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Point discharge current and its dependence on potential gradient and wind speed

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ABSTRACT. The study of point discharge current has been made using an isolated metallic sharp point raised above the earth's surface. A theoretical relation between the point discharge current, wind speed and the potential gradient has been derived and the results have been compared with experimental-observations.

1. Introduction

The point discharge current plays an important role in the transfer of charge between thunder clouds and earth specially in stormy weather when electric potential gradient varies widely and rapidly. The various relations proposed between point discharge current, potential gradient and wind speed have been enumerated by different workers, but no relation can be said to have been finally decided.

2. Apparatus

The potential gradient has been measured by means of an agrimeter (Kamra and Varshney 1968), differential type of D. C. amplifier and a strip chart pen recorder. The point discharge current has been recorded using an isolated metallic sharp point raised at a height of about 20 metre above the earth, an amplifier and a strip chart pen recorder. The wind speed has been measured with a four-cup anemometer using a separate strip chart pen recorder.

3. Earlier Researches

Without considering the effect of wind on point discharge current, Whipple and Scrase (1936) found for field dependance of the current, the relation,

$$I = a\,(F^2 - M^2) \tag{1}$$

where I is the point discharge current through a sharp point at a certain height, F is the potential gradient at the ground and a, M are the constants.

Milner and Chalmers (1937-39) measured point discharge currents through trees and studied their characteristics in charging and discharging processes. They found that trees behave as a parallel-resistance-capacitance circuit with a time constant of about 90 seconds.

Simpson and Robinson (1934), Chiplonkar (1940), Hutchinson (1951) and Yriberry (1954) found their observations in agreement with the result at Eq. (1). However Hutchinson suggested the result to have first power law for high potential gradient. The same result was theoretically derived by Chalmers (1952) who considered one point from one of the rectangular set of points and assumed that the ions are removed solely by the electric field, *i.e.*, he ignored the effect of wind. Kirkman and Chalmers (1957), suggested a general relation—

$$I = B\,[F - M]^n \tag{2}$$

with constants B and M and a power n. They suggested that $n = 1\cdot07 \pm 0\cdot13$ for Whipple and Scrase's results and $n = 1\cdot09 \pm 0\cdot06$ for Hutchinson's results.

Chalmers and Mapleson (1955) using a single isolated point found that the wind speed is bound to produce an effect and derived theoretically:

$$I = k\, W^{3-q}\, V^{q-1} \tag{3}$$

where V is the potential difference between the point and its surroundings, W the wind speed, q is a number not determined by theory and k is constant. They attached a point to a captive balloon, sent it to different heights and found:

$$I = k\,(Fh)^{7/4}\, W^{1/4} \tag{4}$$

where h is the height of the point. The relation (4) agrees with Eqn. (3) because in the absence of space charge $V = Fh$ and thus it gives a value of $2\cdot75$ to q and $0\cdot015$ to k.

Kirkman and Chalmers (1957) measuring the currents through sharp points at high mast, wind speed at about 2 metre below the point and the potential gradient at gound and also at 7 m below the point, suggested a relation :

$$I=k\,(W+C)\,(F-M) \tag{5}$$

where k, M and C, are constants. The value of M being 250 $\mathrm{Vm^{-1}}$, the relation holds valid upto about $F=1000\ \mathrm{Vm^{-1}}$.

From some laboratory experiments and studying the discharge from points attached to aeroplanes Chapman (1956) suggested a relation:

$$I=k\,(V-V_0)v \tag{6}$$

where V is the potential of the point relative to the surroundings, V_0 is the minimum value of V for discharge, *i.e.*, threshold potential to start discharge, v is the velocity of approach of the ions and k is a constant. Champan from wind tunnel experiment suggested that the above relation should contain two terms corresponding to low and high winds, respectively. Large and Pierce (1957) measured current through a point kept in atmosphere at high potential and verified the result using Eqn. (6). This relation (6) agrees with relation (1) if there is no wind, since in that case v is proportional to F. It also agrees with Eqn. (5) for high wind speed, *i.e.*, $v=W$, with $C=0$.

Chalmers (1957) considered v as the vector sum of the wind speed and a term $a\,v$ and obtained.

$$I=-G\,(V-V_0)\,(W^2+a^2v^2)^{1/2} \tag{7}$$

where $a\,v$ represents the ion speed in the field and G is a constant.

