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Simulation of the thermodynamic structure of atmospheric boundary layer over Calcutta with a one-dimensional TKE closure PBL model

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सार—मानसून द्रोणी के पूर्वी क्षेत्र में कलकत्ता में परिसीमा स्तर की उष्मागतिक संरचना का परीक्षण किया गया। यह परीक्षण उष्णकटिबंधीय क्षेत्र के लिए एक विमीय टी०के०ई० क्लोज़र भूमण्डलीय परिसीमा स्तर मॉडल को एकीकृत करके किया गया, जिसमें कि मेघाच्छन्नता और विकिरण के साथ उष्मा, आर्द्रता और संवेग के काउंटर ग्रेडिएंट परिवहन का टकराव शामिल है। इस कार्य के लिए, 1988 में मांटब्लेक्स के पायलट प्रयोग के चरण के आंकड़ा समुच्चयों का प्रयोग किया गया। दो विशेष अवस्थाओं पर विचार किया गया है। पहली अवस्था में तरल जल विद्यमान होता है और दूसरी परिस्थिति में अतितीव्र पवनें प्रचालित होती हैं। प्रक्षोभ गतिज ऊर्जा की दैनिक परिवर्तिता, टी०के०ई० बजट तथा टी०के०ई० के उर्ध्वाधर प्रोफाइल और भंवर विनिमय के गुणांक न केवल उष्मा और आर्द्रता के काउंटर ग्रेडिएंट परिवहन के महत्व, अपितु संवेग की महत्ता को भी प्रदर्शित करते हैं। उत्पलावित उत्पत्ति में काउंटर ग्रेडिएंट और तरल जल की मौजूदगी और प्रतिबल उत्पत्ति में संवेग के काउंटर ग्रेडिएंट की संयुक्त भूमिका की पुष्टि की गई है।

ABSTRACT. The thermodynamic structure of boundary layer over Calcutta on the eastern sector of the monsoon trough has been examined by integrating a one-dimensional TKE closure planetary boundary layer model for tropics which inlcudes interaction of cloudiness and radiation with turbulence and counter gradient transports of heat, moisture and momentum. Data sets of pilot-experiment phase of MONTBLEX in 1988 have been used for this purpose. Two specific situations, one when liquid water is present and the other when very strong winds are prevailing in the boundary layer are considered. Diurnal variation of turbulent kinetic energy, the TKE budget and the vertical profiles of TKE and eddy exchange coefficient have revealed the importance of counter gradient transports not only of heat and moisture but also of momentum. Combined role of presence of liquid water and counter gradients in buoyant production and role of counter gradients of momentum in shear production have been established.

Key words — Planetary boundary layer, Turbulent kinetic energy, Counter gradient, Sensible and latent heat flux, Cloud liquid water.

1. Introduction

The southwest summer monsoon over Indian subcontinent is characterised by a large scale synoptic feature, known as the *monsoon trough* which is an elongated zone of low pressure extending from the desert heat low on the west through the Gangetic plains and dipping into the Bay of Bengal on the east. The rainfall activity of the sub-continent is largely associated with the southward to northward migration of the monsoon trough, leading to active and break conditions of the monsoon. Studies on the boundary layer processes over land regions of tropics are very limited and those along the monsoon trough have been reported only in recent years.

Holt and Raman (1986) reported observed marine boundary layer characteristics different from land type, using the MONEX-79 ships' data over the Bay of Bengal. Holt and Raman (1987) have also analysed the boundary layer structure over the Indian seas during active and break phases of monsoon and have reported (Holt and Raman 1988) simulation studies on the marine boundary layer over the Bay of Bengal. However, similar such studies have not been reported over land stations along the monsoon trough region until the emergence of MONTBLEX programmes. The pilot experiment phase of MONTBLEX in 1988, as a preliminary step, has provided radiosonde/slow rising balloon observations taken four times a day (0530, 1130, 1730 and 2330 IST) at standard pressure levels and at other significant heights in the vertical from 20 July to 1 August at Calcutta, Bhubaneswar, Ranchi and Patna in the vicinity of the eastern sector of the monsoon trough.

