

Harmonic analysis and modelling of annual soil temperature variations

A. LIAKATAS

Agricultural University of Athens, Athens (Greece)

(Received 27 February 1991, Modified 19 November 1992)

सार—धरातल से एक मीटर की गहराई तक वार्षिक मृदा तापमान में परिवर्तनों के लिए सीरीज का प्रयोग करके हारमोनिक विश्लेषण का अनुप्रयोग किया गया है। केवल प्रथम हारमोनिक लगभग 99% परिवर्तन को स्पष्ट करती है।

यह भी दर्शाया गया है कि शुष्कतापीय अवस्थाओं में मृदा के माध्यम में, विशेष रूप से लगभग 20 से 0मी० की सबसे ऊपरी परत में उष्मा संचालन उपयुक्त नहीं है, क्योंकि न तो डैम्पिंग गहराई के द्वारा व्यक्त तापीय गुण, और न ही माध्य मृदा तापमान गहराई के साथ समान रहते हैं। दोलन का आयाम भी चरघाताकी से बहुत भिन्न नहीं होता है। अधिक गहरी परतों की तुलना में मृदा नमी काफी कम पाई गई है।

अन्ततः मासिक अथवा दस दिनों की अवधि के माध्य मृदा तापमान की प्रागुक्ति के मॉडल का विकास मौसम प्राचलों (वर्षा और पान वाष्पन के मध्य अन्तर) पर आधारित है। इससे मृदा नमी में कमी और डैम्पिंग गहराई के आकलन के लिए एक सूचकांक प्राप्त होता है। साथ ही, माध्य मृदा तापमान और दोलन के आयाम में गहराई के साथ होने वाले परिवर्तनों की समस्या का भी निवारण करता है।

ABSTRACT. Harmonic analysis using series is applied to the annual variations of soil temperature from the surface down to 1 m depth. The first harmonic alone explains about 99% of the variation.

It is also shown that heat conduction is not valid in a soil medium under exothermic conditions, especially in the uppermost layer of about 20 cm, as neither the thermal properties, expressed by the damping depth (D), nor the mean soil temperature (T) remain constant with depth, and the amplitude of oscillation (A_s) does not vary strictly exponentially. This is assumed to be considerably reduced soil moisture of this in comparison with deeper layers.

Finally, construction of a model to predict monthly or ten-day period mean soil temperatures is based on weather variables (difference between precipitation and pan evaporation), providing an index for the soil moisture deficit and an estimate of D , and overcomes the difficulties of changing T and A_s with depth.

Key words— Soil temperature, Soil moisture, Harmonic, Temperature oscillation, Temperature gradient, Damping depth, Thermal diffusivity, Heat flow, Pan evaporation, Modelling.

1. Introduction

Net radiant input or output (R_n) is respectively dissipated or supplied by thermal conduction in the soil (G)-determining, together with the soil thermal properties, the soil thermal regime—and transfer of sensible (C) and latent heat (λE) in the atmospheric boundary layer, expressed by the heat balance equation:

$$R_n = C + \lambda E + G \quad (1)$$

The diurnal and annual variations of the insolation cause a variation of the soil temperature with the diurnal period superimposed on the annual, though strictly true periodicity is disturbed by weather variability (Van Wijk 1965). Due to this and the fact that changes of soil moisture and density may

significantly alter the soil thermal properties, analysis of soil heat conduction is rather complex (Monteith 1973) and so is soil temperature forecasting.