Later on, Chalmers (1962) making some fairly justified assumptions, derived a theoretical relation for point discharge current through an isolated point which is given by

$$I=2\,\pi\epsilon_0\,(V-V_0)W \tag{8}$$

where ϵ_0 is the permittivity of free space, this relation was further improved in the form

$$I=2\,\pi\epsilon_0\,(V-V_0)\,(W^2+\omega^2F^2)^{1/2} \tag{9}$$

where ω represents the mobility of ions. Chalmers further attempted to obtain the exact nature of the relation of point discharge current, wind speed and potential gradient but he obtained partial success only.

Kamra and Varshneya (1967) during fast changes of potential gradient observed that the point discharge current flows even when the potential gradient is below the critical value and explained the phenomenon qualitatively as well as quantitatively, on the basis of a transient reversed local potential gradient developed round the point. It was shown that the Chalmers relation (7) can be used partially to correlate this point discharge current with potential gradient and wind speed.

Keeping the above in view, it is now clear that the exact nature of variation of the point discharge current with potential gradient and wind speed cannot be said to have been finally decided. Hence a further attempt (experimental as well as theoretical) in this direc'ion may give fruitful results. We have in the following derived theoretical relation between the point discharge current, wind speed and potential gradient.

4. Derivation of the expression for Point Discharge current.

The presence of a sharp point in a previously uniform vertical field, E_0, will change the course of the lines of force such that they concentrate near the point, (Fig. 1). Taking the vertical as the z direction the field so produced will have azimuthal symmetry. We can, therefore, consider a picture of field in the x-z plane to give us the complete field on rotation along z-axis passing through the sharp point, which we suppose situated at the origin.

Let us find the field at a point P located near the origin (Fig. 2). Its coordinates are given by

$$\mathbf{r}=x\,\mathbf{i}+z\mathbf{k}$$
$$r^2=x^2+z^2 \tag{10}$$

where $\mathbf{i}$ and $\mathbf{k}$ are unit vectors in x and z directions, respectively.

The field around the point can be considered to be the superimposition of two fields, namely, the uniform field everywhere in space remote from the point, and a radial field due to the point, assuming it has an induced charge Q. Then the total potential at P will be given by :

$$V_P=E_0z-Q/kr \tag{11}$$

where $k=4\,\pi\epsilon_0=1.1\times10^{-10}$ Farad $\mathrm{m^{-1}}$ (12)

The resultant field $=\mathbf{E}=-\,\mathrm{grad}\,V$

$$=-\left[\frac{\partial V}{\partial x}\,\mathbf{i}+\frac{\partial V}{\partial z}\,\mathbf{k}\right]$$

$$\text{or}\quad E=-\frac{\partial}{\partial x}\left[E_0z-\frac{Q}{kr}\right]\mathbf{i}-\frac{\partial}{\partial z}\left(E_0z-\frac{Q}{kr}\right)\mathbf{k} \tag{13}$$

Since $r^2=x^2+z^2$, on differentiating we get,

$$\frac{\partial r}{\partial x}=\frac{x}{r}\ \text{and}\ \frac{\partial r}{\partial z}=\frac{z}{r} \tag{14}$$

$$\text{and}\quad \frac{\partial V}{\partial x}=\frac{\partial V}{\partial r}\cdot\frac{\partial r}{\partial x},\quad \frac{\partial V}{\partial z}=\frac{\partial V}{\partial r}\cdot\frac{\partial r}{\partial z}$$

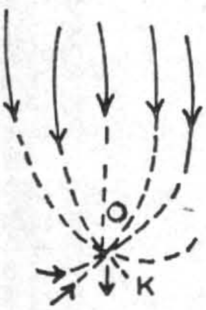

Fig. 1. Lines of force concentrating at a sharp point

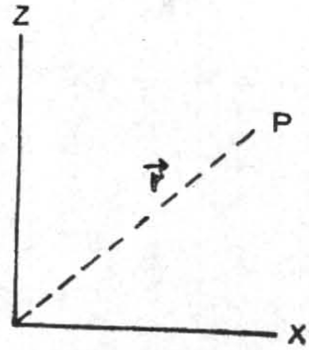

Fig. 2. Position of a point P with respect to the sharp point

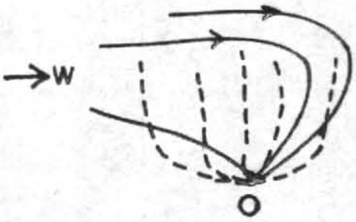

Fig. 3. Path of ions (continuous line) towards the sharp point in presence of the wind