In the present study, the thermodynamic structure of the boundary layer over Calcutta has been examined using a one-dimensional TKE-closure Planetary Boundary Layer (PBL) model. An attempt is made here to establish the role of counter gradient transports of heat, moisture and momentum alongwith the interaction of cloudiness and radiation with turbulence in the PBL. The mathematical formulation of the model is given in section 2. The numerical solution procedure of the model is given in section 3. Section 4 deals with the data used for the purpose. Results are discussed in section 5 followed by conclusions in section 6.

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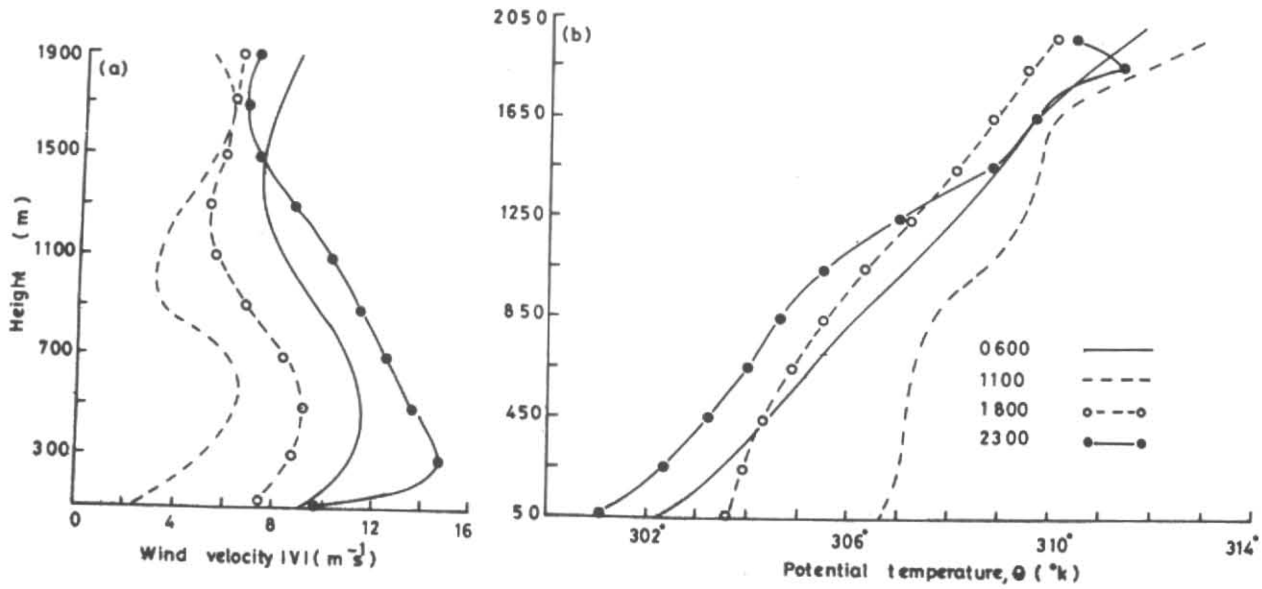
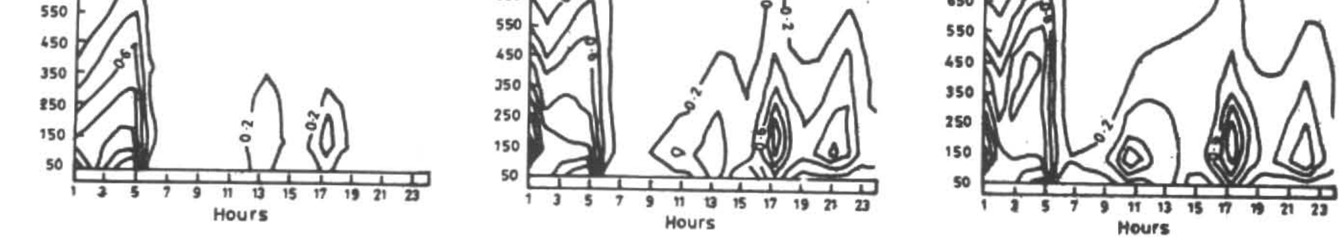


Figs. 1 (a & b). Vertical profiles of: (a) mean wind, and (b) potential temperature on 20 July 1988 at Calcutta



Figs. 2 (a-c). Diurnal variation of turbulent kinetic energy (unit : m^2s^{-2}) : (a) in the absence of counter gradients, (b) in the presence of counter gradients, and (c) in the presece of liquid water and counter gradients

