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Wave spectra and statistics off Godavari during September-October 1980

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सार — जहाज में लगे तरंग अभिलेखों का उपयोग करके 61 तरंगों के अभिलेखों को एकत्रित करके उनका विश्लेषण करने पर स्पेक्ट्रम के आकार में हुए विस्तृत परिवर्तनों का पता चला है। कुछ स्पेक्ट्रम अपेक्षतया संकरे और नुकीले होते हैं, कुछ चौड़े एवं कम नुकीले तथा कुछ में कई नोकें होती हैं। स्पेक्ट्रम के आकार की व्याख्या के लिए स्पेक्ट्रम की चौड़ाई के प्राचल (E_S स्पेक्ट्रमों से और E_Z शून्य अपक्रासिंग विधि से प्राप्त) और स्पेक्ट्रमी शिखरता गुणाकं Q_P की तुलना में अर्ध स्पेक्ट्रमी शिखर बैंड चौड़ाई अधिक महत्वपूर्ण मानों की सृष्टि करते हैं। औसत शून्य-अपक्रासिंग अवधि T_Z 8.1 से 12.9 सेकडं तक हो सकती है, अतः सार्थक तरंगावधि T_S से काफी मिलती है। अन्य औसत अवधियों की तुलना में T_S और T_Z अच्छे आकलन सिद्ध हुए हैं। प्रेक्षण अवधि (सितम्बर-अक्तूबर) में चूंकि मौसम शांत रहता है (बंगाल की खाड़ी में झंझा नहीं होते) तरंग की सार्थक ऊंचाइयों 1 मी० के लगभग होती हैं। उस समय E_S एवं E_Z क्रमशः 0.53 से 0.82 और 0.52 से 0.80 के मध्य होते हैं। तब बैंड चौड़ाई 0.02 से 0.07 तक होती है। शुरूआत के शिखर 8 से 17 सेकन्ड के बीच में प्रेक्षित किए गए हैं। प्रस्तुत अध्ययन में तूफान के न होने पर समुद्री सतह की औसत दशा का अच्छी तरह से वर्णन उपलब्ध है।

ABSTRACT. An analysis of sixty one wave records collected using a shipborne wave recorder (SBWR) showed wide variations in spectral shape. Some of the spectra are relatively narrow and peaked, some are broad with small peaks and some have multiple peaks. Half spectral peak bandwidth (BW) gives a more meaningful value to explain the spectral shape compared to spectral width parameter, (E_S from spectra and E_Z from zero-upcrossing method) and spectral peakedness factor (Q_P). The average zero-upcrossing period, T_Z varies from 8.1 to 12.9 sec and compares well with the significant wave period, T_S. T_Z and T_S are found to be better estimates compared to other average periods. During the period of observation (September - October) since calm weather prevailed (storms were absent in the Bay of Bengal) the significant wave heights are centred around 1 m. E_S and E_Z vary between 0.53 and 0.82 and between 0.52 and 0.80 respectively. BW varies from 0.02 to 0.07. The primary peaks are observed between 8 and 17 sec. The present study fairly describes the average condition of the sea surface in the absence of any storm.

1. Introduction

Studies on surface waves in the Bay of Bengal are scarce and available investigations are mainly based on visually observed data. However, since 1976, considerable amount of wave data have been collected during the cruises of the R.V. *Gaveshani* both in the Bay of Bengal and the Arabian Sea using a shipborne wave recorder. The wave recorder consists of 2 identical sensors installed on the two sides of the vessel at a depth of 2 m below the sea surface. Wave records of 15 minutes duration are generally collected at 3 hours interval when the ship remained stationary. For the present study, 61 wave records collected during the 79th cruise of the R.V. *Gaveshani* during 24 September - 4 October 1980 have been analysed. Fig. 1 shows the location of the two stations G-11 and G-21, where wave records were collected. The depths at these stations are 250 and 500 m respectively and hence waves at these locations may be considered as deep water waves unaffected by the bottom topography. Even though the data have been collected at 2 stations, because of proximity of their locations the data have been treated as if they were collected at a single station continuously.

