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Non-integer (non-harmonic) spectral analysis of a drought index for the American great plains

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सार — गैर-पूर्णांक (गैर-हार्मोनिक) मानावलीय विश्लेषण उपगमन प्रस्तुत किया गया और लगभग 70 वर्षों की अवधि को लेते हुए उत्तरी अमरीका के आन्तरिक मैदानों के लिये अनावृष्टि सूचकांकों में आवर्तिता का पता लगाने के लिये उपयोग किया गया। अनावृष्टि सूचकांक श्रेणियों के विद्युत मानावलीय विश्लेषण से स्पष्ट होता है कि अर्द्धद्वैवार्षिक और अर्द्धत्रैवार्षिक चक्र कई स्टेशनों के लिये अनावृष्टि श्रेणियों में सामान्य रूप से आवर्तित हैं। अनावृष्टि मानावली लगभग 5-6 वर्षों की अवधि वाले शिखरों से प्रभावित होती है। 2-6 वर्षों के तरंग बैंडों में ये सार्थक आवर्तिताएं आकाशी घटनाओं में यत्र-तत्र होती हैं। ये एक स्थान पर प्रत्यक्ष रूप से देखी जा सकती हैं जबकि कुछ सौ किलोमीटर की दूरी पर ये दिखाई नहीं देती हैं।

ABSTRACT. A non-integer (non-harmonic) spectral analysis approach is presented and used to search for periodicities in the drought indices for the Interior Plains of North America, covering a period of at least 70 years. Power spectrum analysis of the drought indices series revealed that the quasi-biennial and quasi-triennial cycles are common periodicities in the drought series for many stations. The drought spectrum is also dominated by peaks with periods of about 5-6 years. These significant periodicities in the 2-6 year wave bands are sporadic in spatial occurrence; they can be evident at one place while being absent at another a few hundred kilometres away.

1. Introduction

Drought hazard has plagued mankind since antiquity and is one of the most serious social problems in many countries. It affects wide ranges of regions on the earth than flood hazard. In order to alleviate the drought problems, various countermeasures are in practice. Many of these countermeasures are in one way or another based upon the empirical knowledge of droughts that have been accumulated from the past for long range planning purposes. These countermeasures could be more effective if drought could be predicted for at least short durations (for example, one to three years in advance).

The first step in predicting drought with time series analysis technique involves a search for periodicities in the historical record. Periodicities, if present, represent potential predictive power for anticipated occurrence or non-occurrence of drought during a future time period. Major searches for periodicities in drought occurrence and their association with periodicities in solar-terrestrial phenomena are, therefore, common in climatological investigations.

Most of the studies on drought's cyclic or periodic characteristics use either of the two classical techniques of spectral analysis, namely the periodogram (Jones 1965) and the lagged products method of Blackman and Tukey (1959) to estimate frequency and phase components of drought. Others (*e.g.*, Currie 1981, 1984a, 1984b) have used the maximum entropy method (MEM) to search for periodicities in drought indices. The lagged products method is a spectrum analysis of the autocorrelation function using different maximum lags depending on the frequency of oscillation of interest. Although computer facility has aided considerably in the use of autocorrelation technique, its main problem is the difficulty in choosing the right maximum lag which can influence the resulting cycles. In the periodogram method, the squared magnitude of the discrete Fourier transform (DFT) of the data is computed, usually by means of the fast Fourier transform (FFT). While this direct method using FFT is more accurate than the lagged products method, the record length must be an exact multiple of the cycle in which one is interested. In this case the resulting cycles are strongly influenced by the record length n. Because the spacing of DFT values is related soley to $1/n$, many of the true peaks in the spectra which may be significant, are never sampled. MEM has been found to have a higher resolution and much lower sidelobes than either the periodogram or the Blackman-Tukey method (*e.g.*, Lacoss 1971, Ulrych 1972). However, the variance of MEM estimates of power density is not properly understood and there is still no known way to obtain variances on the spectral estimates. In addition, MEM does not yet have a consistent statistical test for the evaluation of significant peaks, and the few confidence limit procedures proposed are complicated and rather computationally expensive (*e.g.*, Baggeror 1976, Reid 1979).

Recently, Schickedanz and Bowen (1977) developed a non-integer (NI) or non-harmonic spectral technique for estimating spectra at frequencies which are not integer multiples of $1/n$ using by multiple regression approach. Unlike the periodogram, the NI method does not require any smoothing of raw estimates of the spectrum since the regression used in estimating spectra smooths them considerably. Moreover, the statistical significance of the spectral peaks can be tested from readily available statistical tables. Since its introduction, the NI spectral method does not appear to have been widely used to estimate spectra of climatological time series, even though it appears relatively easier to use than the MEM and it is aimed at resolving frequencies in the range of 5 to 25-yr period; periods which are common in many climatological time series. This paper examines the spectra in the derived growing season Bhalme-Mooley drought index series (Bhalme and Mooley 1980) for the Interior Plains of North America using the NI spectral method in order to detect periodic or quasi-periodic components in the series. The NI method is used because it has been shown to be capable of performing a very thorough search for periodicities (Neil 1981) and its large amount of resolution is claimed to lead to the detection of more hidden peaks in the spectra of time series than the conventional method of DFT (Schickedanz and Bowen 1977). Some mathematical and computational aspects of NI are presented in section 2. In section 3, the derived drought index is introduced, while the results of the application of NI to the drought index series are presented and discussed in section 4. The main conclusions of the paper are given in the last section.

