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A primitive equation spectral model for study of planetary-scale atmospheric flow*

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सार — हाल ही के वर्षों में वायुमण्डल के लिये आंकिक निदर्शण में विकास से पता चला है कि ग्रहीय/भूमण्डलीय-मापी वायुमण्डल के सामान्य परिक्रमा अध्ययनों के लिये मानावलीय निदर्श अधिक उपयुक्त हैं। पिछले दशक में विकसित रूपान्तर विधि के उपयोग ने अभिकलनीय गति में मानावलीय निदर्शों को ग्रिड प्वाइन्ट निदर्शों की तुलना में ला खड़ा किया है।

मानावलीय सूत्रों का उपयोग करते हुए एक भूमण्डलीय पूर्वग समीकरण निदर्श भारत मौसम विज्ञान विभाग में विकसित किया जा रहा है। मानावलीय अंकन सतही गोलाकार हार्मोनिक फंक्शनों की छिन्न शृंखला के रूप में है। वर्तमान अभिकलन संबंधी सुविधाओं के द्वारा एक उच्च वियोजन निदर्श का विकास नहीं किया जा सकता। इसलिये क्षैतिज वियोजन क्षेत्रीय दिशा में छः फौरिअर तरंगों तथा रेखांशीय दिशा में लेजांड्रे फंक्शन के छः शून्यों तक सीमित है। ऊर्ध्वाधर में, निदर्श में सिग्मा निर्देशांकों की तीन सतह हैं।

निदर्श विकास प्रारंभिक अवस्था में है। इस लेख में, निदर्श संरचना, इसके साथ किए गए कुछ प्रयोगों के परिणामों तथा भावी विकास योजनाओं पर चर्चा की गई है।

ABSTRACT. The developments in numercial modelling for the atmosphere in recent years have shown that the spectral models are well suited for general circulation studies of the planetary-scale atmosphere. The use of transform method developed during last decade has made the spectral models comparable in computational speed with the grid point models.

A global primitive equation model using spectral formulation is under development in the India Meteorological Department. The spectral representation is in terms of a truncated series of surface spherical harmonic functions. The computational facilities available at present do not permit the development of a high resolution model. The horizontal resolution, therefore, is limited to six Fourier waves in the zonal direction and six zeros of Legendre functions in the meridional direction. In the vertical, the model has three layers in sigma coordinate.

The model is in its early stage of development. The model formulation, the results of some experiments performed with it and the future development plans are discussed in this paper.

1. Introduction

The use of spectral methods for integrating the equations of atmospheric motion was started more than two decades ago (Silberman 1954; Lorenz, 1960 Platzman 1960), but the initial progress was slow on account of difficulties in computing non-linear terms. The development of transform method (Orszag 1970; Machenhauer and Rasmussen 1972) for calculating non-linear terms accelerated the pace of development in this field very much. During the last one decade, spectral modelling technique has been developed to a matured stage. Machenhauer (1974) has given a comprehensive review of the mathematical aspects of spectral modelling. Daley *et al.* (1976) and Bourke *et al.* (1977) have given detailed description of operational spectral models in Canada and Australia. Basis functions for spectral representation in these models are the surface spherical harmonics.

This study presents the results of integration of a multilevel global spectral model under development in the India Meteorological Department. The computational facilities available at present in the Department do not permit the development of a high resolution spectral model. Therefore, presently it has been limited to six Fourier waves in the zonal direction and six zeros of associated Legendre functions between poles in the meridional direction. In the vertical, the model extends from the earth's surface to the top of the atmosphere with three equally spaced layers in sigma coordinate system. Although, sigma coordinate system is used, earth's topography is yet not formally incorporated. The model does not include radiation, moisture

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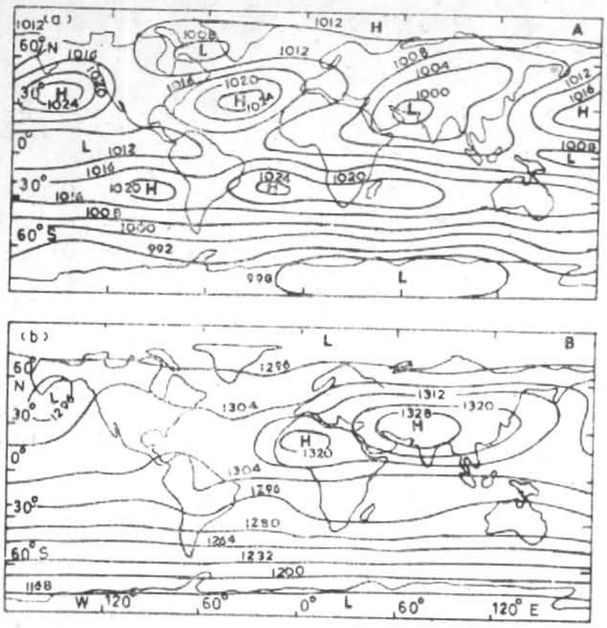