2. Methods of analysis

The wave records have been digitised manually at an interval of 1 sec and later subjected to spectral and zero-upcrossing analyses to obtain the various wave parameters using the computer programmes developed by Varkey and Gopinathan (1980).

(a) *Spectral analysis*

For the present investigation the sea surface over a short-time of 15 minutes is assumed stationary. An idealised sea surface is assumed, *i.e.*, the distribution of sea surface deviations about a mean level is Gaussian

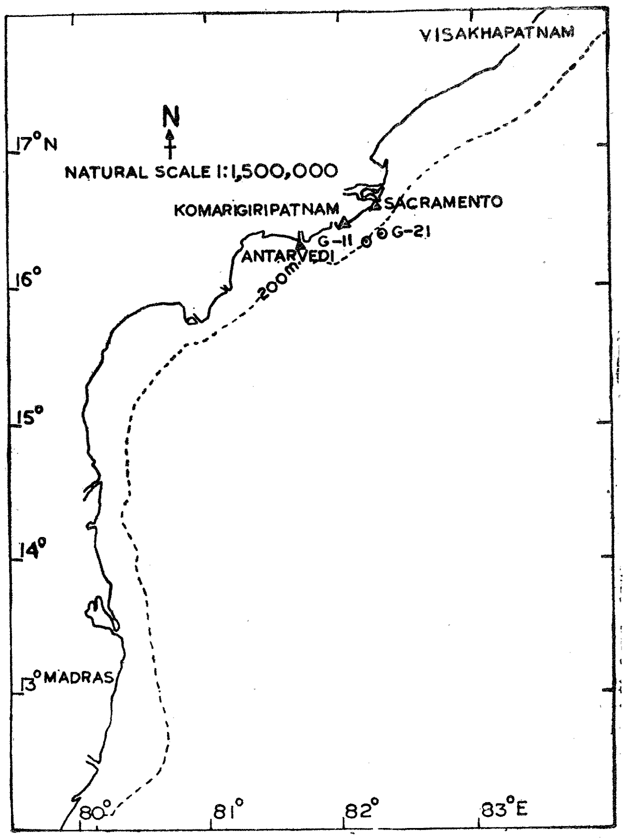

Fig. 1. Station location map

and the energy spectrum is narrow banded. The deviations from the Gaussian distribution is also presented.

The lower limiting component wave period present in the analog records necessarily depends on the depth of installation of the sensors of the wave recorder. A pilot analysis of some records showed that below 3 sec, the energy is negligible for the narrow band spectra and often within the limits of the noise level. The computed spectra have to be compensated for the hydrodynamic attenuation of the component waves. The correction factor is very large for high frequencies as the function increases exponentially towards high frequencies and results in unstable estimates for sea spectra. Hence spectral estimates for frequencies higher than 0.33 Hz (3 sec) were discarded even though their contribution to responses of marine structures may be significant depending on their size (Michel and Nadine 1976). At the upper side, the limiting period was taken at 25 sec. The selected sampling interval ($\triangle t = 1$ sec) does not influence the spectral estimates by aliasing since the frequency, F_D, above which energy densities are negligible is less than or equal to the Nyquist frequency, F_N ($= \frac{1}{2} \triangle t$). To avoid aliasing F_D should be less than or equal to $\frac{1}{2}\triangle t$. In this case $F_D = 0.33 < F_N = 0.50$.

The power spectrum has been computed through autocorrelation. A rectangular lag window is used and Hanning smoothing is employed. Even though the records are of 15 minutes duration, digitisation has been done for approximately 14 minutes only, leaving portions of about 30 sec at the end points of the record to eliminate possible instrumental instabilities. This resulted in about 840 data points and a lag number 60 gives 28 degrees of freedom. This resulted in 0.0166 Hz resolution bandwidth. The noise arising out of digitisation error is distributed uniformly over the complete frequency range. The noise level was determined by finding the mean spectral value for the period range 2 to 2.5 sec. This average noise level (N) was subtracted from the individual smoothed estimates (E_{SP}) for the full frequency range to give the noise-free estimates and then the attenuation correction (C) was applied. The corrected value used for the calculation of spectral moments is:

$$E_C = (E_{SP} - N)\, C^2, \qquad \text{where,}$$

$$C = \{\omega^{-4} \times \exp(2.5\, D\omega^2/g)\} \left[\{(\omega_1^2 - \omega^2)^2 + \alpha_1^2 \omega_1^2 \omega^2\} \times \{(\omega_2^2 - \omega^2)^2 + \alpha_2^2 \omega_2^2 \omega^2\}\right]^{\frac{1}{2}}$$