2. Mathematical formulation

The basic equations, the assumptions under which the equations are derived, the formulation of counter gradient transports, the boundary conditions and the physical processes involved in the one-dimensional Turbulent Kinetic Energy (TKE) closure PBL model are given in the following sub-sections :

2.1. *Basic equations*

The basic equations, of momentum, heat, moisture and turbulent kinetic energy in the atmospheric boundary layer are as given below (Mellor and Yamada 1974 and Lykossov 1990) :

$$\partial u/\partial t = f(v - v_g) + \partial(K_u \partial u/\partial z)/\partial z - \partial(K_u \gamma_u)/\partial z \quad (1)$$

$$\partial v/\partial t = -f(u-u_g) + \partial(K_u \partial v/\partial z)/\partial z - \partial(K_u \gamma_v)/\partial z \quad (2)$$

$$\partial \theta/\partial t = \partial(\alpha_\theta K_u \partial\theta/\partial z)/\partial z - \partial(\alpha_\theta K_u \gamma_\theta)/\partial z + Q_R + Q_L \quad (3)$$

$$\partial q/\partial t = \partial(\alpha_\theta K_u \partial q/\partial z)/\partial z - \partial(\alpha_\theta K_u \gamma_q)/\partial z - Q_F \quad (4)$$

$$\partial q_l/\partial t = \partial(\alpha_\theta K_u \partial q_l/\partial z)/\partial z - \partial(\alpha_\theta K_u \gamma_{ql})/\partial z + Q_F \quad (5)$$

$$\partial e/\partial t = -\alpha_0 K_u \lambda\,(\partial\theta/\partial z + .61\, g/\lambda\, \partial q/\partial z - \gamma_\theta - .61\, g/\lambda\, \gamma_q) + K_u\,[\partial u/\partial z\,(\partial u/\partial z - \gamma_u) + \partial v/\partial z\,(\partial v/\partial z - \gamma_v)] + \alpha_b\, \partial(K_u\, \partial e/\partial z)/\partial z - C_e/l\, e^{3/2} \quad (6)$$

where, u, v are the horizontal wind components. θ is potential temperature, q is specific humidity of air, q_l is specific humidity of liquid water, e is turbulent kinetic energy, l is mixing length, u_g and v_g are geostrophic wind components, K_u is eddy exchange coefficient, γ_u, γ_v, γ_θ, γ_q and γ_{ql} are the counter gradient transports of momentum, heat, moisture and liquid water. λ is convection parameter ($=g/\theta$), Q_R and Q_F are source/sink terms of radiation and moisture. C_e and α_0 are constants. In Eqn. (6), the first term on the right hand side represents the buoyancy production/loss, the second term is the shear production and the third, the energy transport and the fourth, the rate of dissipation of turbulent kinetic energy.

2.2. *Assumptions*

The system of PBL Eqns. (1-6) is obtained under the following general assumptions :

(i) The mean state of atmosphere is in hydrostatic equilibrium.

(ii) The horizontal variations of turbulent fluxes are relatively smaller as compared to vertical variations.

(iii) The molecular contributions are negligible as compared to the small scale exchange terms (except in the lowest millimetre of the atmosphere).

(iv) The flow is horizontally homogeneous.

The turbulent length scale (l) is given by the following equation (Blackadar 1962) :

$$l = kz/(1 + kz/l_\infty) \quad (7)$$

where,

$$l_\infty = 2.7 \times 10^{-4}\, \frac{|v_g|}{|f|} \quad (8)$$

The constants α_b and C_e are assumed to be 0.73 and 0.07 respectively.

2.3. *Counter gradients*

The role of counter gradient transport of heat in nearly neutral or weakly stable situations has well been studied in literature (Budyko and Yudin 1946, Deardorff 1966, 1972; Lykossov 1991). From a study on the jet during winter in the fair weather trade wind boundary layer to the northeast of Puerto Rico, negative wind shear within the boundary layer was seen (Pennel and Lemone 1974) and at the same time, the direction of momentum transport was towards the surface, *i.e.*, counter gradient (Lykossov 1991). In a laboratory simulation of wall jet (Narasimha 1984) it was also found counter gradient transport of momentum below maximum wind velocity. Thus mathematical expressions for the momentum (Wilson and Shaw 1977, Li *et al.* 1985 and Lykossov *et at.* 1991) and moisture counter gradients (Lykossov 1991) are developed, based on generalized Boussinesq approximation.