Fig. 1. Initial flow patterns
A. Surface (mean sea level), unit—mb
B. Top level ($\simeq$160 mb), unit—tens of gpm

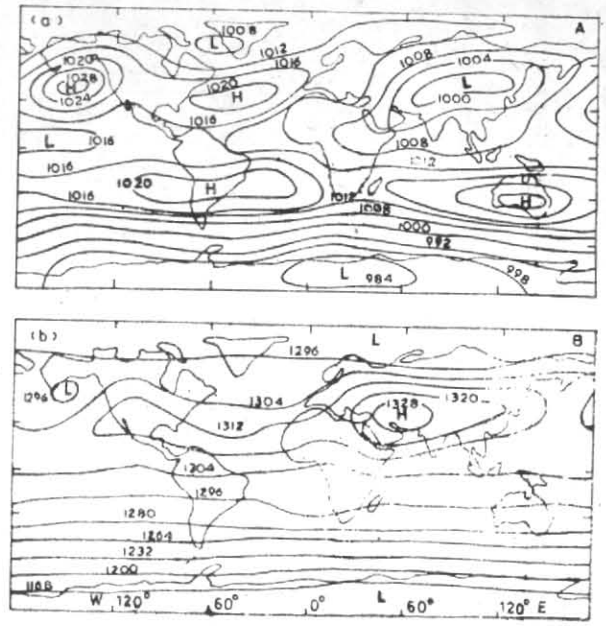

Fig. 2. 24-hr forecast patterns
A. Surface (mean sea level), unit—mb
B. Top level ($\simeq$160 mb), unit—tens of gpm

and boundary layer physics also. Originally, the model was written with equations for horizontal flow in momentum form. Subsequently, it has been modified with these two equations in vorticity and divergence form. The study presents briefly the formulation and the results of integration of this model which is yet in its preliminary stage of development.

2. The model equations

2.1. *Symbols not defined in the text*

a — radius of the earth

D — divergence

q — ln p_s ; p_s = surface pressure

t — time

T — temperature

u — zonal component of wind

v — meridional component of wind

J — $n-m$ = number of zeroes of associated Legendre function at which series is truncated

m — order of associated Legendre function : M is its highest value at which series is truncated