$\alpha_1 = 1.916$, $\alpha_2 = 1{\cdot}241$, $\omega_1 = 0{\cdot}09498$ rad/sec, $\omega_2 = 0{\cdot}1065$ rad/sec, $\omega = 2\pi/T \approx 2\pi/T_Z$, $g = 9{\cdot}81$ m/sec^2, D=transducer depth in metres (2 m).

From the spectral analysis, wave parameters such as average periods, spectral peak period, estimate of significant wave height, spectral width parameter, half spectral peak bandwidth, etc were derived.

(b) *Zero-upcrossing analysis*

From the same digitised data, the number of maxima in the record was first counted. Then the individual waves are identified by the zero-upcrossings. The exact wave height is calculated from the interpolated crest height and trough depth. After removing waves of less than 3 sec the attenuation correction is applied. Later, waves with steepness greater than 0.167, if any, are filtered out. Parameters like maximum period, height and steepness, spectral width parameter, etc have been computed.

3. Results and discussion

Spectral energy distribution, occurrence of multiple peak spectra, wave period, wave height, etc are discussed briefly. Some of the spectra are found to increase steeply towards high frequencies ($\approx$0.33 Hz). This is due to the large attenuation correction of high frequencies and this part of the spectra should be viewed with caution.

(a) *Spectral energy distribution*

While a comprehensive analysis of the 61 wave records is presented elsewhere (Vethamony, Gopalakrishna and Varkey 1982), a few typical wave spectra are presented in Fig. 2. Peak spectral density varies from 0.16 to 1.81 m^2 s. Energy is concentrated in a narrow range of frequencies in the swell region (Fig. 2e) and is spread over a wide range of frequencies in the sea region (Fig. 2g). The swell region is roughly the frequency range covered by the primary peak and the sea region is the high frequency range. In some spectra energy distribution is found to be nearly the same in both the parts (Fig. 2d).

Fig. 3 gives an overall picture of the growth and decay of the wave energy for the 10 days. Spectral

