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Modelling water resource system of Punjab — Certain approaches and challenges*

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सार — पंजाब के संदर्भ में सतह पर उपलब्ध तथा भूमिगत जल संसाधनों के संयुक्त उपयोग के लिये एक निदर्श के विकास में कुछ कार्यविधियों तथा चुनौतियों पर इस शोधपत्र में प्रकाश डाला गया है । इसमें (1) निवेशी वियोजना/अपघटन के द्वारा क्षेत्रों के नदी-घाटियों में अपघटन की विधियां, (2) जलभरों के लिये संजाल सिद्धांत पर आधारित बहुमुखी निदर्श तथा अपघटन विधियों के उपयोग की सम्भाव्यता तथा (3) इष्टतमकारी निदर्श प्राप्त करने के लिये उपरोक्त दोनों निदर्शों का समन्वय, सम्मिलित है ।

ABSTRACT. The paper deals with certain approaches and challenges in evolving a model for the conjunctive use of surface and ground water resources in the context of Punjab. This includes (*i*) methods of decomposition of the regions into basins through input decomposition, (*ii*) network theory based versatile models for aquifer and the possibility of using decomposition methods, (*iii*) integration of the above two models to yield the optimisation model.

1. Introduction

The Punjab water-energy system (Fig. 1) when considered along with the aquifer subsystem is complex enough to warrant the import of large systems techniques. Although the earlier study (Minhas *et al.* 1972) makes use of certain statistical decision making techniques, it pertains to the post-Pong context. The work (Rao 1976) though recent, treats the problem as a deterministic one and stops at giving a preliminary treatment based on certain control theoretic ideas. Interestingly, the task of anyone modelling the system as it obtains now is much more complicated, as compared to the earlier workers mainly due to the following considerations :

(*i*) The structural complexity of the system has increased due to the recently created links *e.g.*, Beas-Sutlej Link (BSL) between the rivers.

(*ii*) The necessity to consider the aquifer system along with the surface reservoirs since large scale utilisation of ground water is resorted to (consequent on the installation of over 5 lakhs of irrigational pump sets).

(*iii*) The interaction of the energy system with the Northern Regional Grid and the water system with those of the neighbouring states (primarily due to mandatory releases).

The situation is to some extent depicted by Fig. 1 (Satsangi *et al.* 1977).

2. The proposed method of decomposition into basins

Let the two regions created by the line B in Fig. 3 be called (1) Sutlej basin (with Bhakra as the major reservoir), (2) Ravi-Beas basin (with Pong as the major reservoir). We make the following observations :

(a) Since the motivation behind the creation of the Beas-Sutlej Link (BSL) was to divert water into Dehar to highly increase its power generating capacity (while at the same time not affecting the flexibility in its use otherwise), it is reasonable to consider its optimum operational policy to be : to divert as much water through it as possible, subject of course to the natural constraints. Assumption of such a rule of operation provides us the following equations :

$$y^1(t) = \begin{cases} z^1(t) - z_{\max} + z^3(t) & \text{if } z^1(t) \geq z_{\max} \\ z^3(t) & \text{if } z^1(t) < z_{\max} \end{cases} \quad (1)$$

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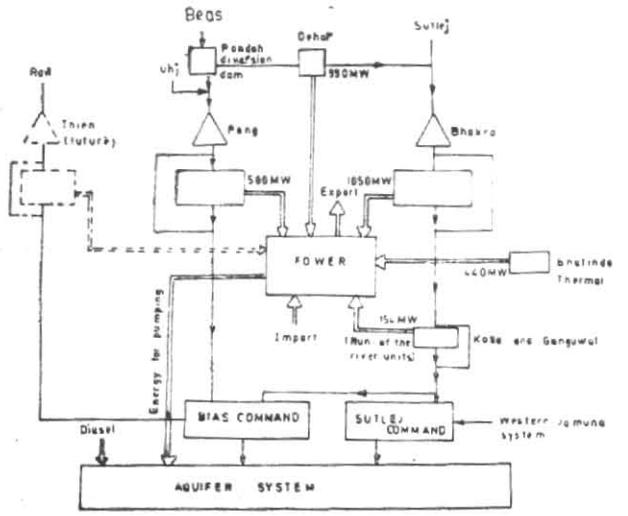

