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On the numerical simulation of large scale atmospheric motion†

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सार — वर्तमान अध्ययन का उद्देश्य यह मालूम करना है कि क्या कोई अन्तर सिग्मा निर्देशांक प्रणाली में मॉडल ए तथा मॉडल बी नामक समीकरणों की दो संरचनाओं में निहित होता है । मॉडल ए कुल ऊर्जा के संरक्षण तथा स्थितिज पूर्ण ऊष्मा की निम्न स्तरीय गतियों पर आधारित हैं । मॉडल बी कुल ऊर्जा तथा एक प्रदत्त स्वैच्छिक स्तरण के चारों ओर क्षोभ ऊर्जा के संरक्षण पर आधारित है । दोनों मॉडलों से प्राप्त पूर्वानुमान एक दूसरे के बहुत करीब थे जिससे निष्कर्ष निकलता है कि इस प्रकार के मॉडल संरचना में संवेदन/शीलता की कमी आंकड़ों में परिशुद्धता की कमी के कारण हो सकती है तथा इस बात की आवश्यकता है कि ऐसे प्रयोग एफ. जी. जी. ई. आंकड़ों के बड़े सैट पर किए जाएं ।

ABSTRACT. The purpose of the present study is to find out that within the sigma coordinate system, any difference exists in two formulations of the equations referred to as model 'A' based on the conservation of total energy and lower order moments of potential enthalpy and model B based on the conservation of total energy and perturbation energy around a given arbitrary stratification. Both the models yielded forecasts close to each other which leads to conclude that thus lack of sensitivity of the model formulation may be due to lack of precision in the data and, therefore, needs performing of similar experiments on a wide set of FGGE data sets.

1. Introduction

Numerical modelling of large-scale atmospheric motion for purposes of weather prediction, general circulation studies or climate simulation, involves a number of purely technical problems : space discretisation, for instance, can be performed using either finite differences or finite elements or else spectral techniques, the details of which can be worked out in many ways; discretisation in time can be done using various explicit, split or semi-implicit schemes with different properties; and so on. However, before going to the technical stage, one must consider a number of questions which turn out to be more fundamental from the methodological standpoint. Let us leave aside at first the parameterisation problems and concentrate for a while on the set of known properties of the atmosphere as a (frictionless, adiabatic) fluid. The main question to be asked this respect is the following : can all these properties be preserved in numerical modelling ? and if not, what choice should we then make ? (or rather : is there in some sense, what we may call a best choice ?) Historically, general circulation models have evolved in a more or less contingent way, following mainly easy paths plus a few random inventions : and no one knows for sure that kind of qualitative improvement may be obtained from radically different approaches.

One example of property usually violated in the current general circulation models is particle-wise conservation of Ertel's potential absolute vorticity for adiabatic (not necessarily hydrostatic) motion. This property, which from an Eulerian point of veiw means invariance of all moments of this quantity on any isentropic surface, seems to bear quite strong dynamical implications : nevertheless, it has been overlooked for a long time in numerical modelling, and even now general circulation models are not designed in such a way as to take advantage of it, except within very crude barotropic approximations (Arakawa 1966 ; Sadourny 1975a). In fact, a satisfactory solution of this problem requires the use of entropy as a vertical coordinate (as performed originally in Eliassen 1962 ; Eliassen and Raustein 1968) which in turn, requires a satisfactory solution of the moving boundary problem. On the contrary the present generation models mostly use normalised pressure (sigma) as vertical coordinate, which is the easiest way to treat the lower boundary condition by reducing it to a coordinate surface. Indeed, no one knows how much this general agreement on the easiest method has really impeded progress in modelling the flow dynamics: possibly the time has come for systematic investigations in this direction.

The purpose of the present work is much more modest. We choose to stand within the sigma coordinate system, and simply compare two formulations of the equations referred to as model A and model B as different from each other as we could think of, in terms of conserva-