Assuming that the soil thermal conductivity (d') and volumetric specific heat ($p'c'$) are independent of depth (z), the time change of temperature in a soil layer ($\theta T/\theta t$) is a function of the change with z of the vertical temperature gradient ($\theta^2 T/\theta z^2$) and the soil thermal diffusivity ($x' = d'/p'c'$) as follows:

$$\theta T/\theta t = x' (\theta^2 T/\theta z^2) \quad (2)$$

Considering, then, that temperature at the soil-air interface follows a sinusoidal variation during a daily or a seasonal cycle, the temperature at time t and

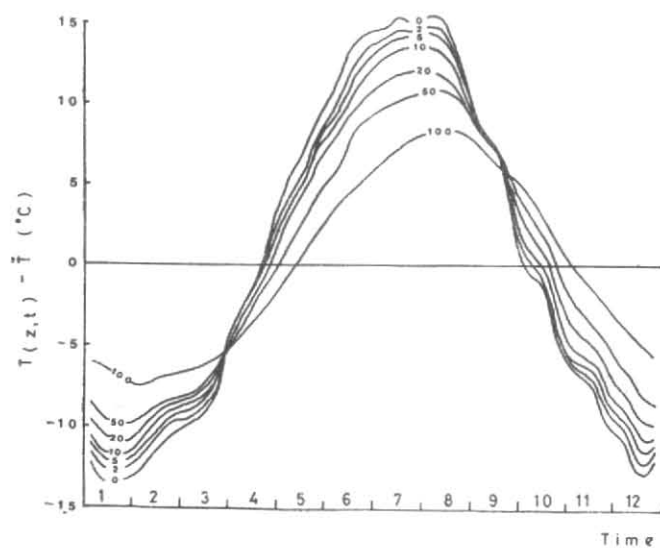


Fig. 1. Yearly variation of ten-day mean soil temperature at various depths (N. Philadelphia, Greece)

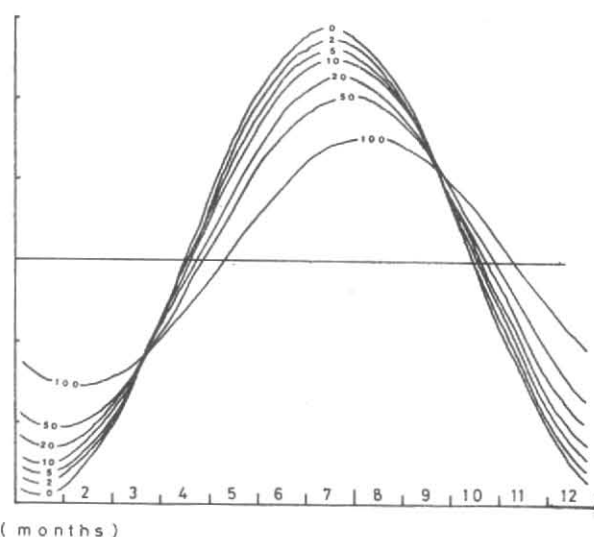


Fig. 2. Profiles of T (---) and $\ln(A_1/A_0)$ (—) in soil (N. Philadelphia, Greece)

depth z of a homogeneous soil can be estimated by the solution of Eqn. (2) as follows:

$$T_{z,t} = T + A_z \times \sin(\omega t - z/D) \quad (3)$$

that satisfies the following boundary condition (at $z=0$):

$$T_{0,t} = F + A_0 \times \sin \omega t \quad (4)$$

where, the temperature amplitude at z is an exponential function of that at $z = 0$ is given by:

$$A_z = A_0 \times \exp(-z/D) \quad (5)$$

$D = (2x'/\omega)^{1/2}$ is the damping depth, *i.e.*, the depth where $A_z = A_0 \times \exp(-1) = 0.37 A_0$, $\omega = 2\pi/P$ is the angular frequency of variation depending on period length (P), and T is the mean temperature during P independent of z (Monteith 1973).