In general, the counter gradients are expressed as follows (Lykossov 1991) :

$$\gamma_D = \frac{\lambda[\gamma\partial\theta/\partial z - (\gamma - l)\gamma_\theta]\partial D/\partial z + \mu\, \omega^2\, \gamma_{DN}}{\lambda\, \partial\theta/\partial z + \mu\, \omega^2} \quad (9)$$

where, D is a variable, ω is turbulence frequency, γ_{DN} are characteristic values of γ_D, and

$$\gamma = 1,\ \mu = \mu_0 \text{ for } D = \theta$$

$$\gamma = 2,\ \mu = 2\mu_0 \text{ for } D = q \quad (10)$$

$$\gamma = \mu_2,\ \mu = \mu_1 \text{ for } D = u \text{ or } v$$

The values of μ_0, μ_1 and μ_2 are 0.05, 0.11 and 1.56 respectively.

2.4. *Boundary conditions*

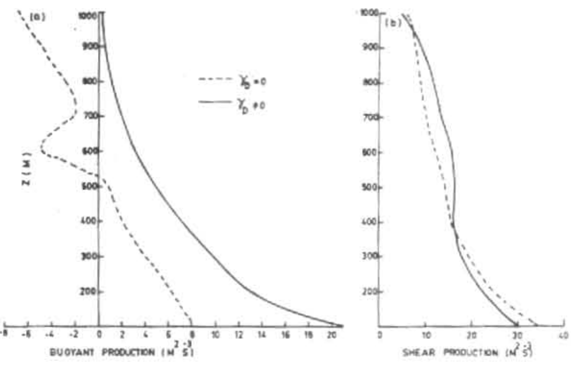
At the lower boundary which is treated as the top of the constant flux layer (50 m), the turbulent fluxes of u, v, θ and q, based on similarity theory are equated in terms of Bulk-aerodynamic relations. The turbulent flux of q_l is set to zero so that low-level fog like cloud formation is eliminated. The turbulent kinetic energy is derived from the closure assumption on K_u. Accordingly,

$$K_u\,(\partial u/\partial z - \gamma_u) = C_D\, Vu \quad (11)$$

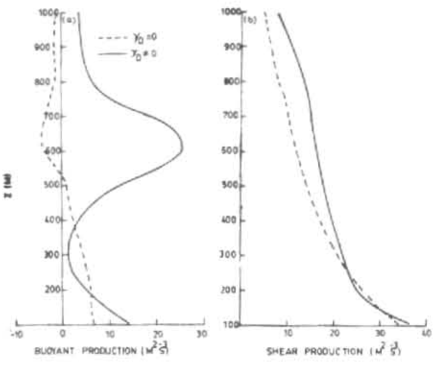
$$K_u\,(\partial v/\partial z - \gamma_v) = C_D\, Vv \quad (12)$$

$$\alpha_0 K_u\,(\partial\theta/\partial z - \gamma_\theta) = C_\theta\, V\,(\theta - \theta_s) \quad (13)$$

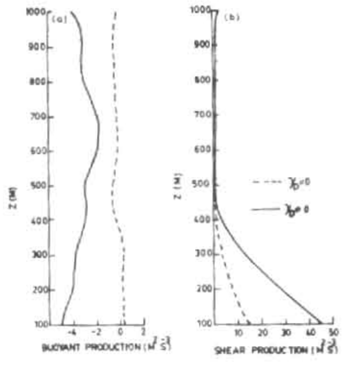
$$\alpha_0 K_u\,(\partial q/\partial z - \gamma_q) = C_\theta\, V\,(q - q_s) \quad (14)$$



Figs. 3 (a & b). (a) Buoyancy, and (b) shear terms of turbulent kinetic energy budget at 0900 IST in the absence of liquid water. (unit : $10^{-4} m^2 s^{-3}$)



Figs. 4 (a & b). As in Fig. 3 but in the presence of liquid water



Figs. 5 (a & b). As in Fig. 3(a) but at 2300 IST

$$\alpha_\theta K_u \,(\partial q_l/\partial z - \gamma_{ql}) = 0 \tag{15}$$

$$e = (K_u/l)^2 \tag{16}$$

where, C_D, C_θ are exchange coefficients of momentum and heat respectively. The eddy exchange coefficient, the heat and drag coefficients are determined from Monin-Obukhov similarity theory using Businger-Dyer relations and 1/3—law of asymptotics for thermal stratification.