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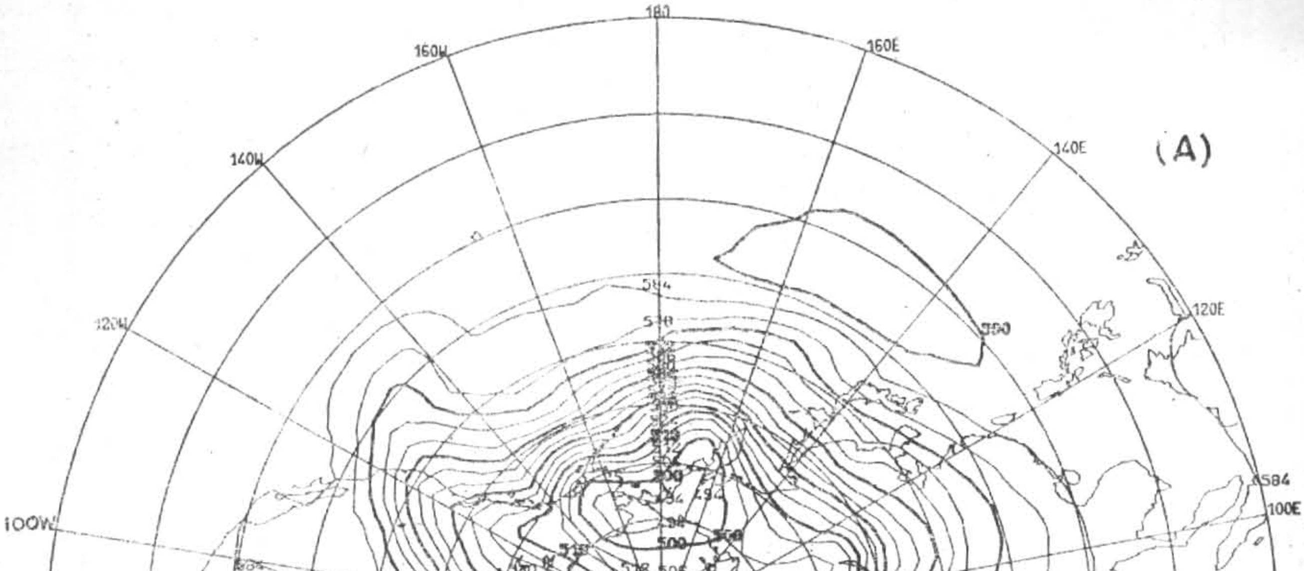

Fig. 1 (a). Four-day predictions for the 500 mb geopotential field of 8 November 1969 using model A, compared to the corresponding GFDL analysis from the GARP Basic Data Set (C). It is based on the conservation of total energy and lower order moments of potential enthalpy.

TABLE 1

σ	s	$(\sigma-Z)^{\kappa}$
0.000		
	0.300	0.322
0.038		
	0.471	0.471
0.106		
	0.582	0.589
0.208		
	0.688	0.691
0.340		
	0.775	0.777
0.489		
	0.850	0.849
0.638		
	0.906	0.905
0.770		
	0.948	0.945
0.873		
	0.975	0.972
0.941		
	0.990	0.988
0.979		
	0.997	0.997
1.000		

tion properties. (Model A is based on the conservation of total energy and lower order moments of potential enthalpy and Model B is based on the conservation of total energy and perturbation energy around a given arbitrary stratification). We thereby hope to estimate to what extent the dynamics can be improved without going to some other coordinate system ; or in other words, get some kind of measure of the variability of the dynamics within the sigma system.

2. Model A : Conservation of total energy and lower order moments of potential enthalpy

We choose to write the thermodynamic equation directly as a flux form of potential enthalpy ; for energetic consistency, the pressure gradient term in the equation of motion has to be formulated as the product of potential enthalpy by the gradient of Exner's function. The corresponding approximation for the primitive equations read :

$$\underline{\delta}\,\phi + [\theta]\,\underline{\delta}\,\pi + \left\{ f\mathbf{N}\times\mathbf{V} + \frac{D\mathbf{V}}{Dt} \right\} = 0 \qquad (2.1)$$

$$\delta_Z\phi + [\theta]\,\delta_Z\,\pi = 0 \qquad (2.2)$$

$$\underline{\delta}.\,([\theta]_A\,\mathbf{v}) + \delta_Z([\theta]_A w) + \frac{\partial}{\partial t}\,(\theta\delta_Z P) = 0 \qquad (2.3)$$

$$\underline{\delta}.\,\mathbf{v} + \delta_Z\left(\frac{\partial P}{\partial t} + w\right) = 0 \qquad (2.4)$$

respectively : the horizontal equation of motion, the hydrostatic equation, the thermodynamic equation and the continuity equation. The symbols used are defined as follows :