n — degree of associated Legendre function

$P_n^m(\phi)$ — Normalised associated Legendre function of order m and degree n

$Y_n^m(\lambda, \phi)$ — normalised spherical harmonic of order m and degree n

c_p — specific heat for air at constant pressure

R — gas constant for air

λ — longitude

ϕ — latitude

Φ — geopotential

σ — p/p_s ; p = level pressure and p_s surface pressure

$\dot{\sigma}$ — $\frac{d\sigma}{dt}$

Ω — angular velocity of the earth

Ψ — stream function

χ — velocity potential

ζ — relative vorticity

$\overline{(\,)}$ — $\int_{\sigma=1}^{0} (\,)\, d\sigma$

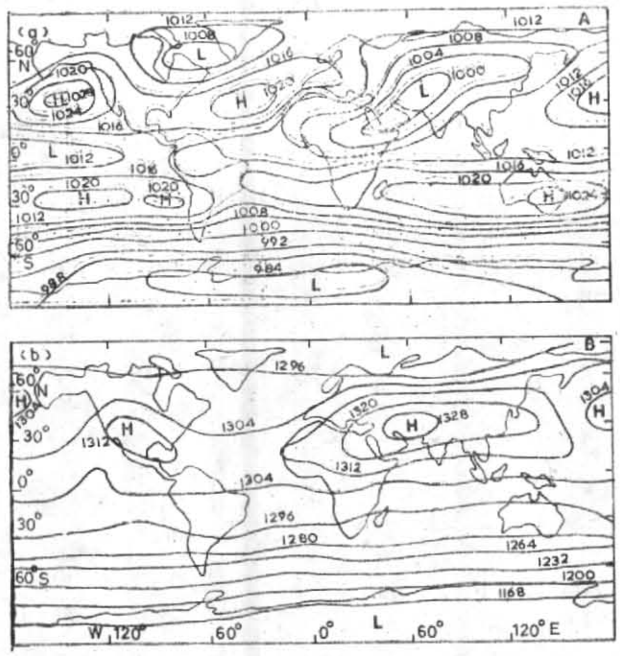

Fig. 3. Same as Fig. 2, but for 48-hr forecast

( ) — $\int_{\sigma=1}^{\sigma} (\ )\, d\sigma$

$(\ )_0$ — horizontal mean

$(\ )'$ — deviation of ( ) from its horizontal mean $(\ )_0$

2·2. The primitive form of equations of motion of the atmosphere in vorticity and divergence form in spherical coordinates in the horizontal and sigma coordinate in the vertical without external forcings and moisture processes can be written as :

$$\frac{\partial \zeta}{\partial t} = -F(A,B) - 2\Omega\left(D\sin\phi + \frac{V}{a}\right) \quad (1)$$

$$\frac{\partial D}{\partial t} = F(B,-A) + 2\Omega\left(\zeta\sin\phi - \frac{U}{a}\right) - \nabla^2 K \quad (2)$$

$$\frac{\partial T}{\partial t} = -F(P,Q) + H \quad (3)$$

$$\frac{\partial q}{\partial t} = -V.\nabla q - D - \frac{\partial \dot{\sigma}}{\partial \sigma} \quad (4)$$

where,

$$A = U\zeta + \dot{\sigma}\frac{\partial V}{\partial \sigma} + \frac{RT'}{a}\cos\phi\frac{\partial q}{\partial \phi}$$

$$B = V\zeta - \dot{\sigma}\frac{\partial U}{\partial \sigma} - \frac{RT'}{a}\frac{\partial q}{\partial \lambda}$$

$$H = T'D - \dot{\sigma}\left(\frac{RT}{\sigma c_p} - \frac{\partial T}{\partial \sigma}\right) + \frac{RT}{c_p}[(\bar{D} + (V+\bar{V}).\nabla q)]$$

$$P = UT'$$

$$Q = VT'$$

$$K = \frac{U^2+V^2}{2\cos^2\phi} + \Phi' + RT_0 q$$

$$F(x,y) = \frac{1}{a\cos^2\phi}\left[\frac{\partial x}{\partial \lambda} + \cos\phi\frac{\partial y}{\partial \phi}\right]$$

Other symbols are defined in Sec. 2.1.

The vertical boundary conditions are

$$\dot{\sigma} = 0 \quad \text{at} \quad \sigma = 1,\ 0$$

On integration Eqn. (4) from $\sigma = 1$ to $\sigma = 0$ we get the prognostic equation for surface pressure as :

$$\frac{\partial q}{\partial t} = \bar{V}.\nabla q + \bar{D} \quad (5)$$

On eliminating $\partial q/\partial t$ between Eqns. (4) and (5) and integrating the resulting equation vertically gives a diagnostic relation for $\dot{\sigma}$ as :

$$\dot{\sigma} = [(1-\sigma)\bar{D} - \bar{D}^{\sigma}] + [(1-\sigma)\bar{V} - \bar{V}^{\sigma}]\cdot\nabla q \quad (6)$$

The hydrostatic relation is given by :

$$\frac{\partial \Phi}{\partial \sigma} = -\frac{RT}{\sigma} \tag{7}$$

The Eqns. (1) to (3) and (5) to (7) form a closed system of equations for the model. They are essentially of the same form as used by Daley *et al.* (1976) and Bourke *et al.* (1977). As suggested by Robert (1966), the continuity of velocity field at poles is achieved by taking U and V in above equations as :

$$U = u\cos\phi = -\frac{\cos\phi}{a}\frac{\partial \Psi}{\partial \phi} + \frac{1}{a}\frac{\partial \chi}{\partial \lambda}$$

$$V = v\cos\phi = \frac{1}{a}\frac{\partial \Psi}{\partial \lambda} + \frac{\cos\phi}{a}\frac{\partial \chi}{\partial \phi} \tag{8}$$

