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Strange attractor dimension of surface radio refractivity over Indian stations

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सार — निर्धारणात्मक अव्यवस्था सिद्धान्त के उपयोग द्वारा भारत के आठ प्रतिनिधि केन्द्रों की रेडियो अपवर्तकता का अध्ययन किया गया है और स्ट्रेंज अट्रेक्टर विस्तार के सम्बन्ध में उस पर विचार किया गया है। इस अध्ययन से यह पता चला है कि अन्य किसी वायुमण्डलीय परिघटना की भांति भारतीय क्षेत्र के लिए रेडियो अपवर्तकता की पूर्व सूचना देने के लिए 6 से 9 प्राचलों की आवश्यकता होती है। इससे यह भी पता चला है कि अंतर्देशी और तटीय केन्द्रों को उनकी जलवायुविक लक्षणों अर्थात भिन्नात्मक विस्तार के आधार पर समूहबद्ध नहीं किया जा सकता।

ABSTRACT. Using the theory of deterministic chaos, the radio refractivity has been studied for eight representative stations in India and discussed in relation to strange attractor dimension. It was found that six to nine parameters are needed to predict radio refractivity over the Indian region similar to that reported for other atmospheric phenomena. Also, inland and coastal stations cannot be grouped in relation to their climatic features *vis-a-vis* fractal dimension.

Key words — Radio refractivity, Deterministic chaos, Strange attractor, Fractal dimension.

1. Introduction

Radio refractive index (N_s) at the ground surface has been extensively used to predict microwave signal strengths in different parts of the world (Bean and Dutton 1966). The main advantage in favour of the use of N_s lies in the fact that very large number of surface observations in the country are routinely taken in India Meteorological Department. Recent studies based on dynamical approach to model atmospheric phenomena have shown that the meteorological observations like the atmospheric pressure and temperature form the example of deterministic chaos and their modelling could be based on 7 to 8 parameters. Satyan (1988) also found that the strange attractor fractal dimension based on seasonal rainfall data over India roughly of the same order as other meteorological parameters. A question, arises whether the radio refractive index which is based on three meteorological variables namely atmospheric pressure, temperature and water vapour can be modelled and if so, the number of parameters needed using the principles of chaos physics.

Since large variations in radio refractive index occur over the Indian region from inland to coastal stations (Johari and Srivastava 1970), we have studied the strange attractor dimension for eight representative stations in the country.

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2. Surface refractivity and data used

Radio refractivity (N) was calculated based on daily values using the formula given below :

