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Temporal fluctuations of seasonal rainfall patterns in east Africa

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सार — पूर्वी अफ्रीका में वर्षा ऋतुओं में वर्षा के निदर्श में कालिक उतार-चढ़ाव के अन्वेषण के लिए लगभग 100 स्टेशनों की 1931–75 की अवधि में हुई वर्षा का वार्षिक आधार पर हार्मोनिक विश्लेषण किया गया है। इस विधि में 1931 से 1975 की अवधि के प्रबल हार्मोनिकों के वार्षिक आयामों एवं कोणाँको से काल श्रेणी बनाई गई है। व्युत्पन्न काल श्रेणी के कालिक उतार-चढ़ावों को उक्त अध्ययन की अवधि में पूर्व अफ्रीका में वर्षा ऋतु में हुई वर्षा के कालिक पैटर्न को स्पष्ट करने के लिए उपयोग में लाया गया है। नम ऋतु की पंचतय (पेनटैड) वर्षा में वार्षिक उतार-चढ़ावों का भी अन्वेषण किया गया है।

हार्मोनिक विश्लेषण से प्राप्त परिणाम पूर्वी अफ्रीका के अधिकांश क्षेत्रों में प्रथम तीन हार्मोनिकों की प्रबलता का संकेत देते हैं। तथापि कुछ शुष्क क्षेत्रों में उच्च हार्मोनिकों से पर्याप्त प्रसरण की पुष्टि होती है।

प्रयुक्त सांख्यिकीय विधियों से पता चलता है कि वर्षा ऋतु की वर्षा के पैटर्न में 95 प्रतिशत विश्वसनीयता स्तर पर कोई सार्थक परिवर्तन या अंतरण नहीं है उधर स्पेक्ट्रल विश्लेषण से 2 से 3 वर्ष और 5 से 6 वर्ष की अवधि में अल्पावधि उतार-चढ़ावों का पता चला है। कुछ श्रेणियों में 10–11 वर्षों के बीच एक दुर्बल शीर्ष का भी आभास मिला है।

ABSTRACT. In order to investigate temporal fluctuations in the seasonal rainfall over East Africa, monthly rainfall records from about 100 stations during the period 1931 - 75 were subjected to harmonic analysis on the yearly basis. Under this method time series were generated from the yearly amplitude and phase angle values of the dominant harmonics for the period 1931 - 75. The temporal fluctuations of the generated time series were used to describe temporal patterns of seasonal rainfall in East Africa during the period of study. Yearly fluctuations in the pentad rainfall were also investigated during the wet seasons.

The results from harmonic analysis indicated the dominance of the first three harmonics over most regions of East Africa. Higher harmonics, however, explained substantial variance in some dry regions.

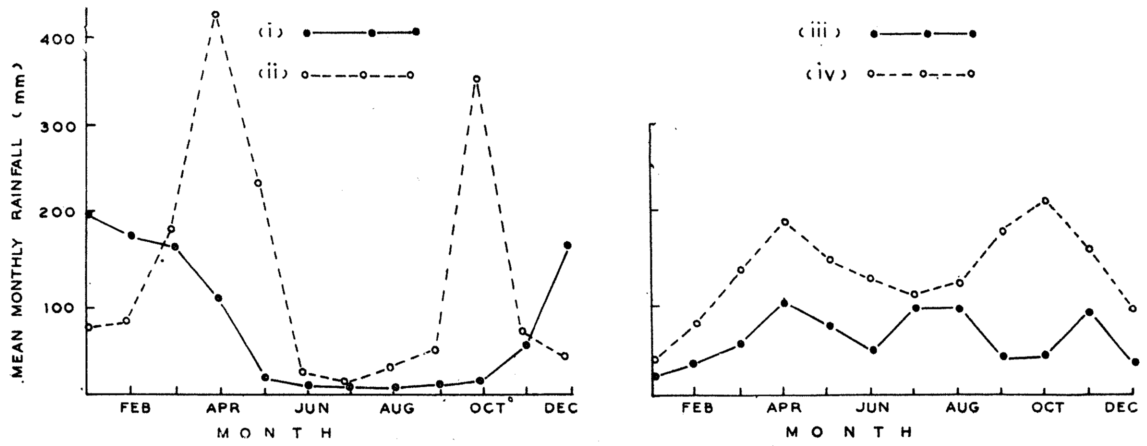
The statistical methods employed indicated that at 95% confidence level, no significant seasonal changes or shifts could be detected in the seasonal rainfall patterns. Spectral analysis, however, displayed short period fluctuations within the ranges of 2 - 3 years and 5 - 6 years. A weak 10 - 11 years peak also appeared in some series.