However, when soil temperature is not simply sinusoidal but a more general periodic function of time, harmonic analysis employing Fourier series can be used to accurately describe the thermal cycle at z as follows:

$$\begin{aligned} T_{z,t} &= C_0 + \sum_{n=1}^{N/2} (A_n \times \cos \omega n t + B_n \sin \omega n t) \\ &= C_0 + \sum_{n=1}^{N/2} C_n \times \sin(\omega n t + \Phi_n) \end{aligned} \quad (6)$$

where, A_n and B_n are the amplitudes of various terms and $(A_n^2 + B_n^2)^{1/2} = C_n$ is the amplitude of the n th harmonic, whereas $C_0 = T_z$ is the mean temperature at z , $\arctan(A_n/B_n) = \Phi_n$ is the phase angle and N is the number of equally spaced points on the temperature wave ($N = 12$ for the annual period of t is in months).

The annual temperature cycle in soil is fairly well described by the, first harmonic alone (*e.g.*, Pearce and Gold 1959, Carson 1963, Van Wijk 1965) in mid and high latitudes with temperate climates, though under tropical climatic conditions the effect of higher harmonics is significant (Krishnan and Kushwaha 1972). However, harmonic analysis of soil temperatures under the semi-arid conditions of the east Mediterranean may deserve some attention.

When only the first harmonic is important, Eqn. (3) can be used to estimate soil temperature provided that x' for a particular soil profile and soil temperature observations at only one depth are available (Monteith 1973, Horton *et al.* 1983). However, the main difficulty in applying Eqn. (3) is the normally observed variation of x' with z and t , due to the water movement, apart from differences in moisture content and porosity (Rose 1966).

Moisture in the soil moves in response to water potential gradients set up by temperature gradients. Liquid (in rather wet soils) and vapour (in drier soils)

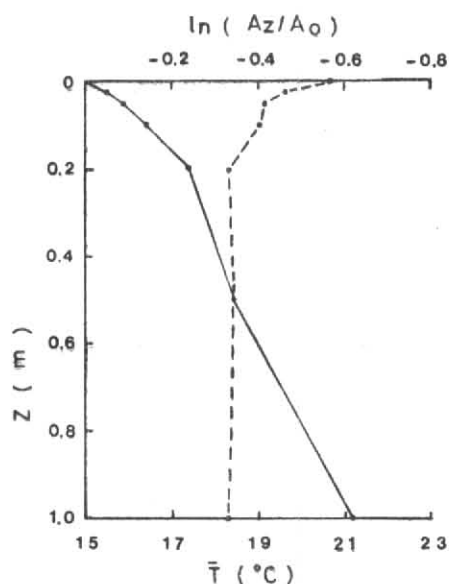


Fig. 3. Monthly soil temperatures at 2 cm depth observed (x) and estimated by the first (—) and second (---) harmonics as well as by their synthesis (—) (N. Philadelphia, Greece)

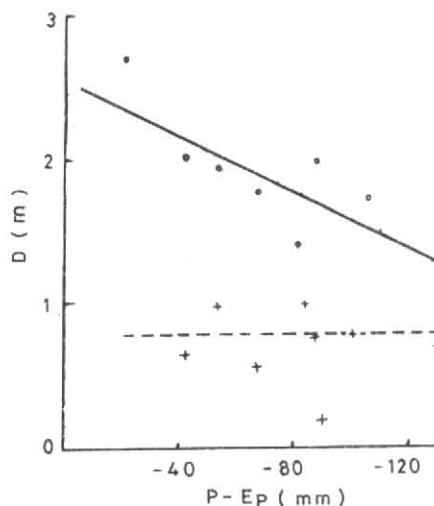


Fig. 4. Soil temperatures at various depths estimated by the first harmonic (N. Philadelphia, Greece)