$$N = \frac{77.6\ p}{T}\left(1 + \frac{7.71\ m}{T}\right) \qquad (1)$$

where,

T — Dry bulb temperature (°K),

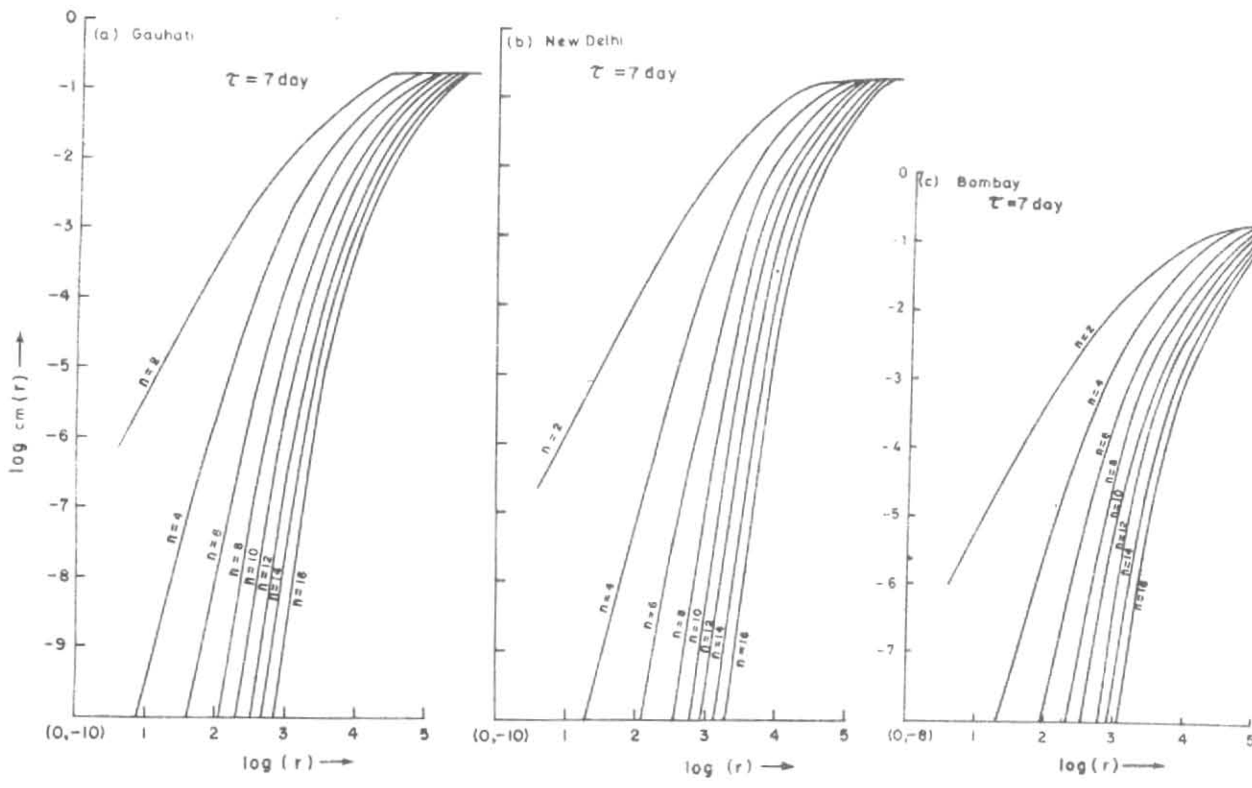
m — Mixing ratio (gm/kg),

p — Station level pressure (hPa).

Daily mixing ratio for each station was computed with the help of formula given by Pruppacher and Klett (1980).

$$\text{Mixing ratio, } m = \frac{0.622\ e_s}{p - e_s} \qquad (2)$$

where, e_s — Saturation vapour pressure (hPa).



Figs. 1 (a-c). log C_m (*r*) *versus* log *r* for : (a) Gauhati, (b) New Delhi, and (c) Bombay

$$e_s = \sum_{n=0}^{6} a_n T_d^{\,n}$$

where, T_d — Dew point temperature (°C) and a_0 to a_6 are constants as given below:

a_0 = 6.107799961, a_1 = 4.436518521×10^{-1},

a_2 = 1.428945805×10^{-2}, a_3 = 2.650648471×10^{-4},

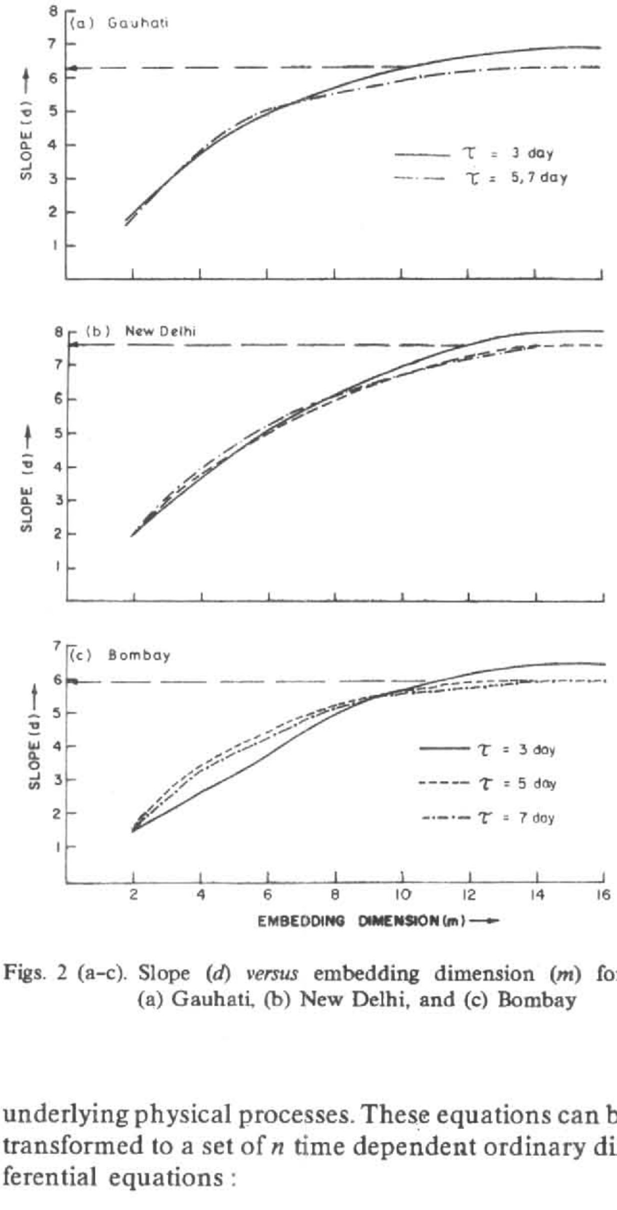
a_4 = 3.031240396×10^{-6}, a_5 = 2.034080948×10^{-8},