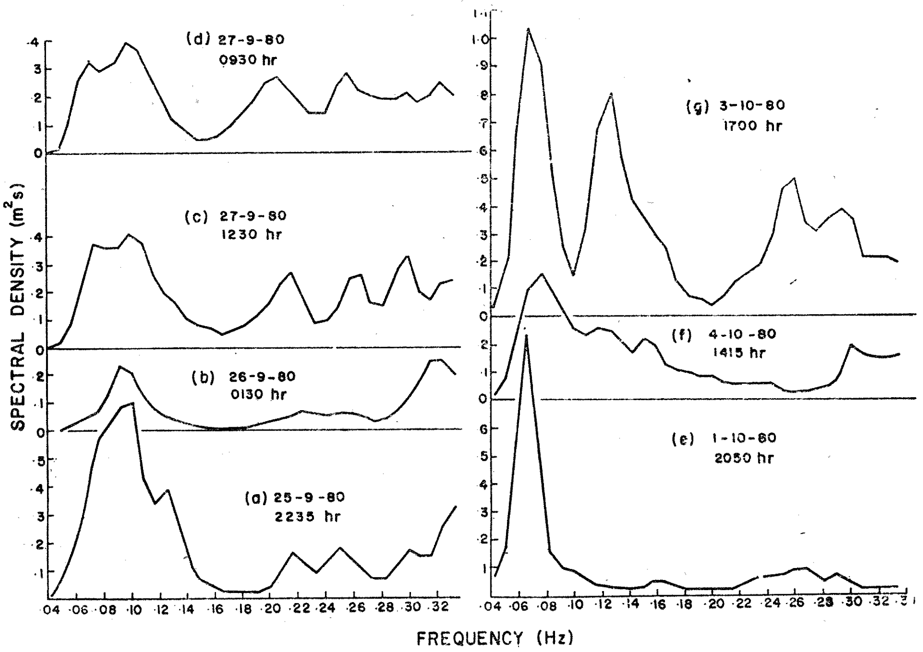

Fig. 2. Typical wave spectra

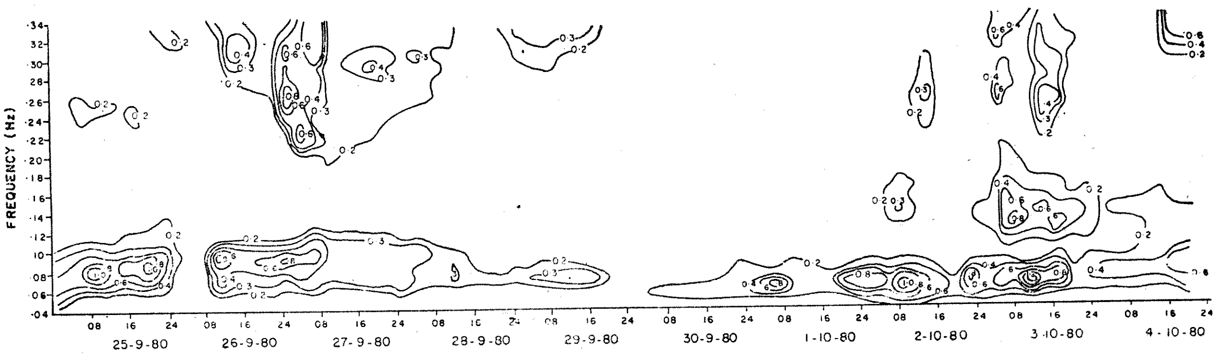

Fig. 3. Evolution of wave spectra

density corresponding to the frequency is plotted against time and contours of spectral density ($\geqslant 0.2$ m^2 s) have been drawn. It is clear from the figure that the highest concentration of energy lies in between 0.05 and 0.12 Hz. The two 'holes' found in the swell region on 26th and on 29/30 September indicate the absence of swells (energy content is very less in the low frequency region). The 'holes' appearing in between the sea region show that sea waves having spectral density 0.2 m^2 s is not present at that time. The contours between 0.10 and 0.17 Hz on 3 October shows the coexistence of swells and sea waves. The decay of swell starts on 27th at 0800 hr and it gradually dies out by 2000 hr on 29 September. Again it developes on 30 September at 0400 hr and gradually increases. On 3 October at 2000 hr it starts dying out.

(b) *Multiple peak spectra*

Most of the wave statistics from the multiple peak spectra differ considerably from that of the single peak spectra. Multiple peak spectra are generally interpreted as an indication of the presence of a local sea and one or more trains of swells from different directions. Two primary peaks in a wave spectrum in the lower frequency side as shown in Fig. 2(g) is due to the superposition of two different wave systems.

83% of the wave spectra have only one primary peak. 60% and 30% of the wave spectra of first and second stations respectively have one or more secondary peaks at the higher frequency side. The primary peaks appear between 8 and 17 sec. The first 15 spectra show a shift of the spectral peak towards higher frequency side, *i.e.*, from 0.076 to 0.100 Hz during the period from 25 to 26 September showing the arrival of distant swells.

(c) *Spectral shape*

The change in wave spectra with time gives an indication of the growth or decay of waves. It has been

