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Weibull and chi-square fits for wind speed distribution at Jessore

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सार — जैस्सोर मौसम स्टेशन के तीन वर्ष के वायु वेग आंकड़ों के साथ वैबुल तथा χ^2 वितरणों को जोड़ा गया, दोनों वितरणों के परिमापकों के मानों का प्रत्येक माह अभिकलन किया गया। ऐसा पाया गया था कि मासिक औसत वायु वेग तथा उपलब्ध ऊर्जा दोनों वितरणों द्वारा पर्याप्त मात्रा में पुनः उत्पादित की गई थी।

ABSTRACT. Weibull and χ^2 distributions were fitted to three years wind speed data of Jessore meteorological station. The values of the parameters of both the distributions were calculated for each month. It was found that the mothly mean wind speed and the available energy were fairly reproduced by both the distributions.

1. Introduction

The wind speed at any locality has a fluctuating nature. The statistical behaviour of the wind has been widely studied in the USA (The U.S. Department of Energy, 1979) and also in some countries of Asia (Exell *et al.* 1981). Two types of distributions, namely the Weibull and χ^2 have been widely used to describe the statistical nature of wind. The parameters of these distributions describe not only the wind speed but also allow one to obtain an estimate of the total available energy. No such study has yet been made in Bangladesh to see the behaviour of the wind of this region. For such a study, the station Jessore was chosen and an attempt has been made to fit the frequency distribution of hourly average data for each month of the year to the above two distributions.

2. Theory

The principal characteristics of the two types of distributions are :

(*a*) *Weibull distribution* — Let $F(v)$ be the cumulative distribution function of the wind speeds giving the probability that the wind at any time has a speed less than v. Such a distribution can be expressed in Weibull form (Corotis *et al.* 1978) as :

$$F(v) = 1 - \exp[-(v/C)^K] \qquad (1)$$

where C and K are the parameters to be determined from the nature of the wind at a particular locality. C is called the scalar factor whose value is slightly greater than the mean wind speed and K is called the shape factor.

(b) χ^2 *distribution* — Here, $F(v)$ can be expressed as

$$F(v) = 1 - \exp - [\pi/4(v/m)^2] \qquad (2)$$

as shown by Corotis *et al.* (1978) where v is the wind speed and m is a parameter of this distribution.

The frequency distribution $f(v)$, whose integral from v_1 to v_2 denotes the probability that the wind speed at any time lies between v_1 and v_2 is obtained by differentiating the distribution function $F(v)$. So,

$$f(v) = dF(v)\,/\,dv$$

$$= \left(\frac{K}{C}\right)\left(\frac{v}{C}\right)^{K-1} \exp\left[-(v/C)^{K}\right] \text{ for Weibull}$$

distribution

and

$$= \frac{\pi}{2}\,\frac{v}{m^2} \exp\left[\frac{-\pi}{4}\left(\frac{v}{m}\right)^2\right] \text{ for } \chi^2 \text{ distribution.}$$

The mean speed $<v>$ is then

$$<v> = \int_0^\alpha v\,f(v)\,dv$$

$$= C\Gamma(1 + 1/K) \text{ for Weibull distribution} \qquad (3)$$

where Γ represents the gamma function and

$$= \frac{m\sqrt{\pi}}{2} \qquad \text{for } \chi^2 \text{ distribution} \qquad (4)$$

As the power in the wind is proportional to v^3, the average value of v^3 over a month or a certain period is of great importance. The value of $<v^3>$ is