a_6 = $6.136820929\times10^{-11}$

3. Computation of fractal dimension of strange attractor

A dynamical system can be described by trajectories in the state space which consist of the variables for describing evolution of the system. The submanifold which attracts the trajectories and has integer dimensions is called the non-chaotic attractor. However, there are other dynamical systems where trajectories remain on an attracting submanifold that is not topological. Such manifold is a strange attractor which is associated with a new geometrical object called a fractal set and has a non-integer dimension (Mandelbrot 1977). Thus following the equations that describe the system, the state of the system after some time can be anything even though the initial conditions were close to each other. This imposes limits on prediction and even if the system is described by equations, the system shows randomness. Such systems are chaotic dynamical systems and their attractors are called strange or chaotic attractors. The dimension D of an attractor (chaotic or non-chaotic) indicates the minimum number of independent variables present in the system (Moon 1987). Thus, if $D=I+p$, where, I is an integer and $0 \leqslant p < 1$, then minimum number of independent variables of the system is $I+1$. Thus determination of dimension of the attractor helps us in modelling the system.

We may consider the dynamics of a system, simulated by partial differential equations describing



Figs. 2 (a-c). Slope (*d*) *versus* embedding dimension (*m*) for: (a) Gauhati, (b) New Delhi, and (c) Bombay

underlying physical processes. These equations can be transformed to a set of n time dependent ordinary differential equations :

$$V_j' = f_j\ (v_1, v_2, \ldots\ldots, v_n);\quad j=1, 2, \ldots\ldots, n \tag{3}$$

where, prime denotes differentiation with respect to time t. The time evolution of the system from an initial condition can be described by trajectories in n-dimensional state space with coordinates $v_1, v_2, \ldots\ldots v_n$, which are n different variables. The system (3) can be reduced to a single differential equation of one of the variables $v_j(t)$, say $v(t)$ if all others are eliminated by differentiation. This gives an nth order non-linear differential equation as :

$$v^{(n)} = f\ (v, v', \ldots\ldots, v^{(n-1)}) \tag{4}$$

so we replace the state space with $v, v', \ldots\ldots, v^{(n-1)}$ without any loss of information about the dynamics of the system.

According to the theorem of Takens (1981) D-dimensional manifolds can be embedded into $m=2D+1$ dimensional space. Thus, for deriving the dimension of an attractor from a single state variable, it is sufficient to embed them into a m-dimensional space spanned by v and its $(m-1)$ derivatives, *i.e.*, $v, v', v'', \ldots\ldots, v^{(m-1)}$. Thus, it is not necessary to know the original state space and its dimension n as long as m is chosen large enough. Ruelle (1981) suggested that instead of continuous variables $v(t)$ and its derivatives, a discrete time series $v(t)$ and its shifts $(m-1)$ time lags by a delay parameter τ can be considered.