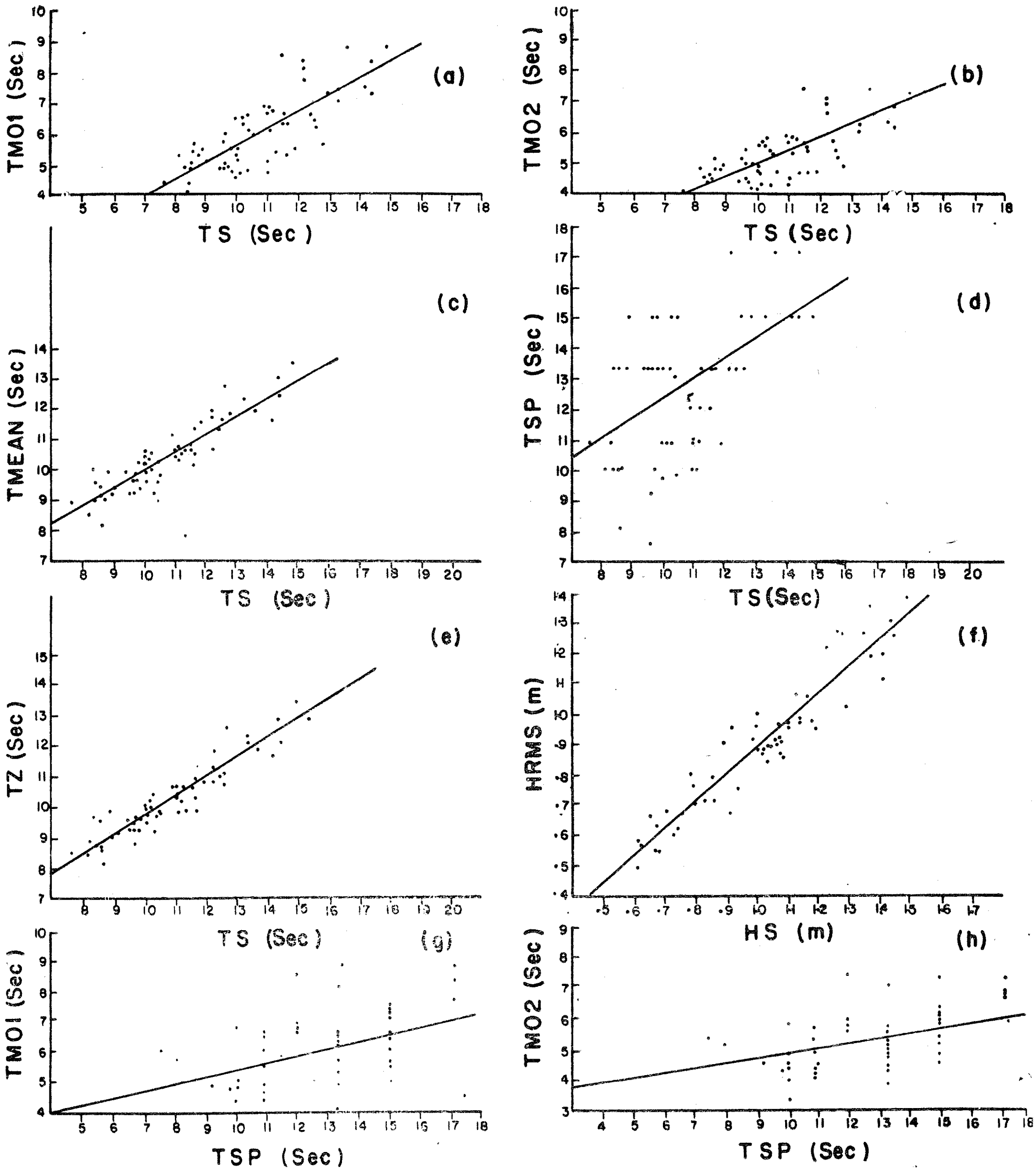

Fig. 4. Scatter diagrams

TABLE 1

Ranges, correlation coefficients and mean ratios of the different periods

Variations of period parameters in sec

T_{MO1}	T_{MO2}	T_{SP}	T_Z	T_S
4.1—11.2	3.9—9.9	8.1—12.9	7.6—15.3	7.5—17.1

Correlation coefficient between

T_{MO1}—T_S	T_{MO2}—T_S	T_{MO1}—T_{SP}	T_{MO2}—T_{SP}	T_{SP}—T_S	T_Z—T_S	T_S—T_{mean}
0.77	0.73	0.44	0.43	0.49	0.92	0.92

Mean ratio of

T_Z/T_{mean}	T_{SP}/T_Z	T_Z/T_S	T_S/T_{mean}
0.96	1.21	0.96	1.04

found that on some occasions there is significant change in the spectral shape of the successive spectra. For example, consider the spectra shown in Figs. 2 (a & b) for the records collected at 2235 hr and at 0130 hr on 25/26 September. The spectral shape parameters Q_P, E and BW for the respective spectra are found to be 1.53, 0.73, 0.03 and 3.01, 0.59, 0.05 respectively. On the contrary, situations in which successive spectra having almost same spectral shape parameters are also observed. For examples, in Figs. 2(c and d), Q_P, E_S and BW for the respective spectra are 1.49, 0.61, 0.06 and 1.55, 0.62, 0.06 respectively. Spectral shape parameters are discussed below in detail.

(*i*) *Spectral width parameter* (*E*)

The nature of wave energy concentration may be studied by the spectral width parameter introduced by Cartwright and Longuet Higgins (1956).

Estimates of spectral widths are :

$E_Z = [1—(T_C/T_Z)^2]^{1/2}$ from zero-upcrossing method and $E_S = [1—(S_{M2}{}^2/S_{M0} \times S_{M4})]^{1/2}$ from spectrum.

Ideally $E = 1$ in the case of 'sea' and $E=0$ in the case of swells. In practice $0<E<1$. The values of E_S and E_Z vary from 0.53 to 0.82 and from 0.52 to 0.80 respectively. For 45% of the spectra, E_S is found to be greater than E_Z. However E_S is found to be low compared to E_Z for spectra having multiple peaks in the higher frequency side. E_S depends on the high frequency cut-off choice while E_Z depends only on T_C and T_Z and as such E_Z is a better estimate for practical engineering purposes.

(*ii*) *Spectral peakedness factor* (Q_P)

Goda (1974) applied spectral peakedness factor (Q_P) as a good measure to describe the sharpness of spectral peak :

$$Q_P = (2/S_{M0}{}^2) \int_{25}^{3} f\, S(f)^2 df.$$

For spectra having single peak in the swell region (*e.g.*, Fig. 2e); Q_P varies from 1.23 to 3.25 and for double-peak spectra (*e.g.*, Fig 2g), it varies from 1.19 to 2.85. Similar results are obtained by Houmb and Due (1978). The differences are due to the fact that double peaked spectra are wider than those with a single peak.