ϕ	geopotential
$\pi = P^{\kappa}$	Exner's function
$\theta = c_p T/\pi$	potential enthalpy
f	Coriolis parameter
$\mathbf{N}$	vertical unit vector
$\mathbf{V} = (u, v)$	horizontal velocity vector
P	local pressure
$\mathbf{v} = (u, v) = \delta_Z P\mathbf{V}$	horizontal mass flux
$w = \delta_Z P\,\dot{\sigma}$	vertical mass flux
$\partial/\partial t$	Eulerian time derivative
D/Dt	Lagrangian time derivative
[ ], [ ]$_A$	Averaging operators
$\underline{\delta} = (\delta_x, \delta_y)$	horizontal finite differencing operator
δ_Z	vertical finite differencing operator

The finite difference approximation of the two last terms in (2.1)—, referred to as a curly bracket—being of no particular importance in the present context, will not be specified. We choose to consider a horizontal grid of the C-type [Fig. 1(a)], the vertical disposition of variables being displayed in Fig. 1(b). For simplicity, the discrete averaging and differencing operators will be given minimal extent, involving two neighbours in x, y or σ only. For the thermodynamic equation, we first choose [ ]$_A$ in such a way that the (discrete) space integrals of θ and an arbitrary function A (θ) are conserved by the flux form :

$$[q]_A = \delta\left(q\,\frac{dA}{dq} - A\right) \Big/ \,\delta\left(\frac{dA}{dq}\right) \qquad (2.5)$$

(Arakawa *et al.* 1974). The problem is then to design the other averaging operators [ ] so that total energy will be conserved.

Multiplying (2.1) by $\mathbf{v}$, (2.3) by π, adding and horizontally integrating yield

$$\sum_{x,\,y}\left\{\mathbf{v}.\,\delta\phi + \pi\delta_Z([\theta]_A\,w) + \pi\frac{\partial}{\partial t}(\theta\delta_Z P) + \mathbf{v}.\frac{D\mathbf{V}}{Dt}\right\} = 0$$

provided the horizontal average [ ] is equal to [ ]$_A$. Integrating by parts in the vertical and in the horizontal and using (2.2), (2.4), one gets

$$\sum_{x,y,z}\left\{\phi\delta_Z\left(\frac{\partial P}{\partial t} + w\right) + w\delta_Z\phi + \pi\,\frac{\delta}{\delta t}\,(\theta\delta_Z\,P) + \right.$$
$$\left. + \,\mathbf{v}.\frac{D\mathbf{V}}{Dt}\right\} = 0$$

again provided [ ] = [ ]$_A$ in the vertical (Here we have used the boundary condition $w=0$ at top and bottom). Again integrating by parts the first term and using (2.2) yields

$$\sum_{x,y,z}\left\{\phi_S\,\frac{\partial P_S}{\partial t} + [\theta]_A\,\delta_Z\,\pi\,\frac{\partial P}{\partial t} + \pi\,\frac{\partial}{\partial t}(\theta\delta_Z P) + \right.$$
$$\left. + \,\mathbf{v}.\frac{D\mathbf{V}}{Dt}\right\} = 0 \qquad (2.6)$$