We, therefore, begin computation with a time series of a dependent or independent variable v of the system. The delay parameter is chosen either 3 days, 5 days or 7 days and then the convergence is checked. Let the series be $v_1, v_2, \ldots\ldots, v_N$ where, N is total number of data and in our case $N=1827$. These values are embedded to construct points X_i in a m-dimensional embedding space :

$$X_i = (v_1, v_{1+1}, \ldots\ldots, v_{1+m-1}) \tag{5}$$

$i=1, 2, \ldots\ldots, N-m+1$ ($=k$, say). For embedding dimension m, the correlation integral $C_m(r)$ as a function of correlation length r is given by (Grassberger and Procaccia 1983).

$$C_m(r) = \frac{1}{N^2} \sum_{\substack{i,j=1 \\ i\neq j}}^{N} H\ (r - |X_i - X_j|) \tag{6}$$

where, $H(x)$ is a Heaviside function

$$H(x) = 1 \quad \text{for } x > 0,$$

$$= 0 \quad \text{for } x < 0.$$

In above equation, $|X_i - X_j|$ is the distance between X_i and X_j and is obtained by conventional Euclidean measure of distance, *i.e.*, by square root of the sum of the squares of components. The correlation of fractal dimension is defined by :

$$D = \lim_{r\to 0} \lim_{m\to\infty} \frac{d\ [\ln C_m(r)]}{d\ [\ln r]} \tag{7}$$

Thus, to obtain correlation dimension of attractors of radio refractivity we have obtained $C_m(r)$ for different values of r using the Eqn. (6). ln $C_m(r)$ is plotted against ln r in Figs. 1 (a–c) for $\tau = 7$ day. To obtain D using Eqn. (7) we require the slope d of the straight line passing through the points corresponding to each embedding dimension m. However, $C_m(r)$ saturates at large values of r due to finite size of the attractor and at small values of r due to finite N.

The computations were made for 2 to 18 embedded dimensions. The results are shown in Figs. 1 (a–c) for three representative stations namely, Gauhati, New Delhi and Bombay corresponding to three distinctive radio climatic regimes (Srivastava and Pathak 1969). As given in Table 1 and Figs. 2 (a–c) there is saturation for $\tau = 3$, 5 and 7 days but only representative curves for $\tau = 7$ are given in the Figs. 1 (a–c).

4. Results and discussion

The fractal dimensions of the strange attractor for Gauhati, Nagpur, New Delhi, Bombay, Calcutta, Madras, Port Blair and Visakhapatnam are given in Table 1.

It may be noted that the fractal dimensions in the Indian region lie in the range of 5.9 to 8.3 implying that 6 to 9 parameters are needed for modelling the radio refractivity. This is roughly of the same order as reported by Fraedrich (1986) for atmospheric pressure.

Satyan (1988) found that six parameters are needed to predict seasonal rainfall over India. Thus the signatures of deterministic chaos are brought out in radio refractivity as well which is based on three meteorological parameters namely, atmospheric pressure, temperature and humidity.

Among the inland stations, only 7 parameters are needed for modelling at Nagpur and Gauhati while at New Delhi, eight parameters are needed. On the other hand, at a coastal station like Bombay where anomalous propagation extending up to coast of Arabian Sea have been reported since World War II, only six parameters are needed for modelling. This could be attributed to the different physical process for the ducting conditions over the inland stations as compared to that over sea (Srivastava and Pant 1979). At Vishakhapatnam, which has the highest value of surface refractivity (Srivastava and Pant 1968) among coastal stations during the southwest monsoon, nine parameters are needed to model the refractivity system. Such a large difference in the modelling parameters between Bombay and Vishakhapatnam is yet to be explained. It may, therefore, be surmised that the inland or coastal stations in India cannot be classified through fractal dimensions *vis-a-vis* climatic features. Nevertheless, the present study brings out the non-linearity in radio refractive index and gives an idea about the number of parameters needed to model radio refractivity.

TABLE 1

Fractual dimension of strange attractor over India

Station	Fractal dimension		
	$\tau = 3$	$\tau = 5$	$\tau = 7$
(a) Inland stations			
Gauhati	[illegible]	[illegible]	[illegible]
Nagpur	[illegible]	[illegible]	[illegible]
New Delhi	[illegible]	[illegible]	[illegible]
(b) Coastal stations			
Bombay	[illegible]	[illegible]	5.9
Calcutta	[illegible]	[illegible]	[illegible]
Madras	[illegible]	[illegible]	[illegible]
Port Blair	[illegible]	[illegible]	[illegible]
Visakhapatnam	[illegible]	[illegible]	[illegible]

5. Conclusions

The above study has brought out the following results :

(*i*) Six to nine parameters are needed to model radio refractivity over the Indian region based on strange attractor dimension. This is of the same order as reported for other atmospheric parameters like atmospheric pressure and rainfall.