1. Introduction

One way of examining seasonal moving effects may be the application of harmonic analysis. Harmonic analysis is based on the idea of Fourier series, and it can be used to decompose any time series into a sum of sinusoidal terms. By examining a seasonal time series year by year, the seasonal components for each year may be identified independently. Yearly fluctuations of these seasonal components may be used to objectively describe the temporal variations of the seasonal time series.

When a time series is subjected to harmonic analysis, the magnitudes of the amplitudes will determine the major harmonics of the time series while the phase angle may be used to describe the time of maximum or minimum for any harmonic considered. The distribution of the first harmonic is unimodal. The second harmonic has bimodal distribution while the third harmonic will have trimodal distribution, etc.

Although the spatial and temporal distributions of rainfall are complex in East Africa, the annual rainfall exhibits strong seasonality. The rainfall seasons migrate with the ITCZ which shifts with the sun's positions. Only parts of Uganda receive substantial rainfall throughout the year. Closer to the equator bimodal distribution of rainfall is well marked with two rainy seasons which are generally known as the long and



Figs. 1(i-iv). Mean seasonal rainfall pattern at (i) Mbeya (BMY), (ii) Meru (MRU), (iii) Rimuruti (RRT) and (iv) Fort Portal (FPL)

TABLE 1
List of the stations and their code names

Country and station	Station's code name	Country and station	Station's code name
KENYA		Sotik	SOT
Garissa	GRS	Tambach	TBC
Gazi	GAZ	Voi	VOI
Kabarnet	KBT	Wajir	WJR
Kabete	NRB	**TANZANIA**	
Kakamega	KKG	Amani	AMN
Kapsabet	KPT	Arusha	ARS
Kericho	KRC	Bagamoyo	BGY
Kiambu	KIB	Biharamulo	BHL
Kilifi	KLF	Bukoba	BKB
Kisii	KIS	Dar es Salaam	DES
Kisumu	KSM	Dodoma	DDM
Kitale	KTL	Kagondo	KGD
Kitui	KTI	Kasulu	KSL
Konza	KNZ	Kigoma	KGA
Lamu	LMU	Kigomasha	KGM
Lodwar	LDR	Kilindoni	KNM
Londiani	LNR	Kilosa	KLS
Machakos	MCK	Kilwa	KLW
Magadi	MGD	Lindi	LND
Makindu	MKD	Lushoto	LST
Makuyu	MKY	Mahenge	MHG
Malindi	MLD	Masasi	MSS
Marsabit	MST	Mbeya	MBY
Meru	MRU	Morogoro	MGR
Mombasa	MBS	Moshi	MSH
Mt. Elgon	MES	Musoma	MSM
Moyale	MYL	Mwanza	MWZ
Naivasha	NVS	Mkokotoni	MKN
Nakuru	NKR	Pangani	PGN
Nanyuki	NYK	Singida	SGD
Narok	NAR	Songea	SNA
Ngong	NGN	Sumbawanga	SWG
N. Kinango	NKP	Tabora	TBR
Nyeri	NYR	Tanga	TNG
Rongai	RGI	Tukuyu	TKY
Rumuruti	RRT	Tunduru	TRN
Simba	SBA	Wete	WET
Solai	SOI		