2. Non-integer spectral method

A sequence of observed climatological time series (x_t) can be expressed mathematically by the following sine and cosine function series:

$$x_t = \bar{x} + \sum_{i=1}^{[n/2]} a_i \sin(2\pi it/P) + b_i \cos(2\pi it/P) \quad (1)$$

or the equivalent

$$x_t = \bar{x} + \sum_{i=1}^{[n/2]} C_i \cos 2\pi\, i\,(t-t_i)/P \quad (2)$$

where P is the length of the observation period, x_t are a sequential series taken at times $t=1, 2, \ldots, n$, $\bar{x}$ is the average of the series, $[n/2]$ is the integer part of $n/2$, C_i are the amplitudes of $[n/2]$ cosine waves, a_i and b_i are the Fourier sine and cosine coefficients given as :

$$a_i = (2/n) \sum_{t=1}^{n} x_t \sin(2\pi it/P),\ i=1, 2, \ldots, [n/2] \quad (3)$$

$$b_i = (2/n) \sum_{t=1}^{n} x_t \cos(2\pi it/P),\ i=1, 2, \ldots, [n/2]-1 \quad (4)$$

$$b_{n/2} = (1/n) \sum_{t=1}^{n} x_t \cos(2\pi it/P) \quad (5)$$

and t_i is the point at which the ith harmonic has a maximum and is given as :

$$t_i = \phi P/(2\pi i) \quad (6)$$

where ϕ, the phase angle is :

$$\phi = \arctan(a_i/b_i). \quad (7)$$

The raw spectral estimate of the ith harmonic is then given as :

$$S_i = C_i^2/2 = \left(a_i^2 + b_i^2\right)/2,\ i=1, 2, \ldots, [n/2]-1 \quad (8)$$

$$S_{[n/2]} = b^2_{[n/2]} = C^2_{[n/2]} \quad (9)$$

The results in Eqns. (8)-(9) are valid because of the orthogonality of sine and cosine functions. The orthogonality property is, however, true only when integer values are used for the harmonic numbers in Eqns. (3)-(5). This means that the fundamental period P must be an integral multiple of the period of harmonics with the harmonics having periods $P/1, P/2, \ldots, P/[n/2]$, or the spectral estimates would occur at equally spaced frequencies, $1/P, 2/P, \ldots, [n/2]/P$. Spectral estimates lying between the harmonic frequencies can be obtained if non-integer values are used for i in Eqns. (3)-(5). Such an approach, however, renders the Eqns. (3)-(5) invalid. The NI method is an attempt to obtain spectral estimates at all frequencies by using non-integer values for the harmonic number i.

Non-integer spectral technique consists of evaluating the Fourier sine and cosine coefficients for non-integer values by performing a multiple regression analysis of the time series under study on the sine and cosine waveforms for any given i such that :

$$x_t = a + (bs_i)(XS_{it}) + (bc_i)(XC_{it}),\ t = 1, 2, \ldots, n \quad (10)$$

where a is the intercept, bs_i and bc_i are the partial regression coefficients, and XS_{it} and XC_{it} are the sine and cosine functions given respectively as :

$$XS_{it} = \sin(2\pi it/P) \quad (11)$$

$$XC_{it} = \cos(2\pi it/P) \quad (12)$$

and are not limited to frequencies that are whole-number multiple of the fundamental frequency. The multiple correlation coefficient R_i and the correlation coefficient r_i between the variables XS_{it} and XC_{it} are now determined for a given value of i. The coefficient of determination R_i^2 which is the fractional amount of variance explained by sine and cosine waveforms approximately gives the normalised spectral density associated with the ith harmonic number. R_i and R_i^2 were tested for significance by the F test which can be expressed as :