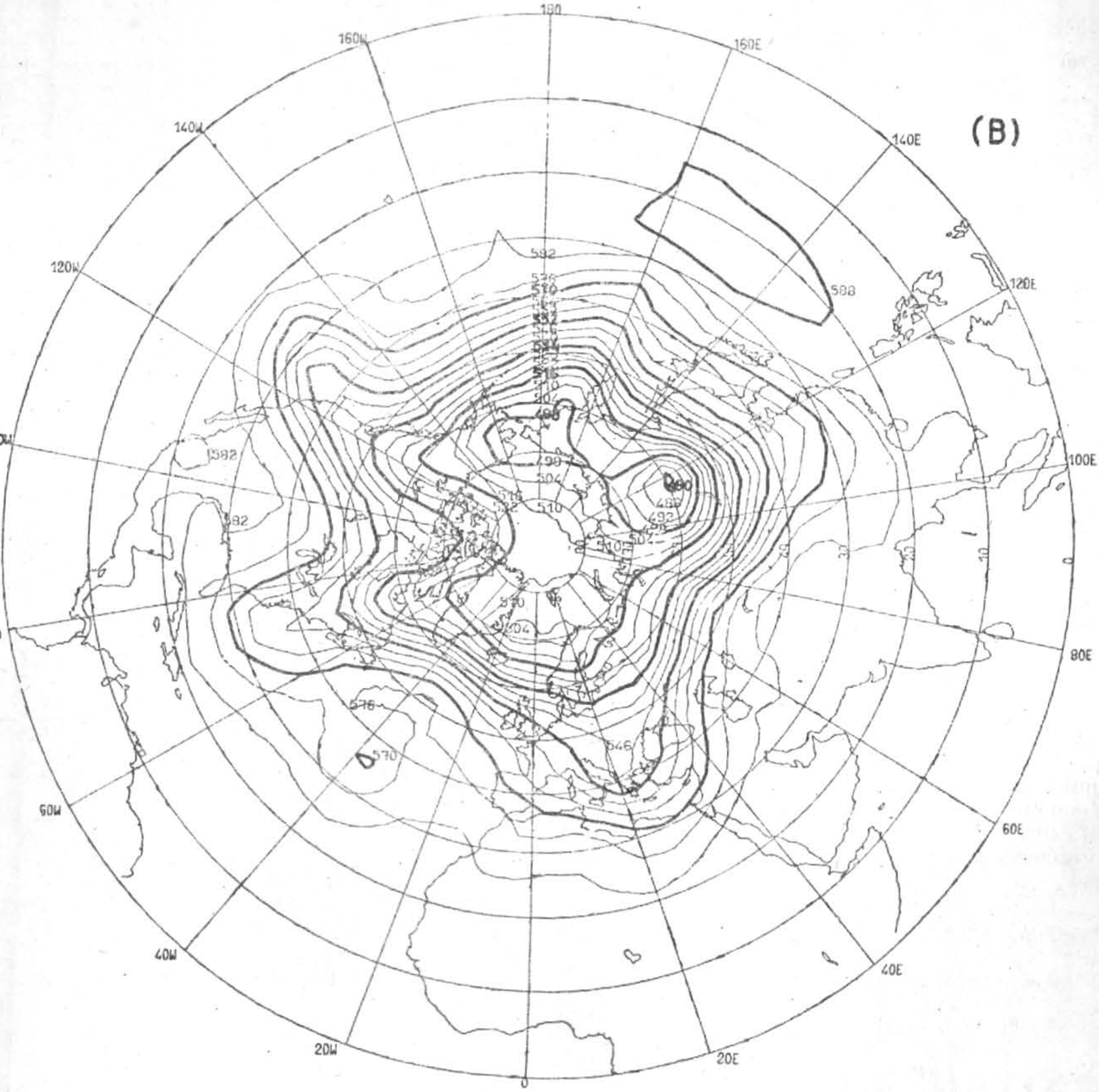

Fig. 1 (b). Four-day predictions for 500 mb geopotential field of 8 November 1969 using model B, compared to the corresponding GFDL analysis from GARP Basic Data Set (C). It is based on the conservation of total energy and perturbation energy around a given arbitrary stratification.

where the suffix S refers to value at the ground. On the other hand, stationarity of the total energy reads

$$\sum_{x,y,z} \left\{ \phi_S \frac{\partial P_S}{\partial t} + \theta \delta_Z P \frac{\partial \pi}{\partial t} + \pi \frac{\partial}{\partial t} (\theta \delta_Z P) + \mathbf{v} . \frac{D\mathbf{V}}{Dt} \right\} = 0 \tag{2.7}$$

Equating (2.6) to (2.7), we get a sufficient condition for formal energy conservation :

$$\sum_{\sigma} \left\{ [\theta]_A \; \delta_Z \pi \frac{\partial P}{\partial t} - \theta \delta_Z P \frac{\partial \pi}{\partial t} \right\} = 0 \tag{2.8}$$

to be satisfied at every horizontal grid point, which is in general not true. To yield a condition of practical useability, (2 8) must be transformed into a relation independent of the particular values taken by variables θ, π and P. In order to get rid of θ, $[\theta]_A$, must be chosen as the arithmetic average, which means that integrals of θ and θ^2 are conserved by the flux form of the thermodynamic equation. In that case $[\;\;]_A$ is symmetric, and (2.8) only requires

$$\overline{\delta_Z \pi \frac{\partial P}{\partial t}}^{z} - \delta_Z P \frac{\partial \pi}{\partial t} = 0 \tag{2.9}$$

at every grid point in the vertical; bar means arithmetic average. If we further assume $P_T = 0$ (or $\sigma = P/P_S$), and pressure Exner's function can be eliminated : denoting σ^κ by s at θ or π locations (*see* Fig. 1b), the final condition for energy conservation reads

$$\overline{\sigma \, \delta_Z \, s}^{z} = \kappa \, s \, \delta_Z \, \sigma \tag{2.10}$$