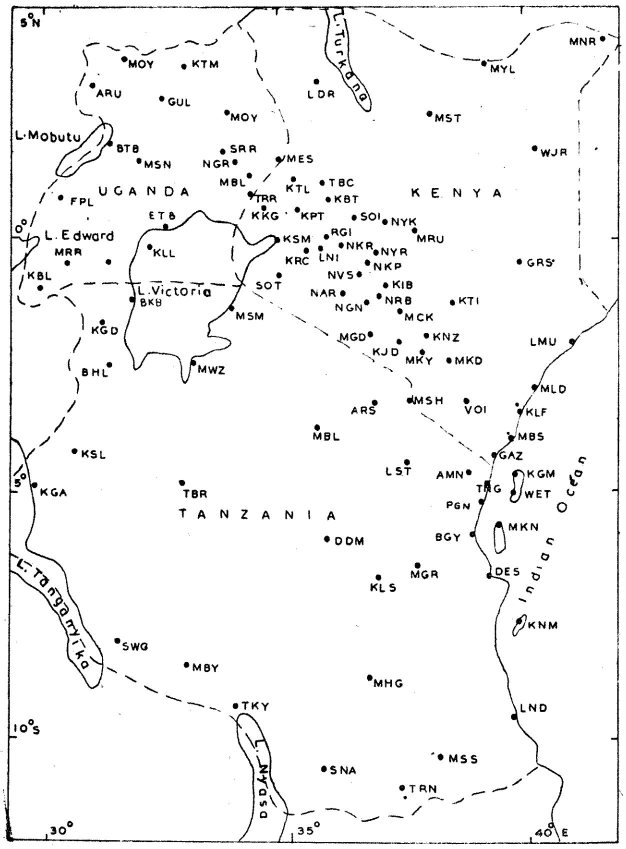


Fig. 2. Network of rainfall stations used

TABLE 1 (*contd*)

Country and station	Station's code name
Uganda	
Arua	ARU
Butiaba	BUT
Fort Portal	FPL
Gulu	GUL
Hoima	HOM
Kabale	KBL
Kalangala	KLL
Kitgum	KTM
Masaka	MSK
Masindi	MSN
Mbale	MBL
Mbarara	MRR
Moyo	MOY
Moroto	MRT
Ngeta	NGT
Ngora	NGR
Serere	SRR
Tororo	TRR

short rainy seasons. Further away from the equator the distributions are dominantly unimodal. In the Rift valley and the central highlands of Kenya, trimodal distribution is observed at some stations. The third rainfall peak is due to moisture incursions from the Congo airmass (EAMD 1962, Nakamura 1967). Some examples of the seasonal rainfall patterns in East Africa is given in Figs. 1 (i-iv). Due to the dominance of these rainfall patterns in East Africa, it is hoped that when seasonal rainfall data are subjected to harmonic analysis, the first three harmonics can adequately explain substantial amount of seasonal rainfall variance.

In this study the monthly rainfall records for the period 1931-75 were subjected to harmonic analysis on yearly basis. The yearly phase and amplitude time series generated from the major harmonics were used to describe the temporal fluctuations of the seasonal rainfall in East Africa. In order to describe the temporal patterns in the seasonal rainfall during the year 1931-75, the generated series were subjected to trend and spectral analyses.

Wet-pentad rainfall records for the period 1931-75 were also subjected to trend and spectral analyses. The wet-pentads were considered as the pentads within the rainy seasons. Since a pentad consists of five-daily rainfall records, each year will have 73 pentads. If time series are generated from the yearly pentad records, then each station will have 73 pentad series. The 12th pentad which extends from 25 Feburary to 1 March will have six days in all leap years when February has 29 days. The records of the 12th pentad were normalised here by dividing all records of the 12th pentad by a factor 5/6 for all leap years.

The data used in this study consisted of daily and monthly rainfall records of about 100 stations during the period 1931-75. The spatial distribution of these stations is given in Fig. 2, while an example of the wet-pentad time series is given in Fig. 3. Table 1 gives the code names of the various stations used.

2. Harmonic analysis

In harmonic analysis, any time series may be expressed mathematically as a sum of simple trigonometric functions whose frequencies are multiples of the fundamental frequency (f). If the rainfall time series Y_t has N data points and is represented by m cosine and sine sums (harmonics) with the arguments equally spaced in the interval (0, 2π), then the mathematical expression for harmonic analysis may be of the form :

$$Y_t = \overline{Y_t} + \sum_{k=1}^{m} \left[a_k \cos\left(\frac{2\pi kt}{N}\right) + b_k \sin\left(\frac{2\pi kt}{N}\right)\right] \quad (1)$$

where $\overline{Y_t}$ is the arithmetic mean of Y_t. Maximum $m = N/2$, hence maximum m is 6 for monthly data, a_k and b_k are constants for each harmonic, and the fundamental frequency $f = 1/N$ for unit time interval.