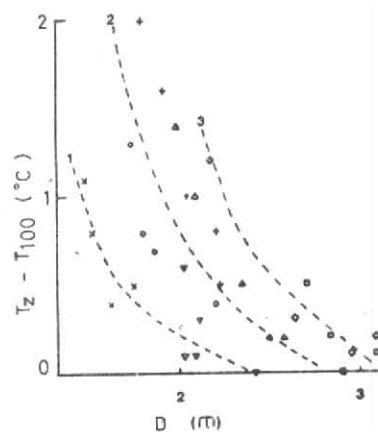


Fig. 5. Damping depths (D) against ($P - E_p$) yearly values, for the soil layers 2-100 cm (—) and 2-5 cm (---)

flow from areas where kinetic energy is high (warm) to areas where it is low (cold) transferring appreciable quantities of heat, invalidating Eqn. (3) which is based on the assumption that heat transport is performed only via sensible heat conduction (Rose 1966, Rosenberg *et al.* 1975). According to Jackson *et al.* (1973) the transport of water by temperature induced flow must be considerable on an annual basis. Therefore, moisture transport, especially in the gaseous phase under the Mediterranean climate, cannot be negligible and rainfall as well as soil water management may profoundly affect soil heat flow and temperature regime.

Thermal diffusivity of the usually drier upper soil layer is often considerably different from that at greater depth (Van Wijk 1965), making determination of its α' very difficult. Lettau (1954), Wierenga and de Wit (1970) and McCulloch and Penman (1956) derived models by which such a non-homogeneous case, (where d' and $p'c'$ are functions of z) can be treated rigorously, using the profiles of soil moisture, composition and structure.

Therefore, though soil temperature patterns can be modelled as a function of ambient weather condition, soil moisture also plays an important role. For short-

term planning, hourly values of weather variables like radiation, temperature and wind speed, as employed by Liakatas (1978) are required to estimate close-to-surface soil temperatures under plastic mulches. For long-term planning, however, more easily accessible weather data like precipitation and temperature are usually sufficient. Oullet (1973) proposed a model for estimating monthly soil temperatures under short grass with variables including rainfall, potential evapotranspiration and soil temperature of the previous month.

Here, construction of a model to predict ten-day period or monthly soil temperatures under bare surface will be attempted, using a soil-moisture indicating combination of weather variables, including soil temperatures at a single depth.

2. Methods and data

Harmonic analysis of soil temperatures at various depths from soil surface down to 100 cm employing Fourier series will be applied using Eqn. (6). In order to decide how many harmonics should be computed for depicting actual data well, the variance of the data below a selected value should be determined as

