Calculation of crosswind integrated concentration by using different dispersion schemes

KHALED S. M. ESSA, REFAAT A.R. GHOBRIAL, A. N. MINA* and MAMDOUH HIGAZY**

*Department of Mathematics and Theoretical Physics, Nuclear Research Centre, Atomic Energy Authority, Cairo, Egypt *Department of Theoretical Physics, Beni-Suef University **Egyptian Environmental Affairs Agency, Maadi* (*Received 24 February 2010, Modified 16 July 2010*)

e mail : essa_ksm@yahoo.com

सार – स्थानीय प्रकीर्णन के लिए गाऊसिन निदर्श व्यापक रूप से प्रयोग में लाया जाने वाला निदर्श है। प्रतिपवन समेकित सांद्रण को प्राप्त करने के लिए गाऊसिन फार्मुले को पाइंट स्त्रोत (जी. पी. एम.) से निरंतर समझने हेतु समाकलित किया गया है। समाकलित सांद्रण को प्राप्त करने के लिए इरविन, पावर लॉ, ब्रिगस, मानक पद्धति और विभक्त सिग्मा थीटा पद्धति नामक अलग—अलग योजनाओं का प्रयोग किया जा सकता है। इसके साथ-साथ कोपनहेगन, डेनमार्क से प्राप्त किए गए प्रेक्षित सांद्रण के आँकड़ों के अनुरूप पाई जाने वाली सर्वश्रेष्ठ योजना के बारे में जानकारी प्राप्त करने के लिए इस निदर्श में पावर लॉ, पल्म राइस और सांख्यिकीय उपायों में अधोपवन की गति का भी प्रयोग किया गया है।

ABSTRACT. The Gaussian model is the most extensively used model for local dispersion. The Gaussian formula for a continuous release from a point source (GPM) is integrated to get crosswind integrated concentration. Different schemes such as Irwin, power law, Briggs, Standard method, and split sigma theta method can be used to obtain integrated concentration. Also downwind speed in power law, plume rise and Statistical measures are used in the model to know which is the best scheme agrees with the observed concentration data obtained from Copenhagen, Denmark.

Key words ‒ Crosswind integrated concentration, Different schemes, Plume rise, Statistical measures.

1. Introduction

Conventional Gaussian plume models are commonly used for the air quality analysis and regulatory purposes. These Gaussian models are based on the solution of an advection diffusion equation derived by assuming that the wind speed and eddy diffusivity don't have spatial and temporal variations (Hanna *et al*., 1982).

For the practical applications of the solutions obtained, the eddy diffusivities are expressed as functions of downwind distance and wind speed is parameterized as a power law function of the vertical height above the ground (Irwin, 1979).

The commonly used power law approximations for the profiles of wind velocity and vertical diffusivity have been employed to derive analytical solutions (Roa, 1981;

Koch 1989; Chitgopekar *et al*. 1990, Essa *et al*. 2008 and Sharan and Kumar 2009).

In this work, a Gaussian plume model using different dispersion schemes, wind speed in power law, and plume rise are used to get the crosswind integrated concentrations. Statistical technique is used to compare between the observed and all predicted concentrations obtained from different dispersion schemes to know the best predicted model.

2. Description technique

The Gaussian plume model is still the basic and most common model used for dispersion calculations. It was derived by Sutton (1953), Csanady (1973), Smith (1973), and Turner (1970). The following formula represents the

Estimates of the power (p) in urban areas for six stability classes based on information by Irwin (1979b)

concentration of a pollutant releasing from a continuous point source;

$$
C(x, y, z, H) = \frac{Q}{2\pi\sigma_y \sigma_z U} e^{\left(-\frac{y^2}{2\sigma_y^2}\right)} \left\{ e^{\left[-\frac{(z-H)^2}{2\sigma_z^2}\right]} + e^{\left[-\frac{(z+H)^2}{2\sigma_z^2}\right]} \right\}
$$
(2.1)

where,

- C is the concentration of the pollutant at a point with coordinates *x*, *y*, *z* (g/m^3),
- *x* is the downwind distance from the source (m),
- *y* is the lateral distance from the source (m),
- *z* is the vertical distance above the ground (m),
- Q is the emission rate (g/s) ,
- U is the downwind velocity (m/s) ,
- *H* is the effective source height above the ground (m), which is generally presented in the form $H = h_s + \Delta h$

where, h_s is the physical stack height, and Δh is the plume rise.

Finally, σ_y and σ_z are the plume dispersion parameters in the horizontal and vertical directions respectively.