From trigonometric algebra the time series Y_t may be expressed as :

$$Y_t = \overline{Y_t} + \sum A_k \sin\left(\frac{2\pi kt}{N} + \theta_k\right) \quad (2)$$

where, $A_k = (a_k + b_k)^{\frac{1}{2}}$ and $\theta_k = \tan^{-1}(b_k/a_k)$ (3)

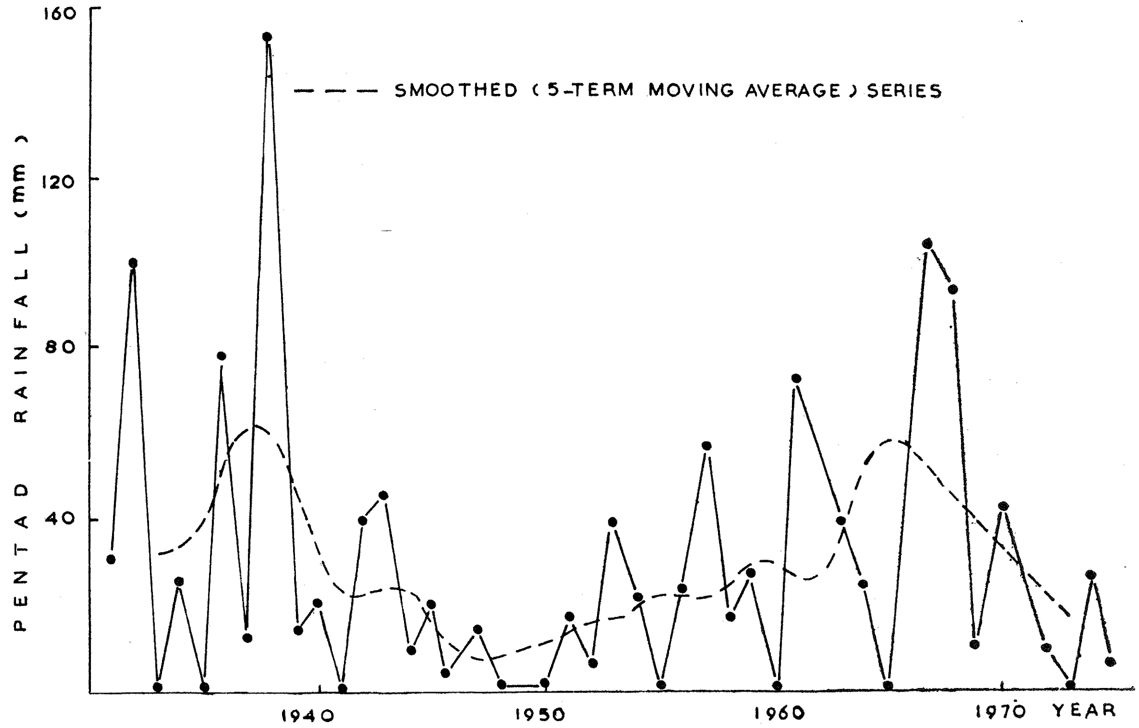


Fig. 3. An example of the wet-pentad (20th pentad) time series at Lodwar

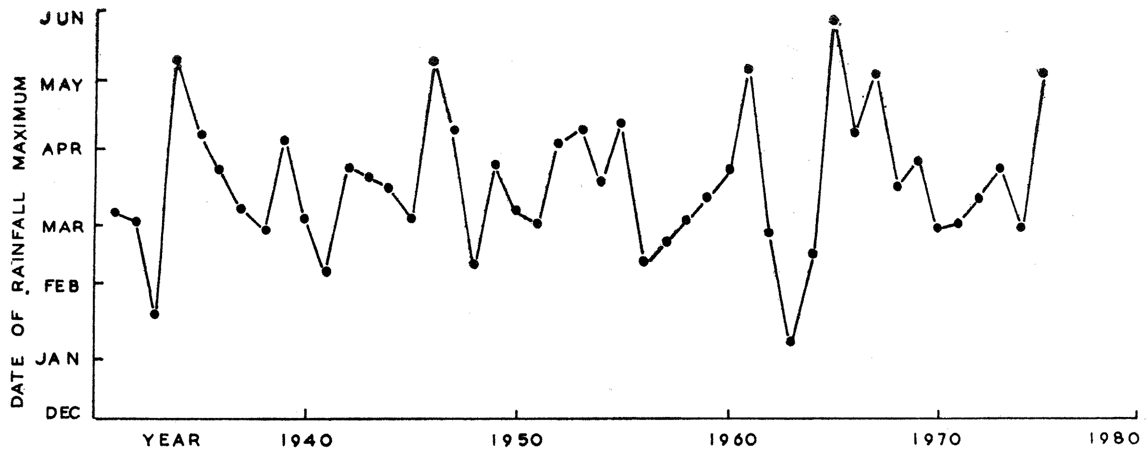


Fig. 4. Yearly fluctuations of the first phase angle at Dar es Salaam

Hence, $$a_k = \sum_{t=1}^{N} Y_t \cos\left(\frac{2\pi kt}{N}\right) \tag{4}$$

$$b_k = \sum_{t=1}^{N} Y_t \sin\left(\frac{2\pi kt}{N}\right) \tag{5}$$

The phase angle (phase difference) θ_k represents the time interval between each harmonic or wave, and can be used to give the best mathematical description for the time of maximum or minimum for any harmonic considered. The ordinate distance from the mathematical mean to the maximum or minimum point is known as the amplitude (A_k). The contribution to the total variance by any harmonic k is proportional to A_k^2. The total variance (S^2) may be expressed as :