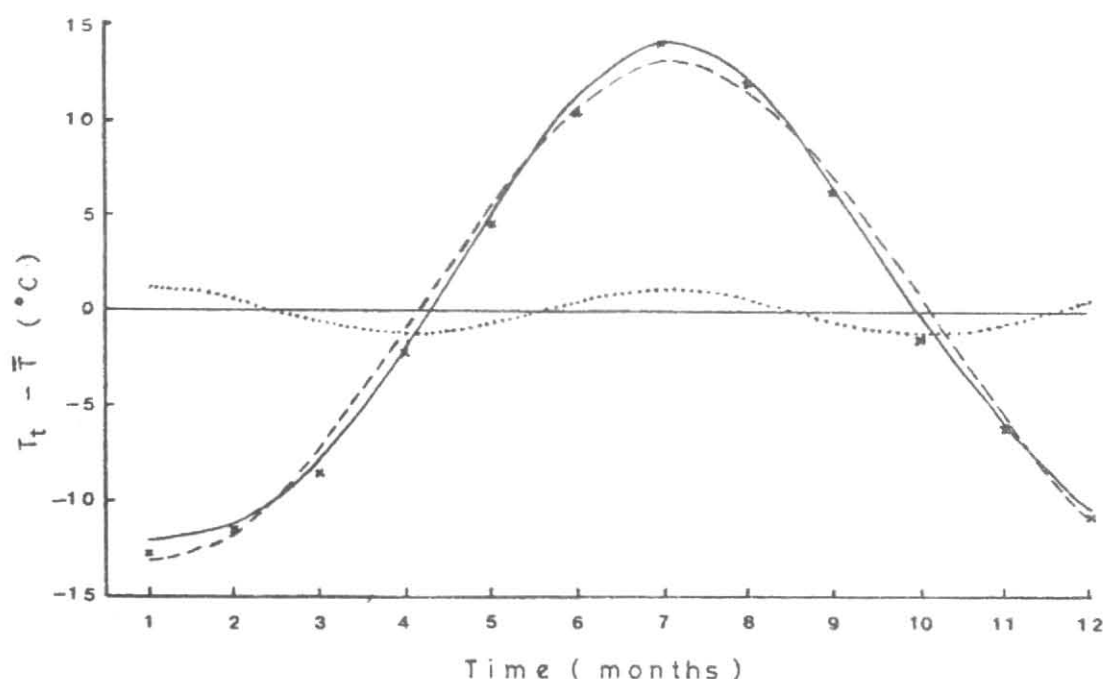


Fig. 6. $(\bar{T}_t - \bar{T}_{100})$ values against D values for various soil layers, confined by z and the reference depth (100 cm), for southern (1), central (2) and northwestern (3) sites

follows using a completeness criterion (Brunt 1937, Panofsky and Brier 1958):

$$\sigma'^2 = 100 - \sigma^2 \times 100 = 100 - (C_n^2 / 2S^2) \times 100 \quad (7)$$

where, $S^2 = \Sigma (\bar{T} - a_i)^2 / N$ is the total variance of the N data values (a_i) about their arithmetic mean (\bar{T}) and the ratio of $C_n^2 / 2$ to S^2 shows the fraction of variability accounted for each harmonic (n).

In case of the first harmonic alone could describe the observed temperature values, considering the depth of 1 m as reference depth (since its daily mean temperature can be sufficiently approximated by a single measurement) and provided that D can be estimated, the annual variation of soil temperature at any depth shallower than 1 m would be given by:

$$T_{z,t} = \bar{T} + A_z \sin [wt + (1-z)/D] \quad (8)$$

where,

$$A_z = A_{100} \exp [(1-z)/D] \quad (9)$$

and A_{100} is the temperature amplitude at 1 m depth.

To accomplish harmonic analysis, monthly and ten-day mean values of soil temperature, at the surface and 2, 5, 10, 20, 50 and 100 cm below, the ground level from ten representative meteorological sites all over Greece for an average period of 10 years, were used. Along with this data, monthly precipitation (P) and class-A pan evaporation (E_p) (their difference considered to provide an indication of the magnitude of soil moisture deficit) were used for modelling soil temperatures.

3. Annual variation pattern

The yearly variations of the ten-day mean values of soil temperatures at various depths for a meteorological site in Attika, Greece (N. Philadelphia $38^\circ 03'N$, $23^\circ 40'E$), presented in Fig. 1, seem to agree with the theory of soil heat conduction in terms of amplitude diminishing and extreme values delaying with depth. It may be noticed that smaller waves of a monthly or bimonthly period are superposed on the annual waves, causing irregularities in the sinusoidal pattern.

The variation with depth of the logarithmic value of the ratio of the amplitudes at a certain depth (A_z) and that at the surface (A_0) is illustrated in Fig. 2. Below 20 cm, it is almost a straight line of $-1/D$ slope (Eqn. 5), suggesting a D value of 2.1 m, whereas, above this depth the slope changes significantly with a mean D value of 0.83 m. Based on data from Van Wijk and de Vries (1963) and Monteith (1973) it is estimated, using the above D values, that for a medium-structured soil the mean annual water content below 20 cm is around 7% and considerably smaller in the soil layer close to the surface.

The profile of average soil temperature against z is also shown in Fig. 2. \bar{T} , though constant below 20 cm in agreement with the theory, becomes increasingly higher as it approaches the surface. At the surface, \bar{T} is by more than $2^\circ C$ higher than at 20 cm and lower depths, though Van Wijk (1965), observed that

variation of \bar{T} within the surface layer rarely exceeds 1°C. This temperature rise towards the surface is probably due to increasing dryness and, thus, decreasing soil heat capacity and conductivity, resulting in easier heating of the uppermost soil layer facilitated by the lower heat flow downwards in day and upwards in night-time, the first case being more effective during the long summer days and the second during the long winter nights.