In a L-layer model, the layer depths $\delta_Z \sigma$ being given, (2.10) can be interpreted as a set of equations which determine the corresponding "optimum" levels s for location of the thermodynamic variables θ and π ; (2.10) is but a particular finite difference analogue of the natural differential relation between s and σ.

The set of equations (2.10) is easily solved for s ; a typical distribution obtained in the case of 11 layers (1975 version of the LMD GCM, referred here as model A), is shown in Table 1. If the distribution of layer depths is not smooth enough, the 3-point centred difference appearing in the left-hand side of (2.10) may excite 2-grid interval noise ; yielding values of s alternatively above and below midpoint values $(\bar{\sigma}^z)^\kappa$; the solution is especially sensitive to the distribution of layers depths near the upper boundary where s and θ diverge markedly from each other. However, if enough care is taken in the design of layer depths the solutions come out quite close to midpoint values, except very near the top.

3. Model B : Conservation of total energy and perturbation energy around a given arbitrary stratification

The fact that the atmosphere is a stratified medium is in itself an important constraint which appears *somewhat distorted* by the use of the σ coordinate system, especially in the vicinity of high mountains. It is not natural to consider differencing operators on artificial surfaces like the σ-surfaces, instead usrfaces which are naturally meaningful for the fluid motion in the interior of the domain, *i.e.*, isobaric or isentropic surfaces, over which the mean stratification is uniform. One consequence of this choice is the possible occurrence of large truncation errors near the mountains (Kurihara 1968; Sundqvist 1975, 1976; *see* also Sundqvist 1979). A related aspect of the problem is the fact that arbitrary resting states, in which all the thermodynamic variables are functions of height only, are not resting states for the model and generate spurious oscillations; in this way the concept of available potential energy looks somewhat ill-defined. Further, one can expect these spurious oscillations to be primarily of gravitational type; then, the inability of the discrete σ-coordinate model to recognize the climatic stratification as a resting state means continuous generation of spurious gravity waves from mountains, asking for the addition of some artificial energy damping process. One, therefore, expects possible distortions, not only in forecasting problems, but also in climate simulations.

It looks obvious at first that only one stratification can be a resting state for a discrete σ-coordinate model. If the model is built in such a way that one such stratification (says **S**) exists, truncation errors in the vicinity of high mountains will remain small for atmospheric states which depart not too much from **S**. Looking a little deeper into the problem, one has to analyse the behaviour of the truncated model around **S** in order to study the structure of its normal modes. Normal modes up to now have been studied for the separable problem without mountains, where the basic state of rest can be considered a function of σ as well as of pressure or temperature. If finite amplitude mountains are given, the linearised primitive equations around a stratification S can be written :

$$\text{grad}\, \phi' - \mathbf{s}H' \,\text{grad}\, \mathbf{m} + f\mathbf{N} \times \mathbf{V} + \frac{\partial \mathbf{V}}{\partial t} = 0. \tag{3.1}$$

$$\partial \phi'/\partial \sigma - \mathbf{s}H' \, \partial \mathbf{m} \,/\, \partial \sigma = 0 \tag{3.2}$$

$$D\mathbf{m}/Dt + \partial H'/\partial t = 0. \tag{3.3}$$

$$\text{div}\, \mathbf{v}' + \partial \left(\frac{\partial P'}{\partial t} + w' \right) \Big/ \partial \sigma = 0 \tag{3.4}$$

where primes refer to perturbation quantities. New notations used here are the following :

$H = c_p T$	enthaply
$\mathbf{g} = \mathfrak{g}(P)$ ·	geopotential of basic stratification
$\mathbf{h} = -dg/d(\kappa \,\text{Ln} P)$	Enthalpy of basic stratification
$\mathbf{m} = \mathbf{h} + \mathbf{g}$	Montgomery potential basic stratification
$\mathbf{s} = -(d\mathbf{m}/d(\kappa \text{Ln} P)^{-1}$	Inverse static stability basic stratification