At the top of the boundary layer (H = 2000 m) in the model, the winds, temperature, humidity are assumed to attain their values in the free atmosphere. The energy flux is assumed to be zero.

Accordingly,

$$u = u_H \tag{17}$$

$$v = v_H \tag{18}$$

$$\theta = \theta_H \tag{19}$$

$$q = q_H \tag{20}$$

$$q_l = q_H \tag{21}$$

$$\partial e/\partial z = 0 \tag{22}$$

2.5. *Physical process*

In the above PBL model the following physical processes are involved :

* Dry and moist convective adjustment
* Large scale condensational heating
* Sensible and latent heat fluxes
* Long wave and short wave radiative fluxes
* Cloud liquid water.

3. Numerical solution

In the above PBL model, the vertical domain, extending from 50 to 2000 m, is divided in to 40 equidistant levels with the lowest level representing the top of constant flux layer (50 m) and the highest level representing the top of the boundary layer (2000 m).

Denoting the levels by an index K, the central difference forms are given by :

$$\frac{\partial D}{\partial z} = \frac{D\,(K+1) - D\,(K-1)}{2\Delta z} \tag{23}$$

$$\frac{\partial}{\partial z}\left(K_u \frac{\partial D}{\partial z}\right) = \frac{1}{\Delta z}\left[\frac{K_u(K) + K_u(K+1)}{2}\,\frac{D\,(K+1) - D(K)}{\Delta z} - \frac{K_u(K) + K_u(K-1)}{2}\,\frac{D(K) - D(K-1)}{\Delta z}\right] \tag{24}$$

$$\frac{\partial}{\partial z}(K_u\,\gamma_D) = \frac{1}{\Delta z}\left[\frac{K_u(K) + K_u(K+1)}{2}\,\frac{\gamma_D(K) + \gamma_D(K+1)}{2} - \frac{K_u(K) + K_u(K-1)}{2}\,\frac{\gamma_D(K) + \gamma_D(K-1)}{2}\right] \tag{25}$$

where, D denotes a dependent variable, and γ_D, the counter gradient associated with the variable D.

The time integration is carried out using the scheme :

$$\frac{\partial D}{\partial t} = \frac{D_K^{n+1} - D_K^n}{\Delta t} \tag{26}$$

A general finite difference form of the equation is as follows :

$$A(K)\, D^{(n+1)}\,(K-1) - B(K)\, D^{(n+1)}\,(K) + C(K) D^{(n+1)}\,(K+1) = -F^n\,(K) \tag{27}$$

where,

$$A\,(K) = \frac{\alpha}{2(\Delta z^2)}\,[K_u\,(K) + K_u\,(K-1)]$$

$$C\,(K) = \frac{\alpha}{2(\Delta z^2)}\,[K_u(K) + K_u(K+1)] \tag{28}$$

$$B\,(K) = A\,(K) + C\,(K) + \frac{1}{\Delta t}$$

$F^n\,(K)$ is the rest of the diagnostic variables.

Here, $\alpha = 1$ for u and v coupled equation, $\alpha = \alpha_\theta$ for θ, q and q_l equations and $\alpha = \alpha_b$ for turbulent kinetic energy equation.

The boundary conditions in finite difference form are expressed in a similar manner. The system of Eqns. (1-6) with the boundary conditions (11-22) is solved numerically using a tri-diagonal matrix method (Richtmyer and Morton 1967).

4. Data

During the pre-MONTBLEX programme in 1988, special upper air observations were taken four times a day, *viz.,* 0530, 1130, 1730 and 2330 IST from 20 July to 1 August at Calcutta, Bhubaneswar, Patna and Ranchi constituting land stations on the eastern sector of the monsoon trough. This sector is considered as a deep-moist convective region (85-90°E) (Goel and Srivastava 1990). In the present study, the upper air data of 20 July at Calcutta have been chosen for its meteorological significance. The coastal station situated at the head Bay of Bengal is significant for the reversal of summer monsoon wind from southwest to northwest and also for its direct influence under tropical cyclonic systems developed in the Bay of Bengal. Incidentally, The period from 15-22 July 1988 was synoptically active with two low pressure systems developed across the Bay.