The different behaviour of this layer, in comparison with deeper soil layers, in terms of mean temperature and thermal properties (expressed by D) may be quite significant when using the heat conduction theory to estimate $T_{z,t}$ as will be seen later.

4. Harmonic analysis

Harmonic analysis, using Eqn. (6), has been applied to the annual variation of soil temperature at various depths. The amplitude (C_n) and the phase angle (Φ_n) of each harmonic (n) up to the fourth were estimated for each depth, including surface, using ten-day mean values of temperature (Table 1). The amplitude of the second harmonic, is found smaller than 10% of the amplitude of the basic harmonic, ranging from a maximum of 10.1% at 5 cm, to 6.3% at 100 cm depth. Also the completeness criterion shows that a little less than 99% of the variation is explained by the first harmonic alone at the surface, the σ^2 increasing with z . The second harmonic could contribute less than 1%. Indeed, as shown in Fig. 3, soil temperatures estimated at 2 cm depth by taking into account both the first two harmonics become slightly closer to the observed values.

Thus, only the first harmonic sufficiently describes the observed annual temperature variation, even at the surface, though the estimated values with ten-day temperature waves (Fig. 4) are free of the irregularities of the observed waves (Fig. 1). In agreement with the theory, maximum and minimum temperatures show an increasing hysteresis as z increases (Fig. 4). The mean absolute differences between the values estimated by the first harmonic and the observed temperatures range between 0.97°C at the surface and 0.34°C at 100 cm depth and indicate the suitability of the method of estimation.

5. Modelling-construction and verification

In applying the heat conduction theory to estimate $T_{z,t}$, one of the major problems is estimation of the thermal properties of the soil, best reflected in its D values usually varying with z . Since D is basically moisture dependent, its evaluation should be possible from

weather elements determining water balance of the soil. The difference between precipitation (P) and pan evaporation (E_p) is considered indicative of the soil moisture deficit as E_p is normally higher than P , on an annual basis, under Mediterranean climatic conditions.

Estimated values of D by Eqn. (9) between the reference, *i.e.*, 100 cm and 2 cm depths as well for the layer 2-5 cm, are plotted against $(P-E_p)$ yearly total values for eight meteorological sites in Fig. 5. The relation between these two variables is almost linear in the layer 2-100 cm. D increases with decreasing $P-E_p$ at a rate 1 m/100 mm. Scattering should be considered as acceptable since the same line passes through points representing eight sites (for which data were available) of quite different climatic and soil conditions. In the layer 2-5 cm, however, there seems to be no response of D to varying $P-E_p$ values and the constant low D value suggests that the top soil layer is, on an average, very dry all over Greece, though in the rainy season (winter) it may be wet for at least short periods.

It should be mentioned that, calculated from temperature amplitudes, D values for a layer confined by the reference depth and a depth shallower than 20 cm may be a little underestimation since the assumption for constant moisture throughout the layer is not valid. For a layer thinner than this, D may be considered as independent of z .

The fact that in the top 20 cm soil layer \bar{T} is higher than at deeper depths and A_z varies with z in a manner not strictly exponential, as already shown in Fig. 2, may suggest that both T_z and A_z depend on soil moisture and thus on D .

In Fig. 6, the differences between T at a certain z and $z = 100$ cm are plotted against corresponding D values for several sites. $T_z - T_{100}$ values become increasingly larger for smaller D , *i.e.*, for lower soil moisture. To reduce scattering, instead of one, three curves were drawn corresponding to southern (dry), central (normal) and northwestern (wet) sites.

Similarly, in Fig. 7 the ratios of amplitudes (A_z/A_{100}) are plotted against D . Despite considerable scattering, there is a satisfactory fit of a single curve to points representing sites differing in climate and soil types. The ratio A_z/A_{100} increases faster as D falls below 2 m, that is when z is close to the surface.