$$S^2 = \tfrac{1}{2} \sum_{k=1}^{m-1} (A_k^2 + A_m^2) \tag{6}$$

Some good account of harmonic analysis have been discussed by Fisher (1929), Jenkins and Watts (1968), Horn and Barryson (1960), Potts (1971), Walker (1975), and many others.

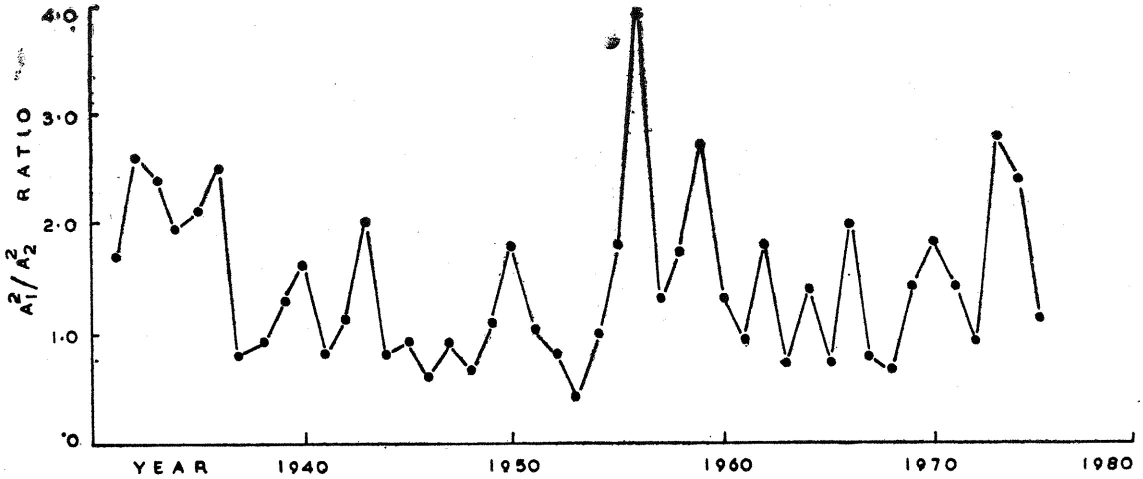


Fig. 5. Yearly fluctuations of the first amplitude at Dar es Salaam

If monthy rainfall data are subjected to harmonic analysis, the first harmonic will describe the tendency towards the annual variation, and it has one maximum and one minimum. The second harmonic has bimodal distribution and it represents the semi-annual tendency. Similarly the kth harmonic will describe the $12/k$ month variations. The mean phase and amplitude of the major harmonics have been used to examine the annual march of precipitation and to delineate precipitation into regimes (Horn & Barryson 1960, Potts 1971). Potts (1971) used mean monthly rainfall records for the period 1931-60.