$$F_i = \frac{R_i^2/k}{\left(1-R_i^2\right)/v} \quad (13)$$

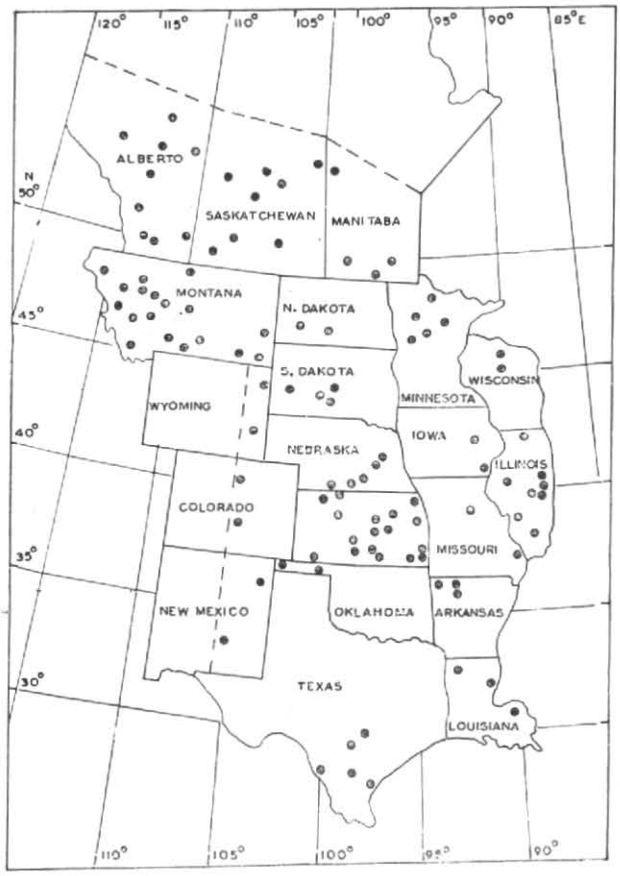


Fig. 1. The Great Plains of North America and the network of selected precipitation stations

where k is equal to the number of independent variables and $v\ (=n-k-1)$ is the number of degrees of freedom of the error mean square. Rearranging (13) gives :

$$R_i^2 = kF_i/(v+kF_i)$$

with the values of F_i at the 0.05 and 0.10 levels of significance complied from the tables of Steel and Torrie (1960). Distribution of R_i^2 with frequency F_i associated with ith waveform is the variance spectrum. NI spectral estimates were, however, determined at equal wavelengths rather than frequency and wavelengths resolvable to 0.1 year were searched for.

3. Data

The data subjected to spectral analysis using the NI method are the growing season Bhalme and Mooley-type drought index series derived from monthly precipitation series for the months of April to September at 407 stations in the Interior Plains of North America with the data dating back to 1875 in some stations. Since our interest was in agricultural rather than hydrological drought, the months of April to September are the most crucial in drought study for the Interior Plains because these are the months when most of the stations in the region receive over 70% of their annual total precipitation. It is, therefore, expected that the occurrence of drought during the growing season will strongly indicate agricultural drought conditions for the whole year. Fig. 1 shows the spatial distribution of the 105 precipitation stations used in this analysis.

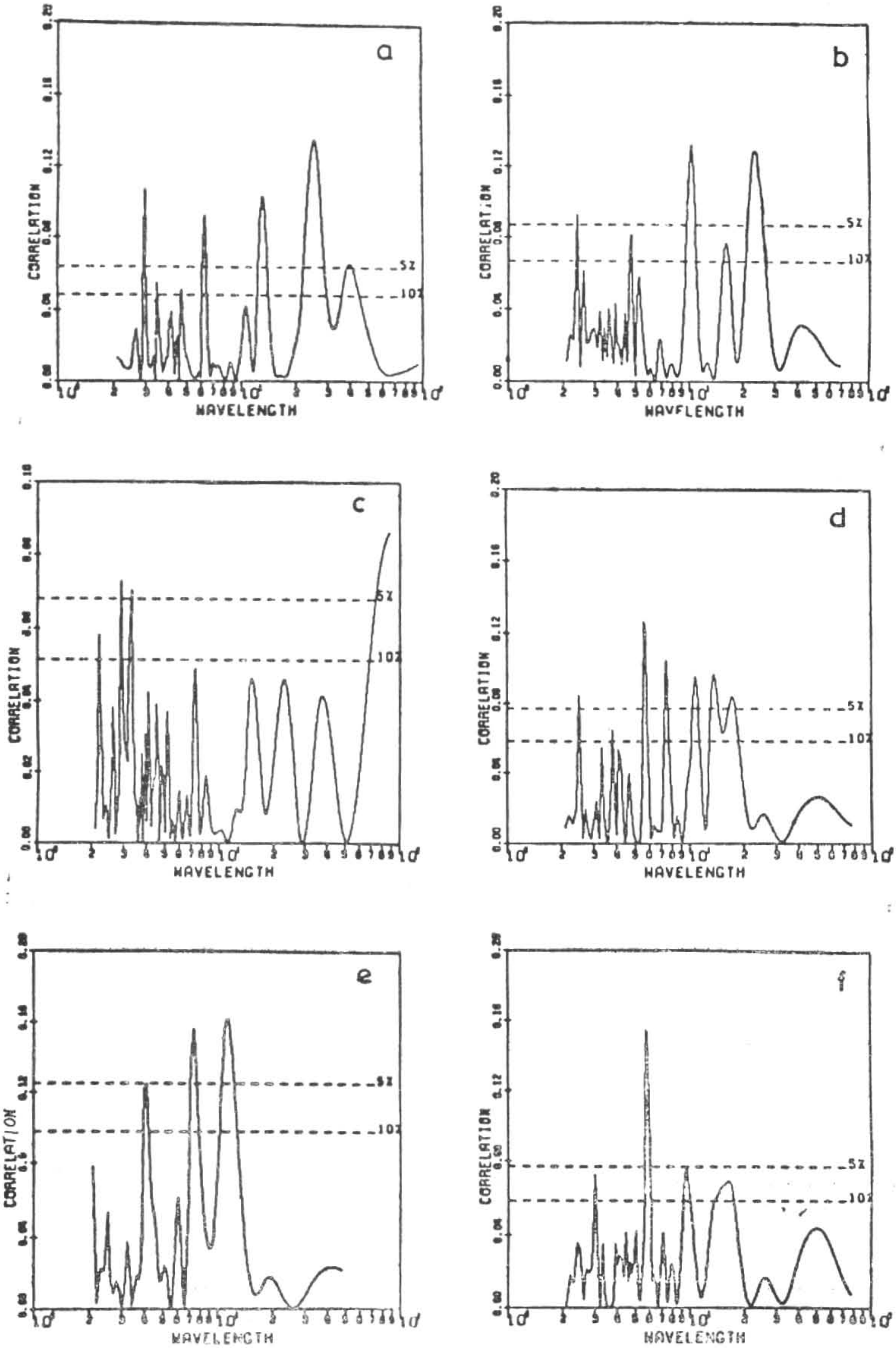
The Bhalme and Mooley drought index (BMDI) is an empirical method based on monthly rainfall for the assessment of drought severity. Its development is based on the assumption that agriculture in a region is adjusted to the average precipitation variability. Details about the derivation of BMDI are given in Oladipo (1985). For the growing season, the derived drought index equation for the region of study is :