Since the system is linear, all stratification functions are considered independent of time; they are functions of x, y, σ only through the basic pressure function $\mathfrak{p}(x, y, \sigma)$, obtained by inversion of the basic geopotential function $\mathfrak{g}(P)$. This basic pressure is naturally used

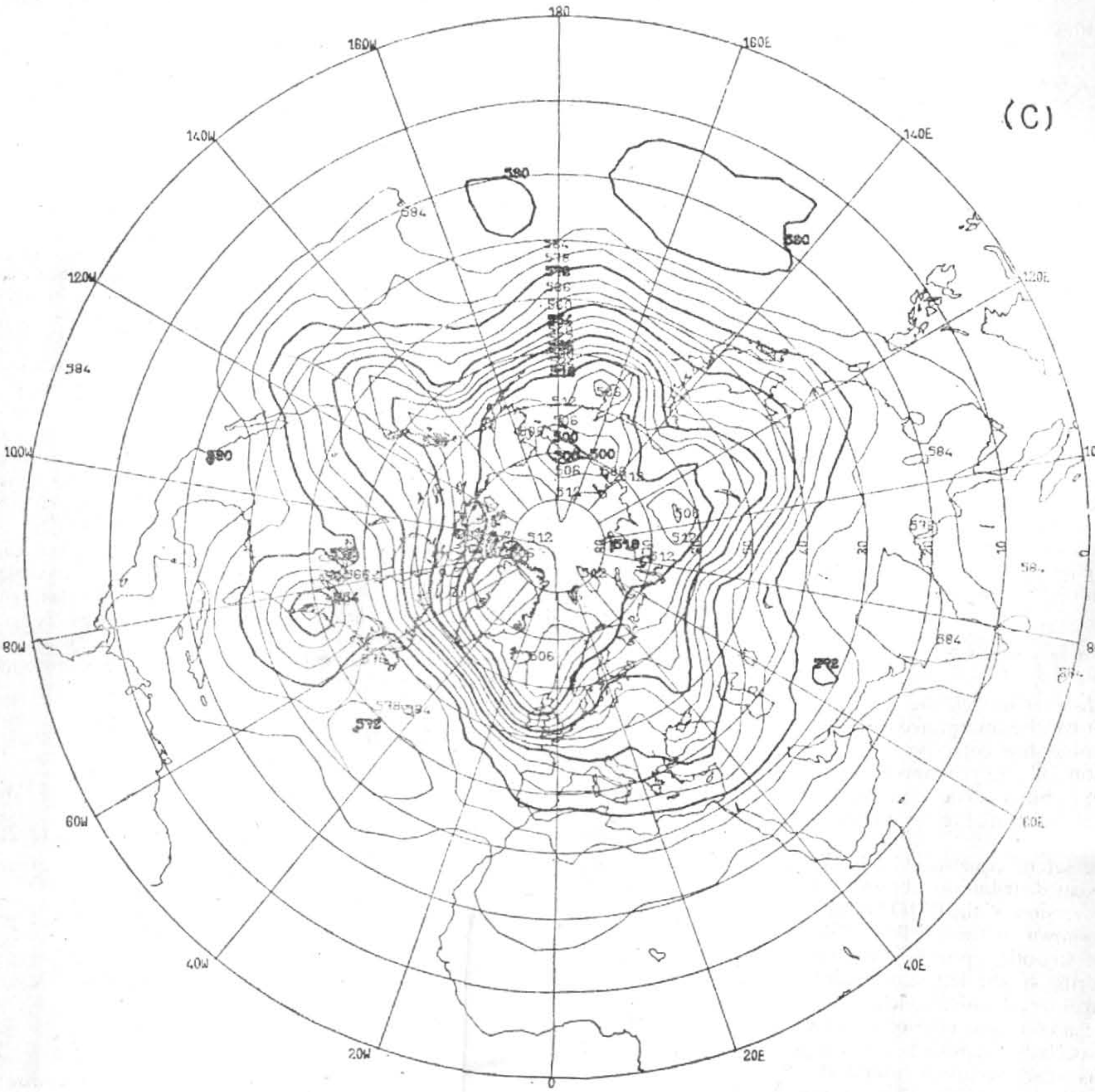

Fig. 1(c). 500 mb geopotential field of 8 November 1969 corresponding to GFDL analysis from GARP Basic Data Set (C)