In the present 24-hour simulation study, the initial fields comprise of vertical profiles of horizontal wind components, the potential temperature, moisture and pressure at 0600 IST on 20 July. Time series of (20-21 July) temperature, pressure and relative humidity at the first level of observations and time series of wind components, temperature and moisture at a height of 2000 m have been used for boundary conditions.

The vertical profiles of mean wind (V) and potential temperature (θ) at 0600, 1100, 1800 and 2300 IST on 20 July have been shown in Figs. 1(a) and 1(b) respectively. The features that can be identified are :

(i) Stronger winds prevailing at 2300 IST, extending to greater heights within 2000 m with maximum of 15 ms^{-1} at 300 m.

(ii) Well mixed layer extending to 400 m developed at 1100 IST on this day.

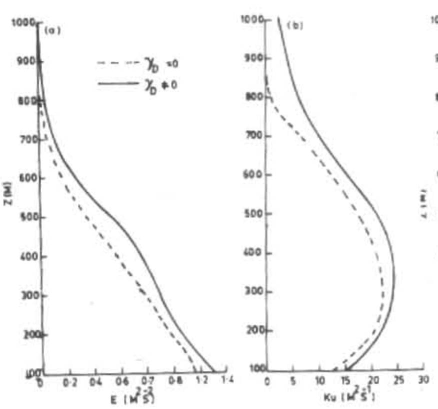
5. Results and discussion

The diurnal variation of turbulent kinetic energy (TKE), the TKE budget and the vertical profiles of TKE and eddy exchange coefficient in the boundary layer over Calcutta obtained on a 24-hour integration of a one-dimensional TKE closure PBL model with initial profiles at 0600 IST as input have been discussed in sub-sections 5.1 to 5.4. In sub-section 5.5, the model simulated profiles of potential temperature are compared with the observed profiles.

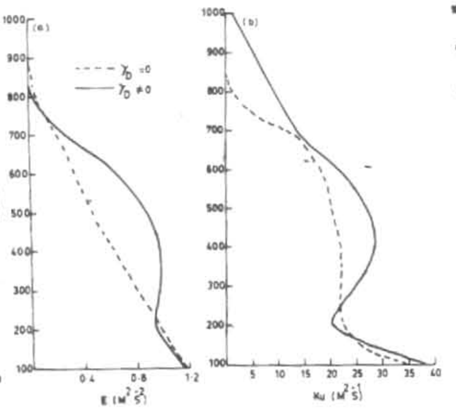
Hourly analysis of the 24-hour model integration revealed the presence of liquid water between 0900 & 1100 IST, with maximum at 1100 IST. The present analysis of TKE-related quantities are limited to two timings, *viz.,* 0900 IST (the time at which liquid water was initially present) and 2300 IST [the time at which the winds were strong in the lower part of the boundary layer, *cf.* Fig. 1 (a)].

5.1. *Diurnal variation of TKE in the absence of counter gradients*

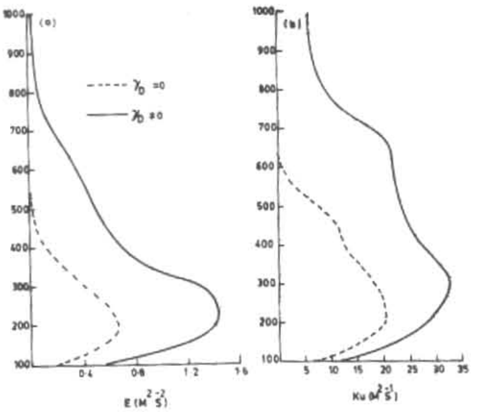
The diurnal variation of TKE for the day 20-21 July in the absence of counter gradients as well as liquid water is depicted in Fig. 2(a). In the vertical, the TKE is distributed to a height of about 700 m at 1100 IST. The vertical distribution is limited to a height of about 300 m during 2300 IST Observed profiles of potential temperature [Fig. 1(b)] suggest a well-mixed layer extending to about 450 m at 1100 IST. Profiles of wind [Fig. 1(a)] show maximum of winds at 300 m at 2300 IST. Liquid water was present in the lower part of boundary layer between 400 & 600 m during 0900-1100 IST on