$$I_k = 0.548\ I_{k-1} + M_k/42.67 \qquad (14)$$

where I_k, I_{k-1} are drought intensities for the kth and (k—1)th months respectively and the moisture index M is :

$$M = 100\ (p-\bar{p})/s \qquad (15)$$

In Eqn. (15), p is the monthly precipitation with mean $\bar{p}$ and standard deviation s. The use of s to standardize the monthly departures should reduce the weight of stations with low rainfall and high annual variability.



Figs. 2 (a-f). Non-integer spectral analysis results for selected stations in the Great Plains : (a) Campsite (Alberta), (b) Melfort (Saskatchewan), (c) Morden (Manitoba), (d) Fayetteville (Arkansas), (e) Canon City (Colorado), (f) Charleston (Illinois)

The more negative is I of Eqn. (14), the greater the severity of drought. Eqn. (14) was used to generate sequence of I's for each growing season month, April-September, for each of the stations used in the study. From these monthly indices, the mean index series for the seven growing season months were calculated for each of the years of record and for each of the stations. The derived growing season BMDI has been shown to perform comparatively as well as the more complicated Palmer drought index (Palmer 1965) in depicting periods and intensity of drought (Oladipo 1985), and its spatial characteristics have been described in a previous publication (Oladipo 1986). Fisher's sample coefficients of skewness (γ_1) and kurtosis (γ_2) of the BMDI series compared with their standard errors indicated that the drought index values were normally distributed at least at the 10 per cent significance level (Oladipo 1986).

4. Results

One hundred and five stations, sampled to be areally representative of the study region, and with a minimum record length of 68 years were selected for detailed analysis. Fig. 2 shows the normalized spectra of the growing season drought index series, plotted as the multiple correlations between the drought index variable and the sine and cosine waveforms *versus* the period (wavelength) of the waveforms, for some selected stations in the Interior Plains of North America. In all the figures, the 0.05 and 0.10 levels of significance, as indicated by the F test are shown. The low values of the correlation coefficients are indicative of the large number of data points used in the series. As the series contain approximately 800 sampled data points, a correlation as low as 0.1 may be statistically significant although its usefulness for prediction may be limited. However, this study is primarily concerned with the spatial coherence (recurrent spatial spectral patterns) which may indicate a strong tendency for temporal covariability over a large area rather than statistical prediction. A list of the significant periodicities identified by the NI method for the 105 series is available on request. Computer outputs of spectral computations were examined for evidence of periodic variations which had similar frequency of occurrence in adjacent stations. Although areal frequency coherence does not necessarily indicate the drought experience over adjacent sub-areas is varying with similar causative factors, such a spatial condition should strengthen the evidence that periodicities exist in the data. Possible large-scale coherence of the periodicities are now examined in the 2.0-2.9 year, 3.0-3.9 year and 5.0-5.9 year significant frequency bands.