From the previous analysis it seems feasible that D could be estimated using P and E_p data. Then, estimation of $T_{z,t}$ is possible either from Eqns. 8 and 9 (first way) or by considering A_z/A_{100} and $(\bar{T}_z - \bar{T}_{100})$ as

TABLE I
Amplitude (C_n), phase angle (ϕ_n) and completion characteristics (σ^2) and C_n/C_1 of the first four harmonics (n) at various depths (z)

z (cm)	00	02	05	10	20	50	100																					
n	C_n	ϕ_n	σ^2	C_n/C_1	C_n	ϕ_n	σ^2	C_n/C_1	C_n	ϕ_n	σ^2	C_n/C_1																
0	20,6	19,6	19,1	19	18,3	18,4	18,3	18,3																				
1	14,6	260	98,8	1	13,8	260	98,6	1	13,2	258	98,7	1	12,6	257	99	1	11,5	254	99,4	1	10,3	248	99,7	1	7,8	236	99,4	1
2	1,4	68	0,9	0,096	1,4	63	1	0,099	1,3	54	1	0,101	1,1	51	0,7	0,087	0,9	52	0,5	0,074	0,7	40	0,3	0,065	0,5	19	0,4	0,063
3	0,4	110	0,1	0,025	0,6	117	0,2	0,042	0,4	121	0,1	0,033	0,3	102	0,2	0,027	0,2	73	0,1	0,021	0,2	69	0	0,016	0,1	48	0	0,015
4	0,3	301	0	0,022	0,5	302	0,1	0,033	0,6	279	0,2	0,043	0,3	274	0,1	0,021	0,2	270	0	0,017	0,1	253	0	0,014	0,1	232	0	0,014

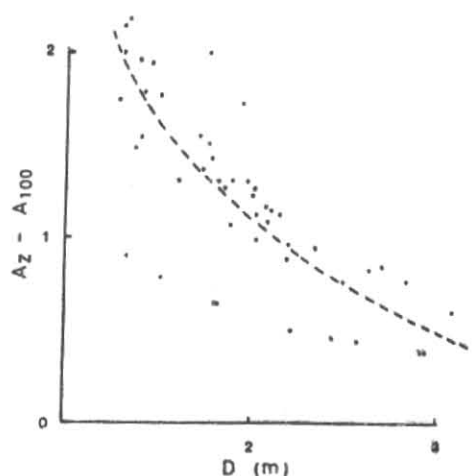
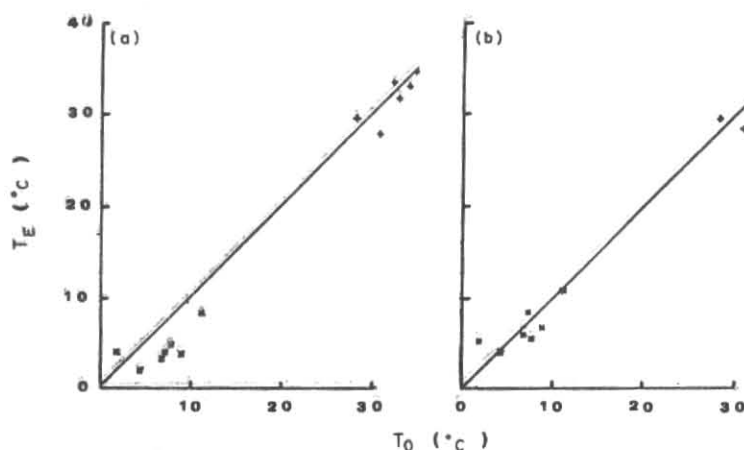


Fig. 7. (A_z/A_{100}) values against D values for various soil layers, defined by z and the reference depth (100 cm)



Figs. 8 (a & b). Estimated temperatures (T_E) by: (a) the first, and (b) the second way (described in text) against observed temperatures (T_0) in the warmest (+) and coldest (x) months at various depths (Aliartos, Greece)

functions of D (second way). Values of maximum and minimum temperatures estimated in both ways for a site in central Greece (Aliartos) are plotted against corresponding observed values in Figs. 8 (a & b). Fitting of a straight line of unit slope to points standing for maximum and minimum temperatures at various depths is slightly better when temperatures are estimated in the second way [Fig. 8(b)]. This is particularly valid for the minimum temperatures which tend to be underestimated in the first way [Fig. 8(a)].