Fig. 1. Punjab water-energy system (Energy flow shown in double lines)

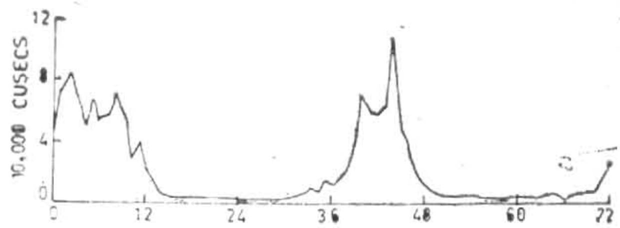

Fig. 2. Average 10-daily inflow into Bhakra (June 1911 to May 1913)

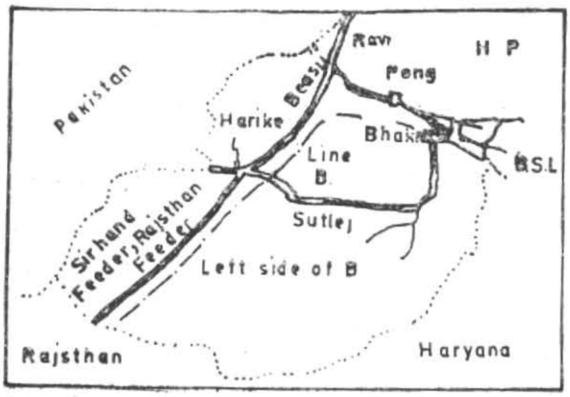

Fig. 3. Proposed physical decomposition (*see* Line B)

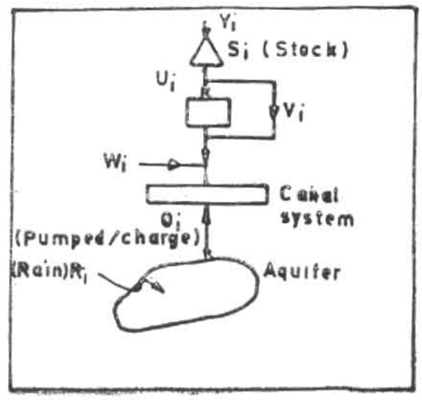

Fig. 4. A typical basin

$$y^2(t) = \begin{cases} z_{\max} + z^2(t) & \text{if } z^1(t) \geqslant z_{\max} \\ z^1(t) + z^2(t) & \text{if } z^2(t) < z_{\max} \end{cases} \quad (2)$$

where the symbols have the meaning as in Figs 1, 4. Further (1) and (2) provide a method of obtaining 'past inflow data' for y^1 and y^2 from the knowledge of that of z^1 and z^2.

(b) Based on flow details and slope conditions, the following observations can be made regarding the interaction among the canal-acquifer sectors :

(*i*) The Sutlej flow into Harike headwork is seen to be insignificant.

(*ii*) The canal system under the Sutlej command is seen to be effective only in the area to the left of B.

(*iii*) The Pong release into Beas is not significant in the irrigational scheme of the Sutlej basin to the left of B.

The validity of (a) and (b) above will imply that each basin can be vizualised as in Fig 4. When the proposed Thien dam comes into existence one might consider the possibility of further decomposing Ravi-Beas basin into two subsystems.

Remark (Modelling inflow as a diffusion process)

Based on the discussion in section 2 we assume the availability of 'past data' for the inflow y^1 and y^2. We propose the diffusion model :

$$dy^i_t = b_i(t, y^i_t)\, dt + \sigma_i(t, y^i_t)\, dB^i_t, \quad i = 1, 2 \quad (3)$$

where B^i_t are independent Brownian motions and b_i and σ_i are respectively the drift and diffusion functions. The details of the identification procedure are explained in (Karunakaran *et al.* 1978) wherein the method has been illustrated by real life data pertaining to the Bhakra inflow.