Figs. 6 (a & b). Vertical distribution of : (a) turbulent kinetic energy, and (b) eddy viscosity coefficient at 0900 IST in the absence of liquid water



Figs. 7 (a & b). As in Fig. 6 but in the presence of liquid water



Figs. 8 (a & b). As in Fig. 6 but at 2300 IST

20 July as revealed by the model. There is, however, no indication of liquid water present during the rest of the day. The diurnal variation of TKE both in the presence and in the absence of liquid water remained the same and is as shown in the figure. This shows that in the absence of counter gradients, processes associated with liquid water within the boundary layer may not have appreciable influence on the turbulent kinetic energy distribution.

5.2. *Diurnal variation of TKE in the presence of counter gradients*

The diurnal variation of TKE for the day 20-21 July in the presence of counter gradients and in the absence of liquid water is depicted in Fig. 2(b). In the vertical, the TKE is distributed to a height of 750 m at 1100 IST which is slightly at a higher level than in the absence of counter gradients. The TKE distribution is continuous during evening and night with vertical extension up to 1300 m at 2300 IST. The maximum of TKE is at 300 m at 2300 IST, which corresponds to the maximum of winds at this hour [Fig. 1(a)]. The TKE distribution in the presence of counter gradients thus differs significantly during evening and night as compared to the distribution in the absence of counter gradients. In the presence of the maximum of liquid water at 400 m at 1100 IST, the TKE has reached its maximum at 400 m at the same time [Fig. 2(c)], thus differing in its vertical distribution from the case of no-liquid water.

5.3. *The TKE budget*

The buoyancy production or loss term and the shear production term of the TKE budget Eqn. (6) are discussed in this section. The compensating terms like the energy-dissipation etc are not dealt with here. The buoyancy production/loss and shear production terms at 0900 and 2300 IST are depicted in Figs. 3-5. From Fig. 3(a), it can be noticed that in the absence of liquid water, there is buoyancy loss above 500 m when counter gradients are absent while it is buoyancy production in this region when counter gradients are present. There is, however, not much change in buoyancy production/loss when counter gradients are absent but liquid water is present [Fig. 4(a)]. In the presence of both counter gradients and liquid water (which is present between 400 & 600 m), the buoyancy production has increased from a level at 300 m and decreased beyond a level at 600 m [Fig. 4(a)]. The increasing trend may be attributed to processes like phase to phase changes and condensational heating that might have affected the counter gradient transports of temperature and humidity.

The shear production at 0900 IST in the absence or in the presence of liquid water has not differred much [Figs. 3(b) and 4(b)] but affected by the counter gradients.

At 2300 IST, the role of the counter gradient transport of momentum in shear production is clearly seen below 500 m in the lower part of the boundary layer [Fig. 5(b)]. A comparison of the almost-zero buoyancy throughout boundary layer in the absence of counter gradients with the negative buoyant production in the presence of counter gradients [Fig. 5(a)] shows the role of counter gradients of heat and moisture.