The extent and significance of the 2.0-2.9 year frequency band is proved by its geographical spread in many parts of the study region. Significant peaks at the approximately quasi-biennial oscillation (2.2 years) are particularly characteristics for stations in the Canadian Prairies, Illinois, Lousiana, Minnesota, Montana, Nebraska, South Dakota and Texas (Fig. 3). The quasi-biennial behaviour is well known to exist in several meteorological phenomena. For example, quasi-biennial oscillation in African rainfall pattern was verified by Rodhe and Virji (1976), Kraus (1977), Klaus (1978) and Ogallo (1979). Similar fluctuations were documented in spectra of Indian (*e.g.*, Bhalme 1975, Bhalme and Mooley 1980), Californian (*e.g.*, Granger 1977), Central America-Caribbean (*e.g.*, Hastenrath and Kaczmarczyk 1981), Brazillian (*e.g.*, Kousky and Chu 1978) and English (*e.g.* Tabony 1979) rainfall series. The fundamental physical causes of this quasi-two-year oscillation is still unclear, but it is generally recognised as a pulsation of the general atmospheric circulation with particular relation to reversals in the tropical stratospheric winds (*e.g.*, Reed *et al.* 1962, Angelle *et al.* 1969). Another possible explanation for the quasi-biennial oscillation appears to be related to large flux-variation of the ultraviolet emissions from the sun because the neutrino flux from the sun has been shown by Sakurai (1981) to vary with a period almost equal to 26 months.

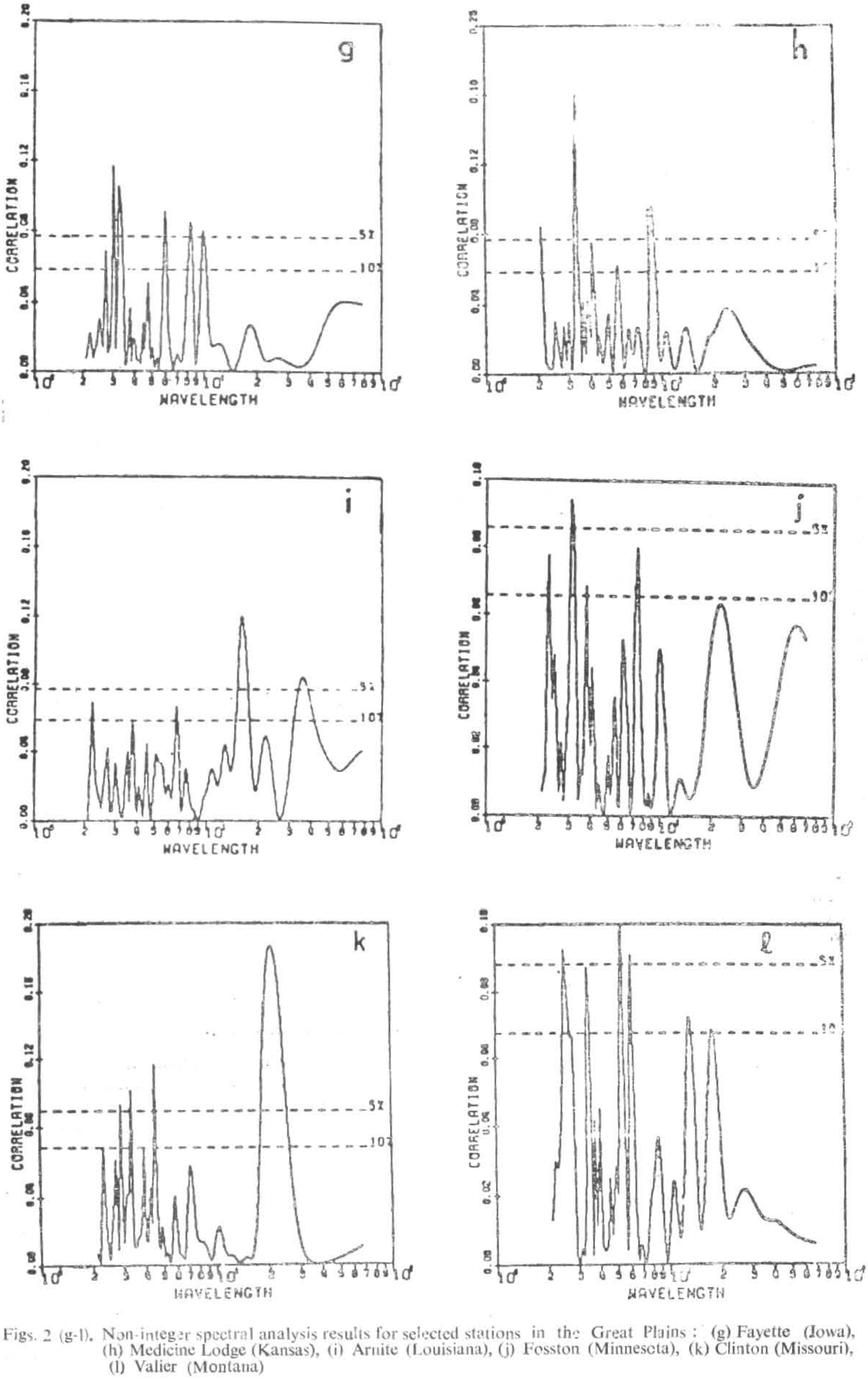
Most of the stations with significant quasi-triennial (3.0-3.9 years), periodicity are concentrated in Iowa, Kansas, Minnesota and South Dakota (Fig. 4). The exact significance of the quasi-three-year oscillation is not known and needs further clarification, but similar periods were observed by Neil (1981) for some sub-areas of the study region (*e.g.*, Iowa, Missouri and Illinois).