Eq. (13) becomes,

$$E = \frac{\partial}{r}\left(\frac{Q}{kr}\right)\frac{\partial r}{\partial x}\,\mathbf{i} - \left[E_0 - \frac{\partial}{\partial r}\left(\frac{Q}{kr}\right)\frac{\partial r}{\partial z}\right]\mathbf{k}$$

$$\text{or } E = -\frac{Q}{kr^2}\cdot\frac{x}{r}\,\mathbf{i} - \left[E_0 + \frac{Q}{kr}\,\frac{z}{r}\right]\mathbf{k}$$

$$= -\frac{Cx}{r^3}\,\mathbf{i} - \left[E_0 + \frac{Cz}{r^3}\right]\mathbf{k} \qquad (15)$$

where $C = Q/k$

Let us now consider the influence of wind. An ion moves under the influence of the wind as well as of the electric field, its path being shown in Fig. 3. Its velocity due to fields is

$$\mathbf{v} = \omega\,\mathbf{E}$$

$$= -\frac{\omega Cx}{r^3}\,\mathbf{i} - \omega\left(E_0 + \frac{Cz}{r^3}\right)\mathbf{k}$$

whereas its velocity due to wind in the upper half space ($z>0$) is $W\mathbf{i}$. So the resultant velocity $\mathbf{V}$ of the ion is given by

$$\mathbf{V} = \left(W - \frac{\omega Cx}{r^3}\right)\mathbf{i} - \omega\left(E_0 + \frac{Cz}{r^3}\right)\mathbf{k} \qquad (16)$$

Let this resultant velocity V make an angle θ' with the **x**-axis, then,

$$\tan\theta' = \frac{V_z}{V_x} = \frac{-\left(E_0 + \frac{Cz}{r^3}\right)\omega}{W - \frac{\omega Cx}{r^3}} \qquad (17)$$

The trajectory of the ion will be the envelope of its resultant velocity at all points along its path on the trajectory given by

$$\frac{dz}{dx} = \frac{-\left(E_0 + \frac{Cz}{r^3}\right)\omega}{W - \frac{\omega Cx}{r^3}} \qquad (18)$$

In this, we neglect the thermal velocity of ions (size $1\,\mu$ m) of the order $0\cdot1$ cm sec^{-1} and negligible as compared to the velocity of the ions under consideration. The component of velocity in the **r** direction is $\mathbf{V}_r = \mathbf{V}\cos\alpha$

$$\text{where } \cos\alpha = \cos(\theta' - \theta)$$

$$= \cos\theta'\cos\theta + \sin\theta'\sin\theta$$

$$= \cos\theta'\cos\theta\,(1 + \tan\theta\tan\theta') \qquad (19)$$

Now $\tan\theta = z/x$

$$\cos\theta = \frac{1}{(1+\tan^2\theta)^{\frac{1}{2}}} = \frac{1}{(1+z^2/x^2)^{\frac{1}{2}}} = \frac{x}{r}$$

Representing dz/dx as z' we write:

$$\tan\theta' = z',\ \cos\theta' = \frac{1}{(1+\tan^2\theta)^{\frac{1}{2}}} = \frac{1}{(1+z'^2)^{\frac{1}{2}}}$$

$$\text{and } (1+z'^2) = 1 + \tan^2\theta'$$

$$= 1 + \frac{V_z^2}{V_x^2} = \frac{V_x^2 + V_z^2}{V_x^2} = \frac{V^2}{V_x^2}$$

$$\text{So, } \frac{1}{(1+z'^2)^{\frac{1}{2}}} = \frac{V_x}{V} \text{ and } \cos\theta' = \frac{V_x}{V} \qquad (20)$$

Putting these values from **Eqn.** (20) in **Eqn. (19)**

$$\cos\alpha = \frac{x}{r}\cdot\frac{V_x}{\mathbf{V}}\left(1 + \frac{z}{x}\,\frac{V_z}{V_x}\right) = \frac{xV_x + zV_z}{r\mathbf{V}}$$

$$\text{or } \mathbf{V}\cos\alpha = \frac{x\,V_x + z\,V_z}{r} \qquad (21)$$

The point discharge current, would be the charge arriving per second at the sharp point. If we imagine a surface around the origin 0, which is at every point normal to the velocity of ion at that point so much distant from the origin that an ion situated on this surface at $t=0$ will reach the origin in one second, then all the ions crossing this surface at any moment will constitute the point discharge current. An element of such a surface is shown in Fig. 7, and its radial distance from the origin R_0 will be determined as follows.