In this study the monthly rainfall records for the period 1931-75 were independently subjected to harmonic analysis on the yearly basis. The yearly phase and amplitude values of the major harmonics at each station were used to generate phase and amplitude time series for the period 1931-75. The corresponding mean regional time series were also generated from monthly rainfall records of some homogeneous rainfall regions which had been defined from empirical orthogonal analysis (Ogallo 1980).

3. The trend and spectral analyses

The series subjected to trend and spectral analyses include phase, amplitude and wet-pentad time series for the period 1931-75.

To examine the trend of the time series, rank correlation tests were used. These tests use non-parametric measures of correlation based on ranks to examine the trend of time series. Detailed description of these methods are provided by Hottelling & Pabst (1936), Friedman (1940), Kendall & Stuort (1961), Siegel (1956), WMO (1966) and many others. The application of the method to climatological time series include the work of Parthasarathy & Dhar (1974), Tyson *et al.* (1975) and Ogallo (1979). In this study the Spearman rank correlation test has been applied in examining the trend of the time series.

To investigate the existence of any periodic or quasi-periodic fluctuations, the series were subjected to spectral analysis. The spectral analysis technique has a very long history during which various approaches have been developed for the computation of the spectral estimates from the observed data. A good account of these methods has been provided by many authors including Cooley *et al.* (1967), Jenkins and Watts (1968), Burg (1970, 1972) and Akaike (1974). The autocorrelation transformation technique has been used in this study. A first difference filter was also included to ensure stationary conditions. The gain $G(f)$ and $R(f)$ response functions for this filter is given by :

$$G(f)=2i\,|\sin(\pi f)| \qquad (7)$$

$$R(f)=2i\exp(-i\pi f)\sin(\pi f) \qquad (8)$$

where f indicates the frequency.

4. Results and discussions

The results from harmonic analysis indicated that in general the magnitudes of the first two harmonics were much larger compared to the other harmonics. The first three harmonics together explained generally more than 90% of the total rainfall variance in all regions of East Africa except in the dry regions where even Potts (1971) noted that the first five harmonics together explained less than 70% of the mean rainfall variance.

Since unimodal and bimodal distributions are common in the annual rainfall patterns in East Africa, the first two harmonics of the harmonic analysis should be expected to explain most of the total rainfall variance. Trimodal rainfall distribution has also been observed at some stations in East Africa. Due to the dominance of the first three harmonics in the East African annual rainfall, only the characteristics of the first three harmonics were examined in this study with exception of the semi-arid parts of Kenya which have high year to year rainfall variability. As many as five harmonics have been used in this region.

The amplitude time series indicated that as expected the seasonal rainfall patterns in some years were significantly different from the mean seasonal patterns. An

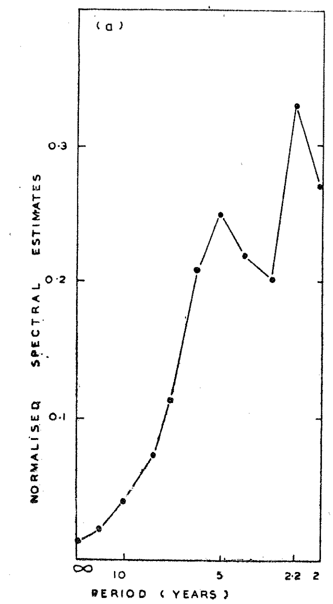


Fig. 6. Spectral patterns of the first amplitude at Voi

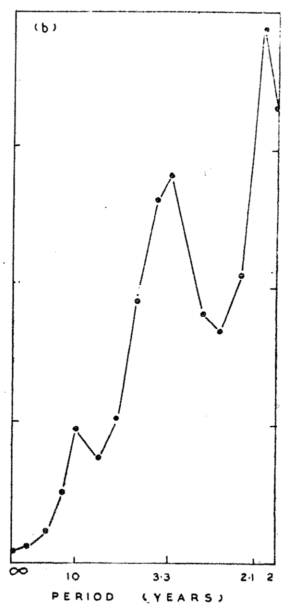


Fig. 7. Spectral patterns of the first phase angle at Voi

example of the yearly amplitude time series is given in Fig. 5 in form of the yearly variance contributions by the first two dominant harmonics (A_1^2/A_2^2). It can be observed from Fig. 5 that although the mean seasonal pattern in Dar es Salaam region is generally unimodal, the ratio A_1^2/A_2^2 was less than one in some years indicating the dominance of two distinct rainfall peaks during these years.

