{V}'$ can be represented by $\mathbf{V}_s$, we note that mean wind vector $\mathbf{V}_m$ is none other than the constant wind vector $\mathbf{V}_c$ used in the present study. Decrease of space mean wind speed and/or increase of speed on the closed zero L_1 isopleth appear to be necessary for the birth of a cyclone and increase of space mean wind speed and/or decrease of speed on the RMW are necessary for its disappearance.

RMW is conspicuously seen in the area of an intense tropical cyclone. Its existence around any spiral centre appears to be a theoretical necessity. Proceeding in any direction from the spiral centre where speed is zero; the speed must initially increase whereafter there are three possibilities :

(a) speed reaches a maximum,

(b) speed monotonically increases and

(c) speed increases and asymptotically reaches a value.

Possibilities (b) and (c) are not tenable in the case of spirals of finite kinetic energy. Hence spiral centres of practical interest will be enclosed by RMW. Any way the applicability of conclusions of this paper is limited to such cases only.

9. Conclusion

The closed zero isopleth of L_1 inside/outside which L_1 is positive/negative is a common property in transformation of cyclones into streamline wave/diffluent/confluent configurations and *vice versa* obtained by superposing constant wind vector.

The ideas presented in this paper are dealt with in greater detail in chapters 5, 6, 7 of "*Theorems on Cyclones*" (1984) which may be referred to with advantage.

References

Lakshminarayanan, S., 1978, *Indian J. Met. Hydrol. Geophys.*, **29**, 4, pp. 655-662.

Lakshminarayan, S., 1984, *Theorems on Cyclones*, published by Indian Meteorological Society, Lodi Road, New Delhi 3.

Palmer *et al.*, 1955, *The Practical Aspect of Tropical Meteorology* No. 76, Air Force Cambridge Research Centre, Bedford Massachusetts.