3. Spectral form of equations

Spectral form of equations is obtained by expanding each of the scalar variables Ψ, χ, T and q etc, in terms of a truncated series of surface spherical harmonics as:

$$\Psi = \sum_{m=-M}^{M} \sum_{n=|m|}^{|m|+J} \Psi_n^m \, y_n^m (\lambda, \phi) \tag{9}$$

where,

$$y_n^m (\lambda, \phi) = e^{im\lambda} P_n^m (\phi)$$

Inserting such representation in Eqns. (1) to (3) and (5) and (7), multiplying both sides by complex conjugate of $y_n^m (\lambda, \phi)$ and integrating over the earth's surface, we get spectral form of equations as :

$$-\frac{n(n+1)}{a^2}\frac{\partial \Psi_n^m}{\partial t} = -F_n^m (A, B) - \frac{2\Omega}{a^2}[(n-1)(n+1) \times$$

$$\beta_{n-1}^m \chi_{n-1}^m + n(n+2)\beta_{n+1}^m \chi_{n+1}^m - im\Psi_n^m] \tag{10}$$

$$-\frac{n(n+1)}{a^2}\frac{\partial \chi_n^m}{\partial t} = F_n^m (B, -A) - \frac{2\Omega}{a^2}[(n-1)(n+1) \times$$

$$\beta_n^m \Psi_{n-1}^m + n(n+2)\beta_{n+1}^m \Psi_{n+1}^m + im\chi_n^m \Big] +$$

$$+ n(n+1) K_n^m \tag{11}$$

$$\frac{\partial T_n^m}{\partial t} = -F_n^m (P, Q) + H_n^m \tag{12}$$

$$\frac{\partial q_n^m}{\partial t} = (\mathbf{V}.\nabla q)_n^m - \frac{n(n+1)}{a^2}\chi_n^{-m} \tag{13}$$

$$\frac{\partial \Phi_n^m}{\partial \sigma} = -\frac{RT_n^m}{\sigma} \tag{14}$$

where,

$$\beta_n^m = \left(\frac{n^2 - m^2}{4n^2 - 1}\right)^{\frac{1}{2}}$$

3. Input

The basic input for the model were the spectral amplitudes of geopotential field. These were obtained by spectral decomposition of geopotential field interpolated at sigma surface at 5° Lat./Long. interval over the whole globe. Linear balance equation on sigma surface was solved to obtain stream function amplitudes from geopotential amplitudes. A quadratic interpolation of geopotential field from regularly spaced latitudes to Gaussian latitudes, and then using a Gaussian quadrature for Legendre transform was found to be a suitable scheme for spectral representation. More details of method for spectral representation and solution of balance equation are available (Bedi 1976).

This initial wind field though balanced with pressure field contained no divergence. Necessary amount of divergence was incorporated through usual initialization process by integrating the model forward by one hour followed by one hour backward integration. Pressure and wind fields were allowed to adjust each other freely during this process. Three such cycles were used to incorporate an appropriate amount of divergence.

4. Results of time integration

With the above initialized input, the model was integrated further. Spectral transform of non-linear terms was achieved by transform method (Orszag 1970; Machenhauer and Rasmussen 1972) using sixteen points for Fourier transform and eighteen Gaussian latitudes for Legendre transform. Fast Fourier Transform (FFT) method was utilised during Fourier transform. More details of the procedure are given by Bedi (1979).

For each time step, first, the spectral components of vertical velocity $\dot{\sigma}$ were calculated from Eqn. (6). Next, the spectral amplitudes of non-linear terms, $F_n^m(A,B)$, $F_n^m(B,-A)$, $F_n^m(P,Q)$, K_n^m, H_n^m and $(\mathbf{V}.\nabla q)_n^m$ were calculated. The right hand side of Eqns. (10) to (13) was then calculated to provide time tendencies for each of the amplitudes.

The time integration consisted of the first time step by Euler's backward scheme, followed by remaining time steps by centred time differencing with a fifteen minute time step. No time smoothing or diffusive smoothing were incorporated during time integration. With initial data for a mean July situation, the model was integrated upto 48 hours.

Fig. 1 shows the initial field at surface (mean sea level) and top level ($\simeq$ 160 mb) which exhibit major features of planetary-scale circulation more dominantly. Figs. 2 and 3 show the 24-hr and-48 hr forecast fields at these levels. The aim of the experiment at present is to test the model coding and stability of time integration. As such only the evolution of forecast patterns in relation to initial field is examined and no attempt is made to compare the forecast field with the actual.