(*ii*) The inland and coastal stations cannot be grouped in relation to their climatic features *vis-a-vis* fractal dimension.

APPENDIX

Concept of deterministic chaos in the study of dynamical systems in recent years has opened up a new method of studying aperiodicity. It is now possible to understand and characterise irregular temporal behaviour in a quantitatively better way using the above method. The study of complex systems for example, turbulent flows, the weather, biological processes, seismic activity etc. suggest that modelling these processes by purely deterministic equations is impossible. The unpredictability in such systems is due to non-linear interactions between the system and some unknown factors modelled as stochastic random noise.

Evolution of dynamical systems can be represented by trajectories in the state space from some initial condition. For a periodic system that develop deterministically, all trajectories initiated from different initial conditions stay on low dimensional smooth topological manifold, called attractor. These attractors are characterised by an integer dimension, equal to the topological dimension of the submanifold. An important property of these attractors is that, trajectories converging on them do not diverge. This guarantees long term predictability of the system. It has been found for many dynamical systems that the trajectories stay on an attracting submanifold which is not topological. These submanifolds are called 'fractal' sets and are characterised by a dimension which is not an integer. The corresponding attractors are called 'strange' attractors. An important property of these attractors is the divergence of initially nearby trajectories. Thus, long term predictability for these systems is not guaranteed. The determination of the dimension of an attractor sets a number of constraints that should be satisfied by a model used to predict the evolution of a system. Higher the value of the fractal dimension more complex is the system. The fractal dimension also gives the number of independent parameters required for modelling the system.

If the mathematical formulation of a system is not available, the state space can be replaced by the so-called phase space. The phase space may be produced using a single record of observable variable $x(t)$ from that system. The physics behind such an approach is that a single record from a dynamical system is the outcome of all interacting variables and thus information about the dynamics of that system should, in principle, be included in any observable variable.

It is assumed that variables present in the evolution of the system in question satisfy a set of n first-order differential equations :

$$\dot{x}_1 = f_1 (x_1, x_2, \ldots\ldots, x_n)$$

$$\dot{x}_2 = f_2 (x_1, x_2, \ldots\ldots, x_n)$$

$$\vdots$$

$$\dot{x}_n = f_n (x_1, x_2, \ldots\ldots, x_n) \qquad (1)$$

where, one dot indicates the first derivatives with respect to time. In such a case the co-ordinates of the state space are $(x_1, x_2, \ldots\ldots, x_n)$. The above system of differential equations can be reduced to one highly non-linear differential equation of nth order, *i.e.*,

$$x_1^{(n)} = f(x_1, \dot{x}_1, \ldots\ldots, x_1^{(n-1)}) \qquad (2)$$

Ruelle (1981) suggested that instead of a continuous variable $x(t)$ and its derivatives. It will be easier to work with $x_1(t)$, and the set of variables obtained from it, by shifting its values by a fixed delay parameter τ we consider from now the phase space defined by the variables.

$$x_1(t), x_1(t+\tau), \ldots\ldots x_1[t+(n-1)\tau]$$

For a typical choice of τ these variables are expected to be linearly independent.

The method of calculating fractal dimension has been included in the text.

The existence of strange attractor shows that the main feature of long term evolution of radio refractive index may be viewed as the manifestation of a deterministic dynamics involving a limited number of key variables. The fact that the attractor has a fractal dimensionality provides a natural explanation of the intrinsic variability of system, despite its deterministic character. It suggests that the actual behaviour of the surface radio refractivity is highly aperiodic. The next integer above the fractal saturation dimension provides the minimum number of independent variables necessary to model the dynamics. The saturation is achieved at embedding dimension < 16 is a good estimate of the upper bound for the number of variables sufficient to model the dynamics of the attractor.

Attractor dimension can be taken as a measure of the turbulence in the refractivity parameter because more turbulence in the refractivity parameter will make the system more complex, hence the value of the fractal dimension become higher. The results can also be utilized to model radio refractive index because fractal dimension indicates the minimum number of parameters involved for modelling.

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