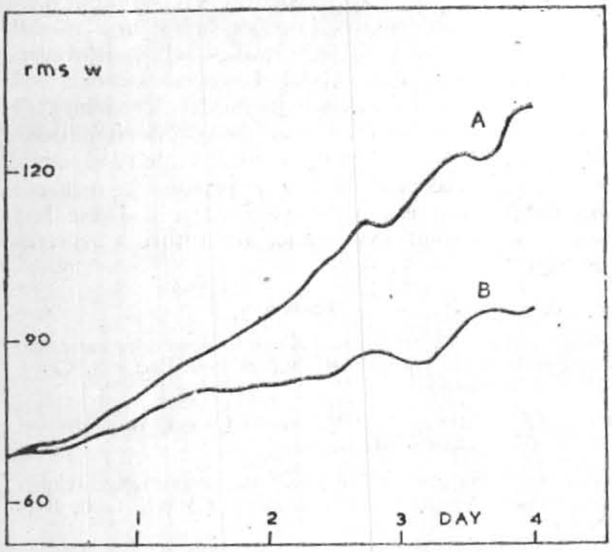

Fig. 2. Evolution of r.m.s. w_{500mb} as a function of time, for models A and B

instead of P in the definition of the perturbation mass fluxes $\mathbf{v}'$ and w'. The perturbation energy — a quadratic invariant of the system — reads

$$\xi' = \tfrac{1}{2} \iiint \left(\mathbf{p}_B (V'^2 + H'^2) + \kappa^{-1} \mathbf{h}_B P'^2_B \right) dx dy d\sigma \tag{3.5}$$

where suffix B refers to bottom boundary (earth surface) values.

The invariance of quadratic form (3.5) is of uppermost importance in establishing the wave-like structure of the eigenmodes, the time dependency of which is, therefore, restricted to pure oscillations. Considering now a discrete, primitive equation model in σ- coordinate, it may be a relevant question to ask how well is this structure reproduced in the underlying linearised model. . Our purpose here is to derive a discrete σ-coordinate model of the full nonlinear primitive equations obeying the usual constraint of energy conservation, which can at the same time be linearised around a prescribed resting state in such a way that the quadratic perturbation energy is then also conserved. This last property means that the model will have well defined normal modes inspite of strong orography, with pure oscillatory character in time.

The nonlinear form of the primitive equations which corresponds to (3.1-3.4) reads as follows :

$$\mathbf{grad}\, \phi' - \mathbf{s}H' \,\mathbf{grad}\, \mathbf{m} + f \mathbf{N} \times \mathbf{V} + \frac{D\mathbf{V}}{Dt} = 0 \tag{3.6}$$

$$\partial \phi' / \partial \sigma - \mathbf{s}H' \,\partial \mathbf{m} / \partial \sigma = 0 \tag{3.7}$$

$$(1 + \mathbf{s}H') \,D\mathbf{m}/Dt + DH'/Dt = 0 \tag{3.8}$$

$$\operatorname{div} \mathbf{v} + \partial \left(\frac{\partial P}{\partial t} + w \right) \Big/ \partial \sigma = 0 \tag{3.9}$$

All stratification functions are now functions of pressure and as such, are bound to vary with time; and perturbations with respect to the stratification functions have been used in lieu of the full variables H and ϕ. The energy invariant can be stated as

$$\xi = \iiint P_B \left(\frac{V^2}{2} + H' + \mathbf{m} + \phi'_B \right) dx dy d\sigma \tag{3.10}$$

The advantage of using $\mathbf{m}$ and $\mathbf{s}$ as basic functions of pressure is that the same argument which leads from (3.1-3.4) to (3.5) also leads from (3.6-3.9) to (3.10). Direct discretisation of (3.6-3.9), if it is energy conserving, will therefore ensure as well conservation of the perturbation energy in the linearised case, which is the property we are looking for.