3. Models for aquifer system

3.1. *Scope of the model*

The formulae which are readily available in hydrology texts are suitable for the computation of water table profile when pumping or charging is carried out at a point. However, from the point of view of water resources modelling for conjunctive water use, the aquifer model has to reflect the dynamics of the water table so that this information can help the scheduling of the operation of large dams like Bhakra and Pong.

Unfortunately the aquifer modelling of Punjab poses certain additional problems also.

For example :

(*i*) Certain areas of Punjab are affected by water-logging and it would be expected of the model to couple the pumping operation in this region with the water requirements of those in the neighbourhood.

(*ii*) There are many districts where the problem of salinity exists. Thus the charging-discharging operations have to take this fact also into consideration.

Our interest will be to evolve models having the following characteristics :

(a) Simple in terms of programming efforts.

(b) Dynamic, with state variables reflecting level of water logging and salinity.

(c) Requires only simple hydrologic parameters.

(d) Amenable to large systems techniques.

(e) Can permit nonlinear dynamic models, particularly since water logging, and salinity factor will demand that.

3.2. *Motivation for network analogy models*

The popularly known finite element method is at the moment not useful for nonlinear dynamic models. However, while building models for aquifer with due attention to salinity and water logging it might be unavoidable to have nonlinear models. This type of more sophisticated nonlinear models might be needed by certain segments of aquifer only, the other segments permitting simpler models. Thus it would be an advantage if the method could permit components of various types. Finally, it would be essential, in the context of large and complex situations, that the modelling effort could be assisted by the computer and the computer oriented large system techniques be also available. The network theory based model suggested below possesses all the above characteristics.

3.3. *Basis for the method*

It is well-known that network analogs can be used for studies in hydrology. This has been possible because, the continuum model of the aquifer used in hydrology, when discretised, will be in terms of variables and parameters and auxilliary conservation equations which can be put in one-one correspondence with variables, parameters and balance equations in electrical network type of systems.

For example, we have (in the case of laminar flow) Darcy equation :

$$q = k.(h_1 - h_2)/L \tag{4}$$

where, $h_1 - h_2$ = the drop in piezometric head between two points, say , from A to B.

q = Darcy velocity (from A to B)

k = hydraulic conductivity

L = distance from A to B.

One has the corresponding equation in electrical network theory :

$$i = gv \tag{5}$$

where, v = voltage drop (say) from A to B

i = current flow from A to B

g = electrical conductivity.

Storage character of soil is responsible for the dynamic behaviour of aquifer. We have,

$$f\, dh/dt = q \tag{6}$$

where, f = fillable porosity

= amount of water that unconfined aquifer can store per unit rise in the water table and per unit area.

h, q as in Eqn. (4)

One might make a pleasant analogy with the equation defining capacitance :

$$C \quad dv/dt = i \tag{7}$$

where, v and i are as in Eqn. (5) and C is defined easily as in Karunakaran (1979).

Let us now briefly review the general theory applying to lumped physical systems (like electrical network) :

(*i*) The variables entering into the description of the model fall into two classes :

(a) across-type variables $\{X_i\}$, (*e.g.*, voltage piezometric head)

(b) through-type variables $\{Y_i\}$, (*e.g.*, current, Darcy velocity)

(*ii*) The X-variables satisfy the 'closed path law' :

"At any time, the sum of the across variable values around any closed path is zero".

Similarly the Y-variables satisfy the following 'junction law' :

"At any time, the sum of the through variables considered at any junction equals zero".

While these laws in electrical network theory are known as Kirchhoff's laws, the corresponding laws in hydrology also exist.

(*iii*) The functions relating the across variables to the through variables are known as constitutive relations (*e.g.*, Eqns. (1)-(4) above).