As seen from Fig. 1, main circulation features truncated at wave number 6 are :

(*i*) *At the surface*—(*a*) The Asian monsoon low, a trough across the Equatorial Pacific, the Pacific and the Atlantic highs and the Iceland low in the northern hemisphere, and

(*b*) A ridge of high pressure girdling round the globe along about 25° S with maximum intensity over South Atlantic and African area (Mascaran High) and the Antarctic low in the southern hemisphere.

(*ii*) *At the top level*—A ridge of high pressure along 25-30° N extending from west Africa to the South China Sea and a low pressure area south of Alaska.

The forecast patterns indicate that the changes that took place in flow patterns during the course of integration are of a right order. However, the centre of Asian monsoon low has shown a northward displacement of about 5–10 degrees Lat. with an east-west extension in 24-hr forecast. In 48-hr forecast the position of low pressure centre gets more closer to its initial position. The Pacific and the Atlantic highs retain their position, the former strengthens by about 4 mb while the latter weakens by the same order. The equatorial trough in the Pacific also shows slight weakening. The southern hemispheric ridge, though maintains its position shows intensification over the Australian area. The Iceland low shows no significant change in its location or intensity while the Antarctic low shows slight deepening. This trend of changes hold good both for 24-hr and 48-hr forecasts, though, larger part of changes have occurred during the first 24 hours.

At the upper level, high pressure ridge over Africa shows weakening accompanied by a westward spread, which gives rise to a separate high pressure area over north America in the 48-hr forecast. Over the Asian region the intensity of ridge remains practically unchanged. The Alaskan low shows progressive weakening and becomes unimportant towards the end of forecast period.

5. Conclusions and future outlook

The results presented above are based on a very low order spectral model in its early stage of development. No physics including topographic effects has yet been incorporated. Inspite of these limitations, the model has been able to retain essential circulation features during the course of integration. A higher resolution in the horizontal and vertical and inclusion of physics like radiation and topographic effects is likely to improve the results of integration. Although, the present computing facilities are not adequate for a very high resolution model, the following developments are possible and are planned in the near future :

(*i*) Increasing the horizontal resolution of the model upto about 12 waves in the zonal direction with 12 zeros in the meridional direction; the vertical resolution to be increased upto 5 layers.

(*ii*) To develop a suitable semi-implicit time integration scheme. This is likely to be three times faster than the present explicit time integration scheme.

(*iii*) To incorporate earth's topography and radiative and diffusive processes.

After this developmental work is perfected, inclusion of moisture processes, boundary layer physics and sub-grid convective processes will be taken up. It is hoped that the model could be used for general circulation studies as well as a supplementary model to the regional primitive equation model to provide it with time varying boundary conditions.

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References

Bedi, H.S., 1976, 'Spectral representation of hemispheric flow patterns', *Indian. J. Met. Hydrol. Geophys.*, **27,** 397-408.

Bedi, H.S., 1979, 'Spectral integration of a hemispheric barotropic model', *Mausam,* **30**, 31-38.

Bourke, W., McAvaney, B., Puri, K. and Thurling, R., 1977, 'Global modelling of atmospheric flow by spectral methods in : *Methods in Computational Physics,* **17**, General circulation models of the atmosphere, Academic Press, 267-324.

Daley, R., Girard, C., Henderson, J. and Simmons, I., 1976, 'Short-term forecasting with a multi-level spectral primitive equation model,' Part I : *Model formulation. Atmos.*, **14,** 98-116.

Lorenz, E.N., 1960, 'Maximum simplification of the dynamic equations, *Tellus,* **12,** 243-254.

Machenhauer, B. and Rasmussen, E., 1972, 'On the integration of spectral hydrodynamic equations by a transform method, Copenagen University, Institute for Theoretical Meteorology Rep., No. 3, 44 pp.

Machenhauer, B., 1974, 'On the present state of spectral method in numerical integrations of global atmospheric models GARP Report on the international symposium on spectral methods in numerical weather prediction, Copenhagen, 1974, 1-21.

Orszag, S.A., 1970, 'Transform method for calculation of vector coupled sums : Application to the spectral form of vorticity equation', *J. atmos. Sci.,* **27,** 890-895.

Platzman, G.W., 1960, The spectral form of the vorticity equation, *J. Met.,* **17,** 635-644.

Robert, A., 1966, 'The integration of a low order spectral form of the primitive equations', *J. met. Soc. Japan,* **44,** 237-244.

Silberman, I., 1954, 'Planetary waves in the atmosphere', *J. Met.,* **11,** 27-34