Stations with significant quasi-five-year (5.0-5.9 year) oscillation are concentrated in Arkansas, Illinois, Montana and Southern Texas (Fig. 5) The approximately five-year periodicity in climatological time series has received little attention, though similar results were reported in other works. Flohn and Fleer (1975) examined the interaction between rainfall fluctuations in the equatorial Pacific and Indonesian-Australian area, which generally fell into the period of five years. The existence of the quasi-five-year oscillation in the Indian, Indonesian, African and South American rainfall series had been reported by others (*e.g.*, Rao *et al.* 1973, Jagannathan and Parthasarathy 1973, Rodhe and Virji 1976, Fleer 1981).

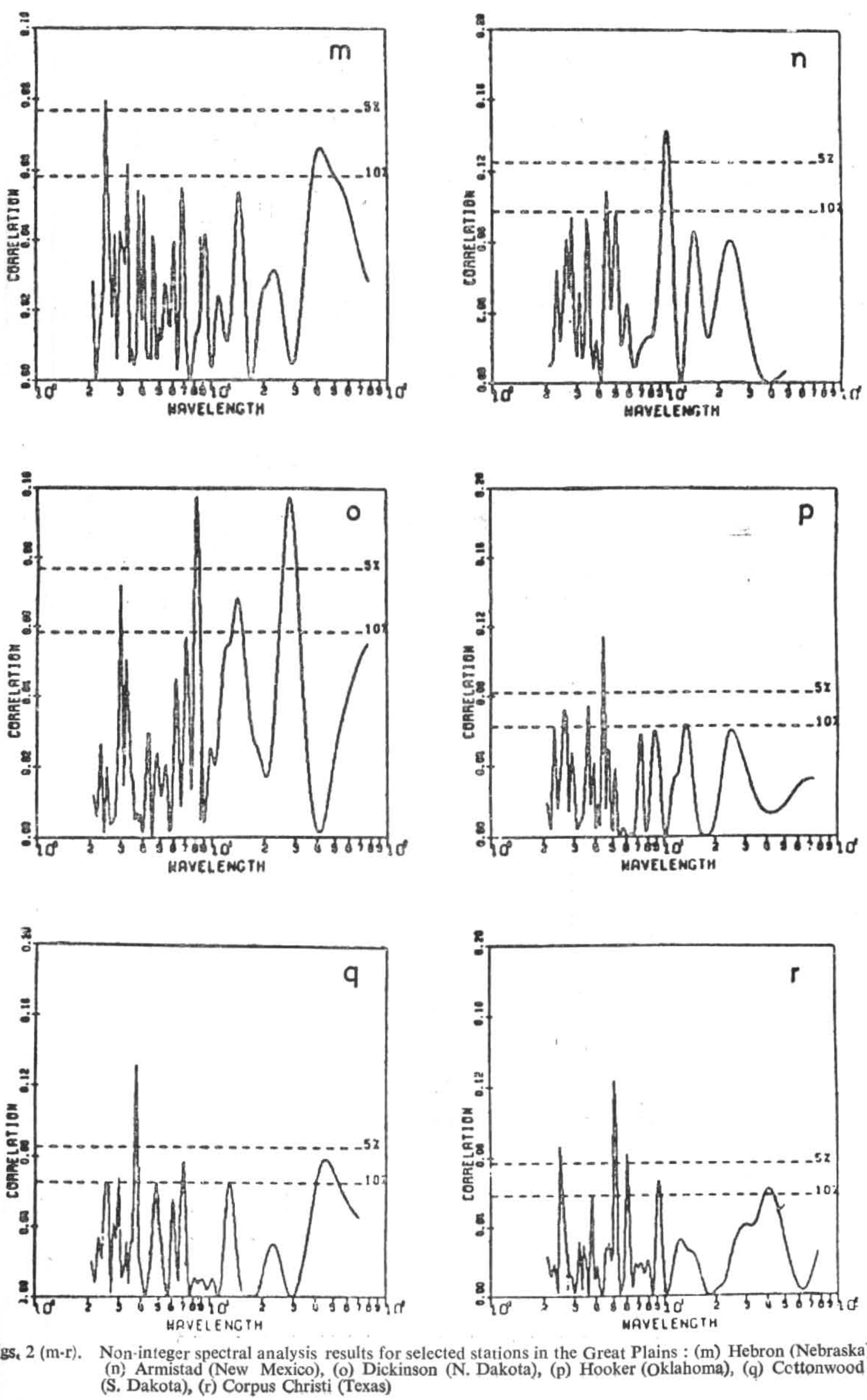
The number of stations with significant periodicities in various frequency bands are summarized graphically in Fig. 6. It is obvious from the figure that periodicities greater than the quasi-five-year frequency bands are not significant in terms of the number of stations and their areal distribution shows no spatial clustering. Only 21% and 15% of the analysed drought series show periodicities in the 10-12 year (sunspot) and 20-21 year (double sunspot) cycles respectively. Even then, these stations are largely randomly distributed over the study region, and the result is, therefore, not interpreted to be a statistically significant evidence of the influence of solar activity on drought occurrence.