By integrating both sides with respect to *y*, we can get;

$$
C_{y}(x, z, H) = \frac{Q}{2\pi\sigma_{y}\sigma_{z}U} \left\{ e^{\left[\frac{(z-H)^{2}}{2\sigma_{z}^{2}} \right]} + e^{\left[\frac{(z+H)^{2}}{2\sigma_{z}^{2}} \right]} \right\} \int_{-\infty}^{\infty} e^{\left(-\frac{y^{2}}{2\sigma_{y}^{2}} \right)} dy
$$
\n(2.2)

Where C_y is the crosswind integrated concentration and the dispersion parameters σ_y and σ_z can be calculated using different schemes, here calculating them by five different methods namely, Irwin scheme, Power-Law, Briggs' scheme, Standard method, and Split sigma theta method.

We can obtain downwind speed U_{115} at height 115m as follows:

$$
U_{115} = U_{10} \left(\frac{h_s}{10}\right)^p
$$
 (2.3)

Where,

- U_{115} is the wind speed at 115m height.
- U_{10} is the wind speed at 10m height.
- *hs* is the physical stack height (115m).
- *p* is a parameter estimated by Irwin (1979), which is related to stability classes, is given in Table 1.

Also the effective height H is gotten by Briggs (1969),

$$
H = h_s + \Delta h \tag{2.4}
$$

$$
\Delta h = 3 \left(\frac{w}{U_{115}} \right) D \tag{2.5}
$$

Where,

- *H* is the effective source height above the ground (m).
- *hs* is the physical stack height (115m).
- Δh is the plume rise (m).

Run No.	Stability	$U_{10}(m/sec^{-1})$	$U_{115}(m/sec^{-1})$	Downwind distance (m)	
$\mathbf{1}$	Very unstable (A)	2.1	3.029172	1900	
$\mathbf{1}$	Very unstable (A)	2.1	3.029172	3700	
$\sqrt{2}$	Slightly unstable (C)	4.9	7.986117	2100	
$\boldsymbol{2}$	Slightly unstable (C)	4.9	7.986117	4200	
3	Moderately unstable (B)	2.4	3.461911	1900	
3	Moderately unstable (B)	2.4	3.461911	3700	
3	Moderately unstable (B)	2.4	3.461911	5400	
4	Slightly unstable (C)	2.5	4.074549	4000	
5	Slightly unstable (C)	3.1	5.052441	2100	
5	Slightly unstable (C)	3.1	5.052441	4200	
5	Slightly unstable (C)	3.1	5.052441	6100	
6	Slightly unstable (C)	$7.2\,$	11.7347	2000	
6	Slightly unstable (C)	7.2	11.7347	4200	
6	Slightly unstable (C)	$7.2\,$	11.7347	5900	
7	Moderately unstable (B)	4.1	5.914098	2000	
7	Moderately unstable (B)	4.1	5.914098	4100	
7	Moderately unstable (B)	4.1	5.914098	5300	
8	Neutral (D)	4.2	7.734349	1900	
8	Neutral (D)	4.2	7.734349	3600	
8	Neutral (D)	4.2	7.734349	5300	
9	Slightly unstable (C)	5.1	8.312081	2100	
9	Slightly unstable (C)	5.1	8.312081	4200	
9	Slightly unstable (C)	5.1	8.312081	6000	

Values of wind speed at 10 m and 115 m and downwind distance through unstable and neutral stabilities in northern part of Copenhagen

w is the exits velocity (4m/s).

D is the internal stack diameter (1m).

The used data set was observed from the atmospheric diffusion experiments conducted at the northern part of Copenhagen, Denmark, under unstable conditions (Gryning and Lyck, 1984; Gryning *et al*., 1987). The tracer sulfur hexafluoride (SF_6) was released from a tower at a height of 115m without buoyancy. The values of different parameters such as stability, wind speed at 10m (U_{10}) , wind speed at 115m (U_{115}) , and downwind distance during the experiment are represented in Table 2.

3. Dispersion parameters schemes

Since the Gaussian plume model is expressed in terms of the dispersion parameters σ_y and σ_z , the appropriate selection of horizontal and vertical dispersion parameters is much targeted. We select five different methods namely, Irwin, Power-Law, Brigg's, Standard, and Split sigma theta methods, for calculating σ _{*y*} and σ ^{*z*} to choose the most accurate one.