(*iii*) *Half spectral peak bandwidth* (BW)

This spectral shape parameter gives an indication whether the spectrum is narrow banded or broad banded and can be calculated from the difference UFR—LFR, where UFR and LFR are upper and lower frequencies at which the spectrum reduces to half the peak spectral density between 25 and 5 sec. In the present analysis BW varies from 0.02 to 0.07. A smaller value of BW indicates a narrow peaked spectrum (Fig. 2e) and a higher value indicates a relatively flat one (Fig. 2f). BW is found to be a better estimate compared to other spectral shape parameters to explain the nature of the spectra.

(*d*) *Wave period*

The spectral moments S_{M0}, S_{M1}, S_{M2} and S_{M4} are estimated from the integral

$$\int_{25}^{3} f^k\ S\ (f)\ df,$$

where f is the frequency and k is the order of moments ($k=0$, 1, 2, 4). From these the mean wave period T_{M01} ($= S_{M0}/S_{M1}$), estimate of zero-upcrossing, period, T_{M02} [$= (S_{M0}/S_{M2})^{1/2}$] and spectral peak period, T_{SP} have been evaluated. The significant wave period, T_S and average zero-upcrossing period, T_Z have been evaluated from the zero-upcrossing analysis. It is noticed that there is large difference between T_{M02} and T_Z ($T_{M02} < T_Z$). This difference is thought to be due to the high frequency cut-off choice T_Z and T_{mean} agree very well and the ratio T_Z/T_{mean} is found to vary from 0.92 to 1.01. Similar result is reported by Goda (1974). T_Z varies with spectral shape too. For broad spectra having multiple peaks T_Z is low and ranged from 8.4 to 8.7 sec. For narrow band spectra T_Z varies from 10.6 to 10.7 sec, T_Z agrees very well with T_S. For most of the cases, T_S is found to be greater than T_{mean} and the average ratio T_S/T_{mean} is found to be 1.04. T_S is found to be more stable than all other estimates of average periods. Relationships of T_{SP} to T_S, T_{M01} and T_{M02} showed wide scatter (Figs. 4d, 4g and 4h). The spectral shape is found to have significant effect on these relationships. The reason is obvious, *i.e.*, T_{SP} represents only the peak period between 25 and 5 sec. It is observed that in 90% of the spectra, T_{SP} falls within the range of 10-15 sec. The range of variations of the different parameters, the correlation coefficient between the parameters and the mean ratio between the parameters are tabulated in Table 1. Regression lines between the parameters are shown in Fig. 4.