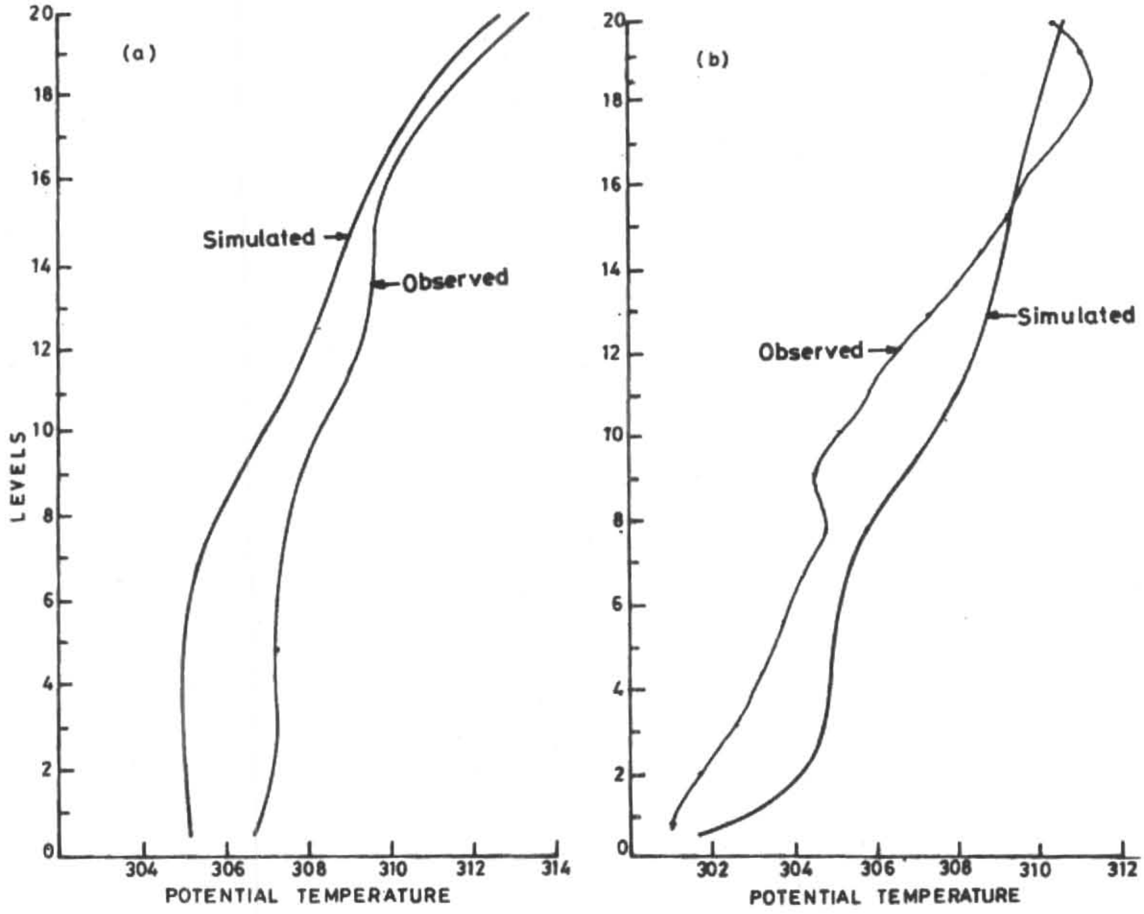


Fig. 9. Profile of potential temperatures at 1100 & 2300 IST on 20 July 1988

5.4. Vertical distribution of TKE and eddy exchange coefficient

The vertical profiles of TKE (e) and eddy exchange coefficient (K) at 0900 and 2300 IST are shown in Figs. 6-8. At these hours, the atmospheric boundary layer was relatively more turbulent on the day 20-21 July. The processes for TKE production on these two timings seem to be different. The TKE production at 0900 IST is mainly due to buoyancy production and that at 2300 IST is mainly due to shear production. At 0900 IST, due to processes like condensational heating and phase to phase changes, the vertical profiles of background potential temperature and specific humidity are relatively more affected than the velocity profiles, which in turn, have affected their corresponding counter gradients. At 2300 IST, due to the strong wind shear that prevailed, the counter gradients associated with velocity components are more affected. the eddy exchange coefficient (K) determined by Kolmogorov's relation ($K=le^{1/2}$) follows the TKE pattern at these hours.

5.5. Simulated profiles of potential temperature

The simulated vertical profiles of potential temperature at 1100 and 2300 IST are compard with the observed profiles in Figs. 9 (a) and 9 (b) respectively. The simulations at these hours as compared to the observed profiles show departures of the order of 2° K in the lower levels. Such departures can be due to the limitation of the one-dimensional PBL model deprived of any possible interaction of advection processes in time.

6. Conclusions

From a 24-hour integration of a one-dimensional TKE closure PBL model using the data sets of 20 July 1988 at the coastal station Calcutta on the eastern sector of the monsoon trough, the following broad conclusions are drawn :

The buoyancy production associated with turbulent kinetic energy within the PBL increases in the presence of liquid water under the influence of counter gradients of heat and moisture than in their absence. Enhanced by the counter gradient transport of momentum under strong wind shear conditions, the turbulent kinetic energy is transported to greater heights within the PBL.

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