The values of the standard deviation of the wind direction in horizontal and vertical directions for different stability classes

Stability Classes	$\overline{1}$	D			E	
σ_{θ} (deg)	25	200	10	10		2.5
σ_{ϱ} (deg)	10		6.5	D.D	2.5	

3.1. *Irwin method*

Irwin (1983) proposed the standard deviations of the horizontal and vertical crosswind concentration distribution of pollutant σ_y and σ_z respectively, as follows:

$$
\sigma_y(x) = \sigma_v t f_y, \text{ and } (3.1)
$$

$$
\sigma_z(x) = \sigma_w t_z, \tag{3.2}
$$

where,

t is the travel time of the pollutant (sec) and equals to $t = x/U_{115}$

 f_y and f_z are non-dimensional function of travel time and are given by Irwin (1983) as,

$$
f_y = \frac{1}{1 + 0.9\sqrt{\frac{t}{1000}}},\tag{3.3}
$$

$$
f_z = 1
$$
 for unstable condition, and (3.4)

$$
f_z = \frac{1}{1 + 0.9\sqrt{\frac{t}{50}}}, \text{ for stable condition.}
$$
 (3.5)

σ*v* and σ*w* are the standard deviation of the wind speed in the lateral and vertical directions respectively. For small angels they can given as,

$$
\sigma_{\nu} = \sigma_{\theta} . U_{115} , \text{ and } (3.6)
$$

$$
\sigma_w = \sigma_\omega \cdot U_{115},\tag{3.7}
$$

where σ_{θ} and σ_{θ} are the standard deviations of the wind direction in the horizontal and vertical, respectively. Specifications of σ_{θ} and σ_{φ} can be found in Gifford (1976) and Hanna *et al*. (1982). Based on the Pasquill stability classes from A to F, they are given in Table 3.

So the values σ _{*y*} and σ *z* are obtained by the following equations,

$$
\sigma_y(x) = \frac{\sigma_{\theta} x}{1 + 0.9 \sqrt{\frac{x}{1000 U_{115}}}}
$$

for stable and unstable conditions (3.8)

or stable and unstable conditions, (3.8)

$$
\sigma_z(x) = \sigma_\omega x
$$
 for unstable condition, and (3.9)

$$
\sigma_z(x) = \frac{\sigma_o x}{1 + 0.9 \sqrt{\frac{x}{50U_{115}}}}
$$
 for stable condition. (3.10)

The final results of normalized crosswind-integrated concentration C_y/Q (10⁻⁴ sm⁻²), after calculating σ_y and σ_z by using Irwin method, are presented in Table 8.

3.2. *Power-Law method*

Smith (1968) worked out analytical Power-Law formulae for σ _{*y*} and σ _{*z*} to be uses easily than using a graph or a table. He used the Brookhaven National Laboratory (BNL) formulas, which are defined by him using wind direction θ recorded over one hour as follows:

A : fluctuations of θ exceed 90°. (Very Unstable conditions)

Brookhaven National Laboratory Parameters

TABLE 5

Formulas produced by Briggs (1973) for $\sigma_y(x)$ **and** $\sigma_z(x)$ **(102 <** x **< 104 m)**

TABLE 6

Stability classes Very unstable (A) Moderately unstable (B) Slightly unstable (C) Neutral (D) Slightly stable (E) Moderately stable (F) r 250 202 134 78.7 56.6 37 s 102 96.2 72.2 47.5 33.5 22 A 0.927 0.370 0.283 0.707 1.07 1.17 P 0.189 0.162 0.134 0.135 0.137 0.134 q -1.918 -0.101 0.102 0.465 0.624 0.70

Values of the dispersion parameters for the Pasquill stability classes

TABLE 7

Values of wind direction fluctuations in the lateral direction $\sigma_θ$ **for the Pasquill stability classes**

The comparison between observed, at ground in diffusion experiment in northern part of Copenhagen, and different calculated crosswind-integrated concentrations C*y***/Q (10-4 sm-2) obtained from the used dispersion schemes**

- B₁: fluctuations of θ from 40 to 90°. (Moderately Unstable)
- $B₂$: fluctuations of θ from 15 to 40°. (Slightly Unstable)
- C : fluctuations of θ greater than 15° with strip chart showing an unbroken solid core in the trace. (Neutral)
- D : Trace in a line, short-term fluctuations in θ less than 15°. (Moderately Stable).