The time of transit $t = R_0/\mathbf{V}\cos\alpha$

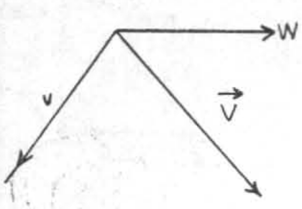

Fig. 4 Direction of resultant velocity of ion in presence of wind and electric field

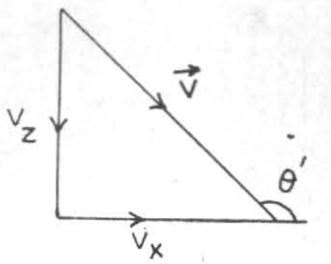

Fig. 5. Component of the resultant velocity along x and z-axis

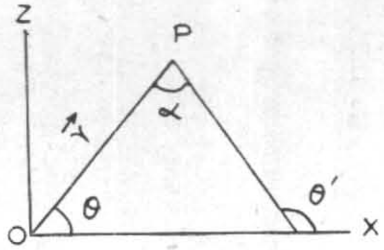

Fig. 6. Angular direction of ion towards the sharp point

Putting $r = R_0$, $x = X_0$, $z = Z_0$ in Eqn. (21) we get

$$t = \frac{R_0}{(X_0 V_x + Z_0 V_z)/R_0} = \frac{R_0^2}{X_0 V_x + Z_0 V_z}$$

$$\text{or} \quad t = \frac{R_0^2}{V_x R_0 \cos\theta + V_z R_0 \sin\theta}$$

since $X_0 = R_0 \cos\theta$, $Z_0 = R_0 \sin\theta$

Putting the values of V_x and V_z from Eqn. (16),

$$R_0 = \left(W \cos\theta - \frac{\omega C}{R_0^2} \cos^2\theta - \omega E_0 \sin\theta - \frac{\omega C}{R_0^2} \sin^2\theta \right) t$$

$$\text{or } R_0^3 + t\,(\omega E_0 \sin\theta - W\cos\theta)\, R_0^2 + \frac{\omega Q}{k}\, t = 0$$

since $C = Q/k$

As R_0 corresponds to $t = 1$ sec, we have,

$$\left[\frac{R_0}{\left(\frac{\omega Q}{k}\right)^{1/3}} \right]^3 + \left[\frac{\omega E_0}{\left(\frac{\omega Q}{k}\right)^{1/3}} \sin\theta - \frac{W}{\left(\frac{\omega Q}{k}\right)^{13}} \cos\theta \right] \left[\frac{R_0}{\left(\frac{\omega Q}{k}\right)^{1/3}} \right]^2 + 1 = 0 \qquad (22)$$

This equation represents the required surface. If we introduce,

$$R_\circ \Big/ \left(\frac{\omega Q}{k}\right)^{1/3} = \xi\,, \quad \omega E_\circ \Big/ \left(\frac{\omega Q}{k}\right)^{1/3} = A\,,$$

$$W \Big/ \left(\frac{\omega Q}{k}\right)^{1/3} = B \qquad (23)$$

We get,

$$\xi^3 + (A \sin\theta - B\cos\theta)\, \xi^2 + 1 = 0$$

$$\text{or} \qquad \xi^3 + P\,\xi^2 + 1 = 0 \qquad (24)$$

where $P = (A \sin\theta - B\cos\theta)$

Consider a surface element at which the velocity normal to it, *i.e.*, the radial veiocity is

$$\mathbf{V} \cos\alpha = V_x \cos\theta + V_z \sin\theta$$

$$\text{Then,} \quad I = \frac{dq}{dt} = \frac{dq}{dr} \cdot \frac{dr}{dt} = \frac{dq}{dr}\, \mathbf{V} \cos\alpha$$