The derivation of an energy-conserving discrete form of (3.6-3.9) is obtained in the usual way and need not be explained in detail. Using variables located as shown in Fig. 2, a possible solution is the following :

$$\underline{\delta} \overline{\phi'}^{Z} - [\mathbf{s}H'] \,\underline{\delta}\, \overline{\mathbf{m}}^{Z} + \mathbf{f}\mathbf{N} \times \mathbf{V} + \frac{D\mathbf{V}}{Dt} = 0 \tag{3.11}$$

$$\delta_Z \phi' - \mathbf{s}H' \,\delta_Z \mathbf{m} = 0 \tag{3.12}$$

$$(1 + \mathbf{s}H') \,D\mathbf{m}/Dt + DH'/Dt = 0 \tag{3.13}$$

$$\underline{\delta} \,.\, \mathbf{v} + \delta_Z \left(\frac{\partial P}{\partial t} + w \right) = 0 \tag{3.14}$$

with

$$D\mathbf{m}/Dt \equiv \left(\overline{\frac{\partial P}{\partial t} + w}^{Z} \delta_Z \mathbf{m} + \mathbf{v} .\, \delta\, \overline{\mathbf{m}}^{Z} \right) / \,\delta_Z P \tag{3.15}$$

Summing up, model (3.1-13.15)

(*i*) is energy conserving ;

(*ii*) allows the arbitrarily chosen stratification $\mathbf{S} = \{\mathbf{s}(P), \mathbf{m}(P), \ldots\}$ as its proper state of rest ;

(*iii*) is perturbation energy-conserving when linearised around stratification $\mathbf{S}$ in presence of finite-amplitude mountains.

This model is hereafter referred to as model **B**.

The simplest case of our formulation is obtained when $\mathbf{S}$ is isothermal : then, $\mathbf{s}$ reduces to a constant and $\mathbf{m}$ to the logarithm of pressure. The corresponding form of (3.11-3.15) has been extensively used (*e.g.*, Corby Gilchrist and Newson 1972 ; Burridge and Haseler 1977). Our formulation is but an extension of this method to the arbitrarily stratified case. A last remark concerning the formulation is that the stratification functions can easily be changed at any time within the integration, in order to follow the mean climatic trend.

4. A preliminary numerical experiment

A thorough comparison of the dynamical behaviours of two General Circulation Models requires such a large number of statistics that the computaional cost would be prohibitive, at least on the scale of a single institution. Therefore, we shall have to restrict our attention to a few selected experiments. First we decided to postpone the most sensitive comparison which would be a comparison of two parallel climatic simulations using the same physical package, and try instead two purely inertial versions of the models on short range forecasts. The first test of the kind which we performed — two 4-day predictions on the Basic Data Set fields — will be described now.

Both models have been used in a stripped-down version involving no boundary layer, no moist convection, no radiative forcing : only lateral diffusion and dry convection were added. The resolution used was 64 points in longitude, 50 points from north to south pole (regularly spaced in sine of latitude), and 11 layers. In model B we took $P_T=1$ mb (remember that $P_T=0$ in model A); the relative depth of the layers was otherwise the same, already described in Table 1. Horizontal differencing in both models follows the potential enstrophy conserving scheme described in Sadourny (1975 b). In model B, the mean stratification is defined by taking $\mathbf{h}(P)$ as a cubic spline interpolation of the globally averaged data on 19 reference pressure levels; $\mathbf{g}(t)$, $\mathbf{m}(P)$, $\mathbf{s}(P)$ are then defined consistently.

Initialisation is performed by cubic spline interpolations of the Basic Data Set fields to our sigma levels. The major defect of the resulting fields is the vertically integrated divergence being too large by an order of magnitude. An adaptation process based on 12-hour time averaging of the solution gets rid of the resulting gravity waves; all initialisation processes are performed in an exactly parallel way for the two models.

The resulting two 4-day forecasts and the reference GFDL analysis for the 500 mb geopotential fields are shown in Figs. 1 (a, b, c respectively). The two forecasts are surprisingly close to each other considering the rather drastic differences in formulation; in fact, the same kind of uniformity in the results was one striking conclusion of the Basic Data Set Experiment comparing a wide range of General Circulation Models (Gadd 1980). Whether this lack of sensitivity of the models' formulation is due to a lack of precision in the data is still a debatable question, until similar experiments are performed on a wide set of FGGE data sets.

Minor points at this stage are the facts that model A seems to produce slightly noisier fields, and model B slightly more energetic long waves. It is, however, interesting to notice that model B generates much less internal inertia-gravity waves than model A — which in a sense, was to be expected from its very formulation: this can be seen in Fig. 2, if the vertical velocity variance at 500 mb is accepted as an approximate measure of internal inertial gravity wave energy. These last hints may bear some importance for future long term integrations.

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