Fig. 1. The Relation between the downwind distances and the observed and predicted concentrations of SF_6

Fig. 2. Relation between observed and calculated concentrations for different Gaussian models

He summarized the BNL formulas which are based on hourly average measurements of diffusion to about 10 km of a nonbuoyant plume released from a height of 108 m:

$$
\sigma_y = ax^b \tag{3.11}
$$

$$
\sigma_z = cx^d \tag{3.12}
$$

Values of the parameters a, b, c, and d are given in Table 4.

Because of the absence of the Very Unstable condition in the solution of Smith (1968), here we use values of the Moderately Unstable condition parameters to calculate cases of the Very Unstable condition. The final results of crosswind-integrated concentration C*y*/Q $(10^{-4} \text{ sm}^{-2})$, after calculating σ _{*x*} and σ *z* by using Power-Law method, are presented in Table 8.

Comparison between our different models according to standard statistical performance measure

3.3. *Briggs method*

Briggs (1973) used theoretical concepts of the related formulas to get set of formulas can be used in common practices. According to these formulas σ _{*y*} and σ *z* is proportional to *x* at all stability conditions. Also σ _{*y*} and σ*^z* are independent of release height and roughness in these formulas. The values of σ_y and σ_z in urban conditions are given in Table 5.

The final results of crosswind-integrated concentration C_v/Q (10⁻⁴ sm⁻²) after calculating σ_v and σ_z by using Briggs method are presented in Table 8.

3.4. *Standard method*

In this method, σ*^y* and σ*^z* can be analytically expressed, based on (*P-G*) curves, using the following forms:

$$
\sigma_y = \frac{rx}{\left(1 + \frac{x}{a}\right)^p}, \text{ and } \tag{3.13}
$$

$$
\sigma_z = \frac{sx}{\left(1 + \frac{x}{a}\right)^q}
$$
\n(3.14)

Where r, s, a, p, and q are constants depending on the atmospheric stability. Their values are given in Table 6.

The final results of crosswind-integrated concentration C_v/Q (10⁻⁴ sm⁻²) after calculating σ_v and σ_z by using Standard method are presented in Table 8.

3.5. *Split sigma theta method*

Here σ _z is estimated as in the Standard method, while σ*^y* is estimated using the wind direction fluctuations in the lateral direction σ_{θ} in the different stability classes through the following relationship:

$$
\sigma_y = x \left[\sinh\left(\sigma_\theta^2\right) \right]^{1/2} \tag{3.15}
$$

The values of the wind direction fluctuations in the lateral direction σ_{θ} in the different stability classes are given it Table 7.

The final results of crosswind-integrated concentration C_y/Q (10⁻⁴ sm⁻²) after calculating σ_y and σ_z by using Split sigma theta method are presented in Table 8.

4. Comparison between the used methods

In this section, we compare between the final results obtained using the five different schemes. We look for which is the most optimum method to be used. Fig. 1 shows the relation between the observed and calculated crosswind concentrations of the tracer sulfur hexafluoride (SF_6) with downwind distances from continuous source.

In this Fig. 1, you can notice that the observed concentrations line is not nearby one identified line. So each model has some points near the observed results while the others are not. In the Fig. 2, we plot the normalized crosswind concentrations calculated using different Gaussian models *versus* the observed concentrations.

Regarding Fig. 2, we can observe that the Standard method and the Split sigma theta method are almost near Zero line. But it stills difficult to know which is the most accurate among Gaussian, Power, Briggs, and Irwin methods.

4.1. *Statistical method*

Here we seek for knowing which method's results are the nearest to the observed concentrations. So to solve this problem, we have used the following standard statistical performance measures that characterize the agreement between model prediction ($C_p = C_{pred}/Q$) and observations $(C_0 = C_{obs}/Q)$:

Normalized Mean Square Error (NMSE) =
$$
\frac{\overline{(C_p - C_o)^2}}{\overline{(C_p C_o)}}
$$
(4.1)

$$
\text{Fractional Bias (FB)} = \frac{\left(\overline{C_o} - \overline{C_p}\right)}{\left[0.5\left(\overline{C_o} + \overline{C_p}\right)\right]}
$$
(4.2)

Correlation Coefficient (COR)

$$
=\frac{1}{N_m}\sum_{i=1}^{N_m}\left(C_{\mathbf{p}i}-\overline{C_{\mathbf{p}}}\right)\times\frac{\left(C_{\mathbf{o}i}-C_{\mathbf{o}}\right)}{\left(\sigma_{\mathbf{p}}\sigma_{\mathbf{o}}\right)}
$$
(4.3)

Factor of two (FAC2) =
$$
0.5 \le \frac{C_p}{C_o} \le 2.0
$$
 (4.4)