The charge enclosed within the shell of infinitesimal thickness, dr at r of the surface element will be given by,

$$dq = ne\, 4\pi \int_0^{\pi/2} r^2\, dr \sin\theta\, d\theta$$

where n is the number of ions per unit volume, and e is the charge on each ion. Then,

$$I = \frac{dq}{dr}\, (V_x \cos\theta + V_z \sin\theta)$$

$$= 4\,\pi\, ne \int_0^{\pi/2} r^2\, (V_x \cos\theta + V_z \sin\theta) \sin\theta\, d\theta$$

Now, let us consider r to be that distance at which an ion will reach the origin in 1 second. Denote it by $r = R_0$ (Fig. 7) obviously then at that surface $\mathbf{V}\cos\alpha = R_0$ in magnitude. Then

$$I = 4\,\pi\, ne \int_0^{\pi/2} R_0^3 \sin\theta\, d\theta$$

Here, we must keep in mind that R_0 is only the numerical value of the radial velocity and therefore the dimensional homogeneity is not impaired. To avoid confusion in further derivations regarding dimensional homogeneity we shall write,

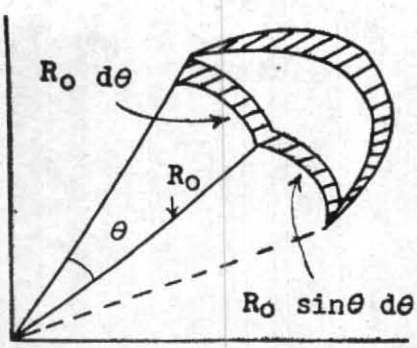

Fig. 7. Element contributing to the point discharge current

$$I = J \int_0^{\pi/2} R_0^3 \sin\theta \, d\theta \tag{25}$$

where the constant J has to take care of the dimensions. In any case, since the value of the constant, J, has to be fixed so as to fit with experimental points, and the expression derived will be employed for the nature of variation of I with E_0 and W, we shall always, in the following keep the constant to take care of the proper dimensions.

Evaluation of the integral

To evaluate $\int_0^{\pi/2} R_0^3 \sin\theta \, d\theta$

we note that R_0 is given by the cubic Eqn. (22) may be further written in an other form (24).

Thus, $$\int_0^{\pi/2} R_0^3 \sin\theta \, d\theta = \frac{\omega Q}{k} \int_0^{\pi/2} \xi^3 \sin\theta \, d\theta \tag{26}$$

We solve the Eqn. (24) for ξ through graphical method. The roots of Eqn. (24) will be given as the points of intersection of the graphs : $y = \xi^3 + \xi^2 P$ and $y = -1$ which have been shown in Fig. 8. The curves (*i*), (*ii*), (*iii*) respectively correspond to $P<0$, $P>0$ and $P=0$. It is clear that a real and positive root is possible only for $P<0$, because for $P>0$, there is only one real but negative root. Thus the case in which we are interested is only that of $P<0$. Also for the positive roots to be real the minima of $y=\xi^3-P\xi^2$ (since $P<0$) must be $|y_m| \geqslant 1$ so that this curve may intersect line $y=-1$.

But $$\frac{dy}{d\xi} = 3\,\xi^2 - 2\,P\xi$$

Then $$\xi_{\min} = \frac{2P}{3} \text{ and } y_{\min} = -\frac{4P^3}{27} \tag{27}$$

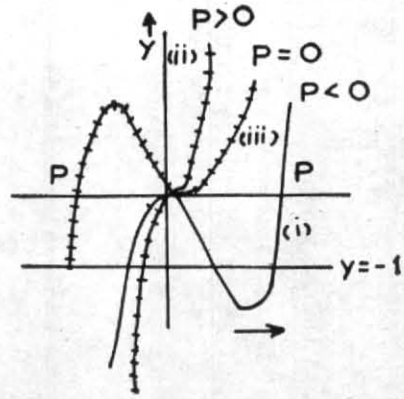

Fig. 8. Curves $y = -1$ and $y = \xi^2 (P+\xi)$ for finding the roots

Hence for positive real roots

$$\left| \frac{4P^3}{27} \right| \geqslant 1 \quad \text{or} \left| P^3 \right| \geqslant \frac{27}{4}$$

$$\text{or} \left| P \right| \geqslant \frac{3}{1.59} = 1.89 \tag{28}$$

Now at $P = 1.89$, the root is $\xi = 1.26$ (29)