$$<v^3> = \int_0^\alpha v^3 f(v)\,dv$$

$$= C^3\Gamma(1 + 3/K) \text{ for Weibull distribution} \qquad (5)$$

and

$$= 3m^3/\sqrt{\pi} \quad \text{for } \chi^2 \text{ distribution} \qquad (6)$$

TABLE 1

The mean frequency distribution of wind speeds for each month of the year

Wind speed (kt)	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
0	449	341	271	97	56	70	83	109	220	393	482	450
1	1	0	0	0	0	0	0	0	0	0	0	0
2	74	66	45	34	21	27	37	44	56	68	16	76
3	84	79	106	72	72	92	93	101	101	90	78	88
4	17	22	37	21	11	39	44	49	35	30	26	22
5	42	64	76	77	96	104	102	108	74	60	44	49
6	15	15	30	22	27	32	108	51	38	22	16	16
7	27	24	45	52	53	66	72	40	41	18	9	18
8	12	23	42	41	52	67	75	71	47	21	11	13
9	13	5	14	26	30	37	18	17	12	3	6	1
10	4	20	31	77	86	79	78	55	51	20	0	9
11	5	0	3	1	1	1	0	1	0	0	2	0
12	0	8	12	26	51	22	36	26	19	5		1
13	0	0	4	9	17	9	9	8	4	1		0
14	1	1	3	4	8	3	2	7	1	0		1
15		0	6	20	15	20	16	14	9	3		
16		1	8	40	71	26	15	14	5	5		
17		0	3	9	7	1	2	2	2	2		
18		1	4	34	31	12	8	9	3	0		
19		0	0	0	6	0	0	0	0	1		
20		0	1	30	19	8	2	9	1	1		
21		0	0	0	0	0	0	0	0	0		
22		0	0	9	3	1	1	2	1	1		
23		0	2	1	1	0	0	0				
24		0	0	0	1	0	0	2				
25		1	1	10	1	1	1	1				
26				0	2		0	0				
27				0	0		0	1				
28				4	2		1	1				
29				3	0		0	0				
30				0	1		1	1				
31				0	0							
32				0	2							
35				0	1							
39				1	1							

TABLE 2

Values of C, K, m, $< v >$, $< v^3 >$ and monthly available wind energy

	Weibull's parameters		χ^2 parameter	$< v >$			$< v^3 >$		Energy (kWh/m²/month)		
	(C)	(K)	(m)	Weibull	χ^2	Direct from data	Weibull	χ^2	Weibull	χ^2	Direct from data
Jan	4.17±.05	1.52±.09	4.35±.08	1.05±.02	1.54±.03	1.76	56±6	56±3	1.6±.2	1.6±.1	1.9
Feb	4.89±.05	1.33±.06	6.68±.25	2.20±.07	2.90±.11	2.40	148±12	247±28	3.8±.3	6.4±.7	4.5
Mar	6.06±.07	1.66±.07	6.35±.13	3.48±0.4	3.60±.07	3.84	239±18	277±17	6.8±.5	7.9±.5	10.7
Apr	10.29±.09	1.57±.05	10.67±.16	8.05±.07	8.23±.12	7.80	1748±105	1789±81	48.1±3.9	49.3±2.2	54.9
May	10.53±.09	1.78±.05	10.59±.23	8.62±.08	8.63±.19	8.55	1640±63	1849±40	46.7±1.8	52.6±1.1	53.0
Jun	7.82±.04	1.58±.04	8.09±.14	6.31±.04	6.44±.11	6.61	785±36	807±42	21.6±1.0	22.2±1.2	23.4
July	7.64±.04	1.79±.04	7.60±.11	6.05±.03	5.99±.09	6.31	602±20	661±29	17.2±.6	18.8±.8	20.4
Aug	7.69±.09	1.51±.06	9.05±.20	5.89±.07	6.82±.18	6.12	765±62	1066±85	21.8±1.8	30.3±2.4	24.1
Sep	6.30±.04	1.63±.05	6.61±.17	3.89±.03	4.04±.14	4.30	299±17	337±26	8.3±.5	9.3±.7	10.0
Oct	4.92±.05	1.28±.06	6.68±.21	2.14±.03	2.78±.09	2.34	157±19	237±22	4.5±.5	6.8±.6	5.1
Nov	4.01±.04	1.80±.09	3.88±.08	1.18±.01	1.13±.02	1.37	32±2	33± 2	0.9±.1	0.9±.1	1.1
Dec	3.91±.03	1.54±.07	4.25±.10	1.41±.01	1.51±.04	1.61	46±4	52± 4	1.3±.1	1.5±.1	1.6

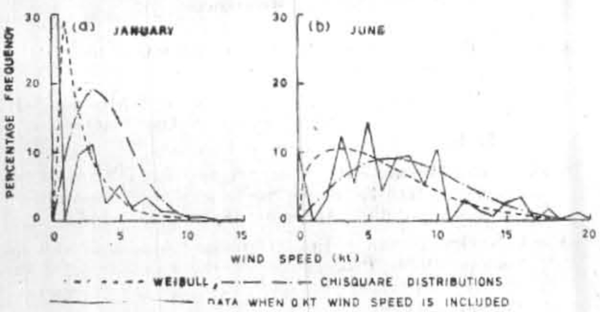

Fig. 1. Fits of the Weibull and χ^2 distributions to the data when 0 knot wind speed is included