6. Conclusions

Under the exothermic conditions of an east Mediterranean country, like Greece, application of Fourier series to soil temperatures at various depths shows that yearly temperature variations can be sufficiently described by the first harmonic alone. This efficiency of the basic harmonic facilitates estimation of soil temperature even close to the surface. However, due to soil moisture heterogeneity, application of the heat conduction theory to estimate seasonal temperature fluctuations in the uppermost 20 cm — soil layer may present some difficulties which could be overcome by taking into account the moisture dependence of the soil thermal properties.

References

- Brunt, D., 1937, "The combination of observation", *Met. Mag.*, 72, pp. 267-269.
- Carson, J.E. 1963, "Analysis of soil and air temperatures by Fourier techniques". *J. Geophys. Res.*, 68, 2217-2232.
- Horton, H., Wierenga, P.J. and Nielsen, D.R., 1983, "Evaluation of methods for determining the apparent thermal diffusivity of soil near the surface", *Soil Sci. Soc. Am. J.*, 47, 25-32.
- Jackson, R.D., Kimball, B.A., Reginato, R.J. and Nakayama, F.S., 1973, "Diurnal soil-water evaporation: Time-depth flux patterns", *Soil Sci. Soc. Am. Proc.*, 37, 505-509.
- Krishnan, A. and Kushwaka, R.S., 1972, "Analysis of soil temperatures in the air zone of India by Fourier techniques", *Agric. Met.*, 10, 55-64.
- Lettau, H., 1954, "Improved models of thermal diffusion in soils", *Trans. Am. Geophys. Union*, 35, 121-132.
- Liakatas, A., 1978, "Thermal microclimate of plastic mulches and responses of maize", Ph.d. Thesis. University of Nottingham.
- McCulloch, J.S.G. and Penman, H.L., 1956, "Heat flow in the soil", *Proc. 6th Internat. Soil Science Congr.*, 1, 275.
- Monteith, J.L., 1973, "Principles of environmental physics", Edward Arnold, London p. 128.
- Outlet, C.E., 1973, "Macroclimatic model for estimating monthly soil temperatures under short grass cover in Canada", *Canadian J. Soil Sci.*, 53, 263-274.
- Panosky, H.A. and Brier, G.W., 1958, "Some applications of statistics to meteorology", Pennsylvania State Univ. Press, 254 pp.

- Pearce, D.C. and Gold, L.W., 1959, "Observations of ground temperature and heat flow at Ottawa." *Canadian J. Geophys. Res.* **64**, 1293-1298.
- Rose, C.W., 1966. "*Agricultural physics*", Pergamon press, London, p. 31.
- Rosenberg, N.J., Blad, B.L. and Verma, S.B., 1975, "Microclimate—The biological environment. Second Edition, John Wiley and sons. New York, p. 94.
- Van Wijk, W.R., 1965. Soil microclimate, its creation, observation and modification. In : *Agricultural Meteorology, Met. Monogr.* **6**, 28, 59-73.
- Van Wijk, W.R. and de Vries, D.A., 1963. Periodic temperature variations. In : *Physics of Plant Environment*, Ed. Van Wijk, W.R. North-Holland Publishing Co., Amsterdam.
- Wierenga, P.J. and de Wit, C.T., 1970, "Simulation of heat transfer in soils". *Soil Sci. Soc. Am. Proc.* **34**, 845-848.
-