(*e*) *Wave height and steepness*

The estimates of significant wave height, H_{RMS} [$=4\ (S_{MO})^{1/2}$] is derived from the spectral moment. The significant wave height (H_S), *i.e.*, average of the highest one third of the wave heights and average wave height (H_{mean}) have been evaluated from the zero-upcrossing analysis.

H_{RMS} and H_S values vary from 0.49 to 1.39 m and from 0.61 to 1.49 m respectively. Correlation coefficient between H_{RMS} and H_S is found to be 0.95 and the regression equation is :

$H_{RMS}=0.90\times H_S—0.02$ (Fig. 4f)

Unless and otherwise storms occur, generally, in the Bay of Bengal during September-October the wind field would be weak and only low 'seas' exist. 90% of the H_S values are found to be greater than H_{RMS}. H_{mean} varies between 0.4 and 1.1 m. The relationship between H_{mean} and H_S is examined and is found that H_S/H_{mean} varies from 1.35 to 1.55. H_{max} varies between 0.91 and 2.81 m. The steepness of the highest waves ($S_{TH\text{max}}$) varies from 0.003 to 0.167. $S_{TH\text{max}}$ is found to be equal to the maximum steepness $S_{T\text{max}}$ in 33% of the cases and the remaining part shows that $S_{T\text{max}}$ is greater than $S_{TH\text{max}}$, and this indicates that the highest wave need not have the maximum steepness.

(*f*) *Skewness and kurtosis*

Skewness and kurtosis have been computed for the sea surface without considering the attenuation effect on the component waves. Skewness varies from —0.22 to + 0.29. 65% of the values are found to be negative. Goda (1968) obtained results ranging from —0.42 to +0.16 but mostly with positive values. Kurtosis varies from 2.72 to 3.80. If the sea surface is ideally

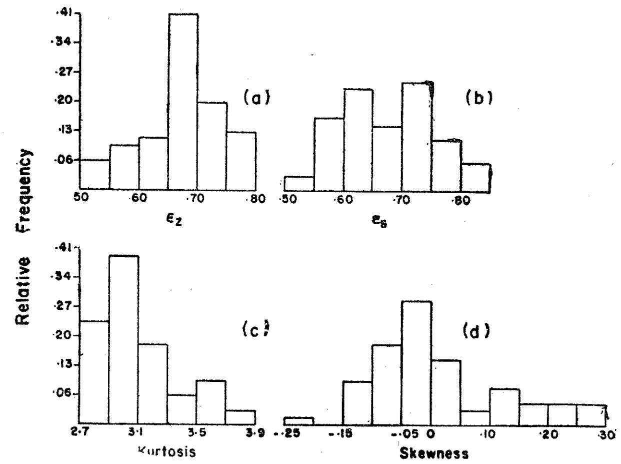

Fig. 5. Frequency distributions of spectral width parameters, skewness and kurtosis

Gaussian, skewness would be zero and kurtosis would be 3 and the deviation from these values indicates the non-normal distribution of sea waves. Kinsman (1960) showed that correction has to be applied to the Gaussian distribution if skewness varies between 0.090 & 0.336. Distribution of kurtosis and skewness are shown in Figs. 5(c) and 5(d). The modal values for the two distributions occur at about 3.0 and 0.0, holding the Gaussian approximation fairly valid.

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