5. Summary and conclusions

The results of the spectral analysis suggest that no significant power in drought indices is present in the Interior Plains of North America at periods greater than about six years. The drought spectra are dominated by three peaks in the quasi-biennial (2.0-2.6 yr), quasi-triennial (3.1-3.5 year) and quasi-five-year oscillations. The strong peaks near the 2.0-yr frequency band should be interpreted with caution because they are close to the Nyquist frequency, and, therefore, energy at higher frequencies may have been folded back into the spectrum at this frequency. Moreover, an examination of the geographical distribution of stations with significant periodicities in the range of 2.0-6.0 years frequency bands shows no clear indication of any large-scale spatial coherence in their oscillations (Figs. 3-5).



Figs. 2 (g-l). Non-integer spectral analysis results for selected stations in the Great Plains : (g) Fayette (Iowa), (h) Medicine Lodge (Kansas), (i) Arnite (Louisiana), (j) Fosston (Minnesota), (k) Clinton (Missouri), (l) Valier (Montana)



Figs. 2 (m-r). Non-integer spectral analysis results for selected stations in the Great Plains : (m) Hebron (Nebraska), (n) Armistad (New Mexico), (o) Dickinson (N. Dakota), (p) Hooker (Oklahoma), (q) Cottonwood (S. Dakota), (r) Corpus Christi (Texas)

Fig. 3

Fig. 4

Fig. 5

Figs. 3-5. Spatial distribution of stations with significant 2.0-2.9 yr (Fig. 3), 3.0-3.9 yr (Fig. 4) and 5.0-5.9 yr (Fig. 5) periodicities

Fig. 6. Distribution of significant periodicities in the 2.0-27.0 year frequency band as indicated by the non-integer spectral method

Because the lowest frequency at which the NI spectra can be estimated is $1/n$, long-period spectra from short records may not be reliable. Experimentation with the non-integer programme suggests that a climatological time series length of 4 to 5 times the period of oscillation is needed for accurate determination of the wavelength (Neil 1981). Therefore, periods greater than 25 years which occur in the drought indices spectra for some of the stations used in the study are taken to be statistically insignificant. Thus, the tendency for the non-integer spectral technique to resolve long period in the short drought indices is taken to be more of an indication of its power to measure long-term persistence in the series rather than the method's effectiveness in spectral resolution of these long wave bands. The 11.3-yr and 22.6-yr sunspot cycles, which are now part of the drought folklore in the region of study, are found to be statistically significant in only a few of the stations analysed. Whether the sunspot cycle or its Hale cycle found in the drought series by other researchers (*e.g.*, Thompson 1973, Hancock and Yarger 1979 among others) are really persistent features in the region of study is questionable. Instrumental climatological records are still too short for detecting changes in the amplitudes of these periodicities. It is, however, possible that lack of evidence of solar-drought connections is a function of analysed sample size used and the period of study.

The purpose of the foregoing analyses is for heuristic interpretation of the spectral results. Spectrum analysis is unable to prove physical relationships and, in this case, it is not clear whether the significant spectral peaks in the 2.0-6.0 yr periodicities will persist in future drought characteristics. Moreover, the areal peaks contained within the quasi-biennial and quasi-triennial oscillations do not indicate any significant spatial coherence that can be used for future studies into the possible causes of drought in the region of study. Accordingly, there is no claim of a definite identification of all periods in the drought indices with any predictive power. The present exploratory study does not provide more than a spectral description of preferred time scales of drought, and, thereby, avoids the problems of prior *versus* posterior interpretation of the significant peaks. However, many of the spectral peaks in the 2.0-6.0 yr periods have been identified by other investigators in many other geophysical time series.

Finally, for all practical purposes, the non-integer spectral method shows that the growing season drought indices in the Interior Plains of North America have large random temporal variation. There are only short-lived time variabilities in the drought indices. These variabilities may not be the same for stations located within relatively short distances of one another. This apparent incoherent spatial and temporal behaviour of the drought indices in the region of study will complicate the theoretical development of its causal mechanisms.

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References

Angell, J.K., Korshover, J. and Cotten, G.F., 1969, Quasi-biennial variations in the "centres of action", *Mon. Weath. Rev.*, **97**, 867-872.

Baggeror, A.B., 1976, Confidence intervals for regression (MEM) spectral estimates, *IEEE Trans. Inf. Theory. IT*-22, 534-545.

Bhalme, H.N., 1975, Fluctuations in the pattern of distribution of southwest monsoon rainfall over Rajasthan and their association with sunspot cycle, *Indian J. Met. Hydrol. Geophys.*, **26**, pp. 57-64.