For all other values of $P < -1.89$ one can try and check that the solution comes nearer to P. Hence the solution is $\xi \approx P$ approximately. For P to have values larger than -1.89 the percentage error will diminish with the increase of the value of P.

$$\text{Then} \int_0^{\pi/2} \xi^3 \sin\theta \, d\theta = \int_0^{\pi/2} P^3 \sin\theta \, d\theta$$

$$= \int_0^{\pi/2} (A\sin\theta - B\cos\theta)^3 \sin\theta \, d\theta$$

$$= \int_0^{\pi/2} [A^3 \sin^4\theta - B^3 \cos^3\theta \sin\theta -$$

$$-3A^2 B \sin^3\theta \cos\theta + 3\,AB^2 \sin^2\theta \cos^2\theta]\, d\theta$$

$$= \left[\frac{3\pi}{16} A^3 - \frac{3}{4} A^2 B + \frac{3\,\pi}{16} AB^2 - \frac{1}{4}\, B^3 \right]$$

Putting the value of A and B from Eqn. (23) we get finally employing Eqns. (25) and (26).

$$I = J \left[\frac{3\,\pi}{4}\, \omega^3 E_0^3 - 3\,\omega^2 E_0^2\, W + \frac{3\pi}{4}\, \omega E_0\, W^2 - W^3 \right] \tag{30}$$

where J is a constant and takes care of the dimension as decided above.

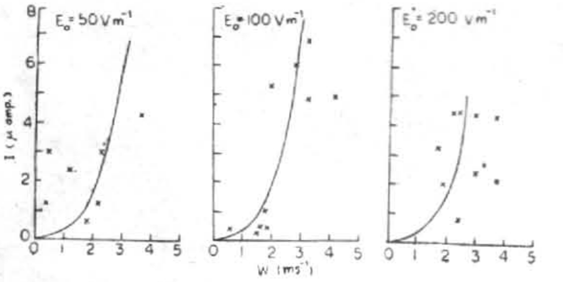

Fig. 9-11. Variation of I with W for $E_o = 50, 100, 200$ V/m showing experimental points (crosses) and theoretical curves

TABLE 1

W (m/sec)	$-I$ in μ Amp. for $E=50$ V/m	$E=100$ V/m	$E=200$ V/m
0·0	$-24{\cdot}8\times10^{-6}$	$-19{\cdot}7\times10^{-6}$	$17{\cdot}9\times10^{-6}$
1·0	0·245	0·241	0·23
1·5	0·845	0·824	0·805
2·0	1·98	1·96	1·93
2·5	3·85	3·82	3·77
3·0	6·71	6.67	6·59
3·5	7·16	7·11	7·00
4·0	15·93	15·86	15·74
4·5	22·44	22·35	22·25

Using relation (30) and putting $E_0 = 50$, 100 and and 200 Vm^{-1}, we calculate the value of I/J.

In order to compare these calculations with the experimental observations, we chose arbitrarily a value of $J = 0.25\times10^{-6}$. Using this value of J we now calculate the values of I for different values of wind speed. These values are given in Table 1. These values of I and W for each value of E have been plotted separately and shown by graphs in Figs. 9-11.

5. Observations and Analysis

We recorded the time variation of point discharge current for April 1970 to October 1970, atmospheric potential gradient and wind speed for this period were also on record as described earlier. To study the effect of wind speed upon point discharge current, we collected the various sets of the values of point discharge currents and wind speed for different potential gradients (within an interval 5Vm^{-1}). Such values of I and W have been plotted (shown by crosses on graphs in Figs. 9 to 11).

6. Discussion and conclusion

Our experimental points showing the variations of the point discharge current with wind speed (shown by crosses in Figs. 9-11) can be well fitted in the theoretical curves traced in Fig. 9-11. They compare well with the experimental values particularly with regard to the nature of variation.

7. Remarks

The result for point discharge current derived above is an approximate expression valid for cases when secondary ionization is not important, *i.e.*, when atmospheric potential gradient is not too high. A comparison of the theoretical results with the experimental values as already discussed above is quite favourable, although at these potentials secondary ionization could not completely be absent.

A complete theoretical description should involve secondary ionization and could be a subject for further investigation.

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