Where σ_p and σ_o are the standard deviations of C_p and C_o respectively. Here the over bars indicate the average over all measurements (N_m) . A perfect model would have the following idealized performance:

$NMSE = FB = 0$ and $COR = FAC2 = 1.0$

From the statistical method, we find that the Gaussian model, Irwin, power law, Briggs methods are factors of 2 with observed data. Regarding to NMSE, the

mentioned methods can be considered as good models except for Irwin which is relatively far. Power-Law method is the best relating to FB, while the Gaussian Model and the Power-Law are the best correlation with observed data. The calculated concentrations using standard and split sigma theta methods are far from the observed concentrations.

5. Conclusions

Gaussian plume models are commonly used for the air quality analysis and regulatory purposes. From the final results, we can conclude that calculated concentrations obtained from Irwin, power law, Briggs as well as Gaussian models are agree with the observed concentrations when the wind speed is greater than 2 m/s. While the calculated concentrations using standard and split sigma theta methods can be acceptable if the wind speed is less than 2 m/s.

References

- Briggs, G. A., 1969, "Plume Rise", U.S. & Atomic Energy Commission, Div. Technical Information, S 1p.
- Briggs, G. A., 1973, "Diffusion estimation for small emissions", ATDL Contrib. 79 (draft), Air Resource Atmospheric Turbulence and Diffusion Laboratory, Oak Ridge.
- Chitgopekar, N. P., Reible, D. D. and Thibodeaux, L. J., 1990, "Modeling Short Range Air Dispersion from Area Source of Non-Buoyant toxics", *J. Air Waste Ass*., **40**, 1121-1128.
- Csanady, G. T., 1973, "Effect of plume rise on ground level pollution", *Atmospheric Environment*, **7**, 1-16.
- Essa, Khaled S. M. and Maha, S. EL-Otaify, 2008, "Atmospheric vertical dispersion in moderate winds with eddy diffusivities as power law functions", *Meteorologische Zeitschrift*, **17**, 1, 013-018.
- Gifford, F. A., Jr., 1976, "Turbulent Diffusion Typing Schemes", A Review, Nucl. Saf., **17**, 68-86.
- Gryning, S. E. and Lyck, E., 1984, "Atmospheric dispersion from elevated sources in an urban area: Comparsion between tracer experiments and model calculations", *J. Climate Appl. Meteor*., **23**, 651-660.
- Gryning, S. E., Holtslag, A. A. M., Irwin, J. S. and Sivertsen, B., 1987, "Applied dispersion modeling based on meteorological scaling parameters", *Atmos. Environ*., **21**, 1, 79-89.
- Hanna, S. R., Briggs, G. A. and Hosker, R. A. Jr., 1982, "Handbook in Atmospheric Diffusion", U.S. Dept. of Energy report COE/ TIC-11223, Washington, D.C.
- Irwin, J. S., 1979, "A Theoretical Variation of the Wind Profile Power Law Exponent as a Function of Surface Roughness and Stability", *Atmospheric Environment*, **13**, 191-194.
- Irwin, J. S., 1983, "Estimating plume dispersion a comparison of several sigma schemes", *J. Appl. Meteorot*., **22**, 92- 114.
- Koch, W., 1989, "A Solution of the Two Dimensional Atmospheric Diffusion Equation with Height-Dependent Diffusion Coefficient Including Ground Level Absorption", *Atmospheric Environment*, **23**, 1729-1732.
- Roa, K. S., 1981, "Analytical solution of a gradient- Transfer Model for Plume Deposition and Sedimentation", 75p, NOAA Technical
- Sharan, Maithili and Kumar, Pramod, 2009, "An analytical model for crosswind integrated concentrations released from a continuous source in a finite atmospheric boundary layer", *Atmospheric Environment*, **43**, 2268-2277.
- Smith, M., 1968, "Recommended Guide for the Prediction of the Dispersion of Airborne Ejluenrs", A.S.M.E.
- Smith, F. B., 1973, "A scheme for estimating the vertical dispersion of a plume from a source near ground level", In Proceedings of the 3 Meeting of an expert panel on air pollution modelling. Brussels: NATO-CCMS Report **14**.
- Sutton, O. G., 1953, "Micrometeorology", McGraw-Hill, 273-295.
- Turner, D. B., 1970, "Workbook of Atmospheric Dispersion Estimates", U.S. Public Health Service, Division of Air Pollution, publication no. **999**, AP 26.