Bhalme, H.N. and Mooley, D.A., 1980, Large-scale droughts/floods and monsoon circulation, *Mon. Weath. Rev.*, **108**, 1197-1211.

Blackman, R.B. and Tukey, J.W., 1959, *The Measurement of Power Spectra from the Point of View of Communication Engineering*, Dover Publications, New York, 190 pp.

Currie, R.G., 1981, Evidence of 18.6 year MN signal in temperature and drought conditions in North America since 1800 AD, *J. geophys. Res.*, **86**, 11055-11064.

Currie, R.G., 1984 (a), Evidence for 18.6-year lunar nodal drought in Western North America during the past millenium, *J. geophys. Res.*, **89**, 1295-1308.

Currie, R.G., 1984 (b), Periodic (18.6-year) and cyclic (11-year) induced drought and flood in Western North America, *J. geophys. Res.*, **89**, 7215-7230.

Fleer, H., 1981, Large-scale tropical rainfall anomalies, *Bonner Met. Abhand.* **26**, 114 pp.

Flohn, H. and Fleer H., 1975, Climatic teleconnections with the equatorial Pacific and the role of ocean/atmosphere coupling, *Atmosphere*, **15**, 96-109.

Granger, O.E., 1977, Secular fluctuations of seasonal precipitation in lowland California, *Mon. Weath. Rev.*, **105**, 386-397.

Hancock, D.J. and Yarger, D.N., 1979, Cross-spectral analysis of sunspots and monthly temperature and precipitation for the contiguous United States, *J. atmos. Sci.*, **36**, 746-753.

Hastenrath, S. and Kaczmarczyk, E.B., 1981, On spectra and coherence of tropical climate anomalies, *Tellus*, **33**, 453-462.

Jagannathan, P. and Parthasarathy, B., 1973, Trends and periodicities of rainfall over India, *Mon. Weath. Rev.*, **101**, 371-375.

Jones, R.H., 1965, A reappraisal of the periodogram in spectral analysis, *Technometrics*, **7**, 531-542.

Klaus, D., 1978, Spatial distribution and periodicity of mean annual rainfall of the Sahara, *Arch. Met. Geophys. Biok. Ser. B.*, **26**, 17-27.

Kousky, V.E. and Chu, P.S., 1978, Fluctuations of annual rainfall for northeast Brazil, *J. met. Soc. Japan*, **56**, 457-465.

Kraus, E.B., 1977, Sub-tropical droughts and cross-equatorial energy transports, *Mon. Weath. Rev.*, **105**, 1009-1018.

Lacoss, R.T., 1971, Data adaptive spectral analysis methods, *Geophysics*, **36**, 661-675.

Neil, C., 1981, Using non-integer spectral analysis in discerning spatially coherent rainfall periodicities, *Time Series Analysis*, D. Anderson and M.R. Perryman, eds. North-Holland Publishing Co., 375-384.

Ogallo, L., 1979, Rainfall variability in Africa, *Mon. Weath. Rev.*, **107**, 1133-1139.

Oladipo, E.O., 1985, A comparative performance analysis of three meteorological indices, *J. Climatol.*, **5**, 655-664.

Oladipo, E.O., 1986, Spatial patterns of drought in the Interior Plains of North America, *J. Climatol.*, **6**, 495-513.

Palmer, W.C., 1965, Meteorological drought, *U.S. Weather Bureau Res. Paper* No. 45, 58 pp.

Rao, K.N., George, C.J., Morey, P.B. and Mehta, N.K., 1973, Spectral analysis of drought index (Palmer) for India, *Indian J. Met. Hydrol. Geophys.*, **24**, pp. 257-270.

Reed, R.J., Campbell, W.J., Rasmussen, L.A. and Rogers, D.G., 1962, Evidence of a downward propagating annual wind reversal in the equatorial stratosphere, *J. geophys. Res.*, **66**, 813-818.

Reid, J.S., 1979, Confidence limits and maximum entropy spectra, *J. geophys. Res.*, **84**, 5289-5301.

Rodhe, H. and Virji, H., 1976, Trends and periodicities in east African rainfall date, *Mon. Weath. Rev.*, **104**, 307-315.

Sakurai, K., 1981, Quasi-biennial oscillation of the solar activity and its control of the earth's climate, Proc. Int. Conf., *Sun and Climate CNES*. Toulouse, 165-172.

Schickedanz, P.T. and Bowen, E.G., 1977, The computation of climatological power spectra, *J. appl. Met.*, **16**, 359-367.

Tabony, R.C., 1979, A spectral and filter analysis of long-period rainfall records in England and Wales, *Met. Mag.*, **108**, 97-118.

Thompon, L.M., 1973, Cyclical weather patterns in the middle latitudes, *J. Soil. Water Conservation*, **28**, 87-89.

Ulrych, T.J., 1972, Maximum entropy spectrum of truncated sinusoids, *J. geophys. Res.*, **77**, 1396-1400.