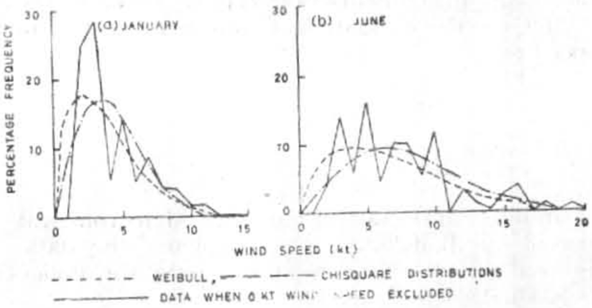

Fig. 2. Fits of the Weibull and χ^2 distributions to the data when 0 knot wind speed is excluded

3. Data and analysis

Hourly wind speed data for the meteorological station at Jessore Airport have been collected from the Bangladesh Meteorological Department for the period 1978-80. Wind speeds were recorded each day at every hour with a cup type anemometer having an accuracy of $\pm$ 1 knot. From the data the frequency distributions of each month for every year were obtained and these were averaged (Table 1) to represent the mean frequency distribution for Jessore. The frequency distribution $F(v)$ was then obtained.

To find the values of the parameters C, K and m the following procedures have been adopted. From Eqn. (1) one can write

$$\ln\left\{-\ln\{1-F(v)\}\right\} = K\ln v - K\ln C$$

which means that when $\ln\left\{-\ln\{1-F(v)\}\right\}$ is plotted against $\ln v$, a straight line fit will be obtained whose slope will be equal to K and the intercept with the Y axis will be equal to $-K\ln C$. Thus the parameters C and K may easily be determined. Similarly from Eqn. (2) one can write

$$\ln\left\{1-F(v)\right\} = -\pi v^2/4m^2$$

Hence, when $\ln\left\{1-F(v)\right\}$ is plotted against v^2, it will also be a straight line passing through the origin having a slope equal to $-\pi/(4m^2)$.

It was pointed out earlier (Exell *et al.* 1981) that for Thailand Weibull distribution shows a good fit when the wind speed exceeds 1 knot. Figs. 1 and 2 illustrate the general behaviour of Weibull and χ^2 fits for Jessore station with and without 0 knot speed. So, the values of v^2, $\ln v$, $\ln\left\{1-F(v)\right\}$ and $\ln\left\{-\ln\{1-F(v)\}\right\}$ for each month were calculated omitting the 0 knot wind speed. Using the least square method, the slope, intercept and the associated errors were calculated with a programmable hand calculator. Once the values of C, K and m were known, the values of v and v^3 could be found out easily with the help of the Eqns. (3)-(6). Since the occurrence of 0 knot wind speed was not included in the determination of the parameters, C, K, and m, the values of $\langle v\rangle$ and $\langle v^3\rangle$ as calculated from the above parameters will have higher values than the average values of v and v^3. To include the effect of 0 knot, the values of $\langle v\rangle$ and $\langle v^3\rangle$ are to be multiplied by a factor of $(1-f)$, where f is the fraction of time that 0 knot wind speed has been reported (Exell *et al.* 1981). With the help of the values of $\langle v^3\rangle$, the available monthly wind energy for a wind machine were calculated using the formula (Sarkar *et al.*)

$$E\,(\text{kWh/m}^2) = 3.8236\times10^{-5}\,D^2V^3H$$

where D is the rotor blade diameter expressed in metres, V^3 is the average value of v^3 (here $\langle v^3\rangle$) expressed in knot^3 and H is the duration in hours for which $\langle v^3\rangle$ has been considered.

4. Results and discussion

The values of C, K, m, $\langle v\rangle$, $\langle v^3\rangle$ and the monthy energy output are displayed in Table 2. The table also contains the average values of v and the monthly available energy calculated directly from the wind speed data as recorded by the Meteorological Department. Corotis *et al.* (1978) observed that for the mean wind speed, χ^2 distribution gives as good a fit as Weibull but for energy, Weibull seems to be better than the χ^2distribution. A close look at the Table 2 shows that the average wind speed and the monthly available energy are reproduced fairly well by both the distributions. The annual energy available is 208 (kWh/m^2) accordingto χ^2, 183 (kWh/m^2) according to Weibull and 211 (kWh/m^2) from direct calculation from individual data. The squares of the deviations of the average wind speed and the monthly energy output from their values obtained directly from the data were summed and it is found that χ^2 distribution has slightly lower deviations for our data. Hence, we find that either of the distributions may be used but χ^2 distribution appears to be marginally better for the estimation of both energy and mean wind speed.

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