The equations in (*ii*) and (*iii*) together enable a solution of the system when a solution exists. The details could be found in Koenig *et al.* (1967). Certain worked examples could be found in Karunakaran (1979).

3.4. *Large systems techniques*

With the use of network-type models as indicated above we have the advantage of powerful large systems techniques available in network theory. While details could be found in Koenig *et al.* (1967), we give below a brief sketch of the mathematical idea behind the method :

(*i*) Let S be the physical system (for example the aquifer system) whose modelling is based on e pairs of across and through variables, the measurement scheme being represented by a oriented graph G. Let this graph have v vertices and e edges. A direct method of solving for the response of the system will be to consider the $v-p$ independent junction constraints (where p= number of components in G), $e-v+p$ independent closed path constraints and e equations relating the across variables with the through variables.

(*ii*) Instead of solving the model in one stroke we can use the following decomposition approach :

Step 1 — Decompose the graph G into l subgraphs $\{G_i\}$ in such a way that there is no 'coupling' among the variables of G_i with those of G_j $(i \neq j)$ other than through the junction and loop constraints. For the sake of simplicity in description we can assume that $l=2$.

Let, then , the subgraph G be torn into two subgraphs G_1 and G_2, to correspond to the subsystems S_1 and S_2.

Let G_1 and G_2 have m vertices in common. Let $\bar{G}_1$ be the graph obtained by superimposing a tree T_1 of $m-1$ edges at the vertices of G_1 which are in common with G_2. Similarly let T_2 be superimposed over G_2 to obtain $\bar{G}_2$. Due to these added $(m-1)$ edges there will be $2(m-1)$ variables entering into the description of the new system $\bar{S}_1$ which is S_1 along with the new variables designed by T_1 considered as unspecified drivers. This will take care of the influence of the system S_2 on S_1. Similarly we get $\bar{S}_2$.

Step 2 — Manipulate the model of $\bar{S}_1$ so that we get a relation between the across and through variables associated with T_1. This relation will express the model of S_1 as viewed by S_2. Let this model be M_1. Similarly obtain M_2.

Step 3 — Consider M_1 and M_2 together and solve the relevant equations. This will yield the boundary conditions imposed by S_2 on S_1 and *vice versa* so that one can go back to the subsystems S_1 and S_2 and achieve a complete solution.

Remark — If G is decomposed into many subgraphs, the recombination can be carried on in many stages resulting in a multistage solution.

3.5. *Extended network analogy models*

For an aquifer model needed for the Punjab system we should also have ideas of salinity and water logging incorporated. This can be incorporated by introducing further variables like salinity level and water logging level and relating (nonlinearly) to the variables water table. These details will be developed in a further report.

4. Optimisation models

The outcome of the aquifer, model discussed in section 3 will be a model of the type :

$$\frac{dh(t)}{dt} = A\,h(t) + B\,u\,(t) \qquad (8)$$

where, $h\,(t)$ can be a vector containing the various aquifer sybsystems and $u(t)$ is the vector of inputs like pumping and recharge, rainfall, evapotranspiration etc.

An aggregate model obtained from Eqn. (8) can provide a scalar version of the same form suitable (and sufficient) for use in the optimisation model. The state variable vector at this stage contains reservoir inflow, reservoir level and the aggregate $h(t)$ from Eqn. (5). The use of dynamic programming based on such a model is indicated in Karunakaran *et al.* (1978).

5. Conclusions

The modelling philosophy proposed in the previous paper (Karunakaran *et al.* 1978) is carried a bit further by proposing an easy method of modelling aquifer. This method is suitable for large systems techniques and also appears to accommodate variables related to water logging and salinity, Numerical implementation related to the aquifer model is being looked into. For situations where network type of models might not suit, an algebraic technique to build state models using space-time water table sequence seems to be feasible. This is inspired by developments in algebraic system theory (Fornasini and Marchesini 1976) and will be presented in a subsequent report under preparation.

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