The yearly phase time series indicated that although the dates of rainfall maximum varied significantly from year to year within the rainy seasons, no significant patterns could be detected from the yearly phase values during the period of study. Fig. 4 presents the yearly phase time series observed at Dar es Salaam for the annual wave during the period 1931-75. It can be observed from this figure that at Dar es Salaam the phase angles of the first harmonic indicated seasonal fluctuations of rainfall peaks between early December to late May during the period 1931-75.

A typical example of the wet-pentad time series was given in Fig. 3. The wet-pentad time series displayed characteristics identical to the seasonal wetness and dryness patterns which were observed over many East African stations during the period of study.

When the yearly time series were subjected to trend analysis, no significant trends could be detected at 5% significance level in most of the time series. No significant trends in both wet-pentad and the phase time series indicate that no significant seasonal shift could be detected from the yearly fluctuations in the timing of the rainfall maximum and minimum during the period of study.

The results of spectral analysis indicated the dominance of short period fluctuations. In general, the most prominent spectral peaks were centred around 2-3 years and 5-6 years. A weak 10-11 years peak also appeared in many time series. Some examples of these peaks are shown in Figs. 6 and 7. Physical significance of the cycles displayed from spectral analysis have not been examined here, although their statistical significance were investigated. Similar patterns have been observed in the seasonal and annual rainfall records in East Africa (Rodhe 1976, Ogallo 1982).

5. Conclusions

The results of the study revealed that the statistical methods employed could not detect any significant trend in the seasonal time series. This indicate that the observed yearly fluctuations in the seasonal rainfall patterns during the period of study were not statistically different from those expected by chance at 5% significance level. Thus, no significant seasonal change or shift could be detected. The prominent features of the seasonal series suggested from the results of spectral analysis were quasi-periodic fluctuations with short periods. In general the 2-3 and 5-6 years spectral peaks were more prominent in most of the time series examined. A weak 10-11 year peak was also common in some time series.

References

Akaike, H., 1974, *IEEE*, **AC6**, pp. 716-723.

Burg, J.P., 1970, *Soc. Explors. Geophys.*, New Orleans.

Burg, J.P., 1971, *Geophys.*, **37**, pp. 375-76.

Cooley, *et al.*, 1967, *IEEE*, **AC15**, pp. 79-84.

EAMD, 1962, East Afric. Met. Dep., 8, pp. 1-4.

Fisher, R.A., 1929, *Proc. Roy. Soc.*, **A125**, pp. 54-59.

Friedman, M., 1940, *Ann. Math. Stat.*, **11**, pp. 89-92.

Horn, L.H. and Barryson, R.A., 1960, *Ann. Ass. Geo.*, **50**, pp. 157-171.

Hottelling, H. and Pabst, P.M., 1936, *Ann. Math. Stat.*, **7**, pp. 29-43.

Jenkins, G.M. and Watts, D.G., 1968, *Spectral analysis and its applications*, Holden Day.

Kendall, M.G. and Stuart, A., 1961, *Advanced theory of statistics*, Charles Griffin & Co., London.

Nakamura, K., 1967, Japanese Prog. Climat., Geog. Report, Tokyo Metrop. University (2).

Ogallo, L. J., 1979, *Mon. Weath. Rev.*, **109**, pp. 1133-1139.

Ogallo, L. J., 1980, *Stat. Climat. Dev. Atm. Sci.*, **13**, 255-266, Elservier, Amsterdamm.

Ogallo, L. J., 1982, *Kenya J. Sci. Tech.* (*A*).

Parthasarathy, B. and Dhar, O. N., 1974, *Quart. J. R. met. Soc.*, **100**, pp. 245-257.

Potts, A., 1971, *J. Trop. Geog.*, **33**, pp. 31-42.

Rodhe, H., 1975, *Mon. Weath. Rev.*, **104**, pp. 307-315.

Siegel, S., 1956, McGrawhill.

Tyson, P.D. *et al.*, 1975, *Quart. J. R. met. Soc.*, **101**, pp. 817-833.

Walker, J.M., 1975, *Indian, J. Met. Geophys.*, **26**, 2, pp. 181-188.

WMO, 1966, *WMO Tech. Note* 79.