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Phase velocity to distance conversion in machine location of array recorded teleseismic events

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ABSTRACT. In on-line deployment of small computer systems with limited machine capability, the object of locating seismic sources atleast in the teleseismic distance range essentially requires, besides azimuth, accurate deduction of epicentral distance from apparent phase velocity in minimum operational steps. It has been shown that a seventh order polynomial in positive powers of apparent *P*-velocity yields reasonably precise distance estimates. The merits of this optimum relation have also been discussed in the light of its application to the Gauribidanur Array real-time processing system.

1. Introduction

The phase velocity due to a plane wave traversing a group of detectors placed in one horizontal plane, *viz.*, an array of seismometers, varies non-linearly with epicentral distance. The velocity tends to increase, generally, with increasing distance owing to decreasing angle of emergence at the surface. For example, the phase velocity of *P* wave due a nearly surface-focus source increases from about 10 km/sec at 20° to about 24½ km/sec at 100° (Herrin 1968).

In off-line analysis, the epicentral distance corresponding to a known value of phase velocity is obtained from the digitized distance-velocity curve with reasonable precision. However, in on-line applications using systems with limited machine capability, *e.g.*, the TDC 12 real-time processing system at Gauribidanur Array (Ram Datt and Dilip Kumar 1973), it is desirable to express the distance-velocity function as a polynomial which can be easily incorporated in the form of a library routine to reproduce the distances as best as possible. Further, to economise on machine time, an optimum polynomial is sought for wherein the low order of the function is compromised with a satisfactory limit of conversion accuracy throughout the range of distance. In the present case, better than ½° accuracy is considered quite satisfactory for our application for distances ranging between 20° and 100°.

An attempt has been made to qualitatively assess the implications of fitting polynomials both in positive and negative powers of phase velocity beginning with a least-squares line upto 12th order expressions. On the basis of merits of the fit, it has been found that the 7th order polynomial in positive powers of velocity represents optimum conditions beyond which order no significant reduction in error (departure from true values) in the deduced distance is achieved. Under this conversion scheme, the coefficient system converges fast so that the source location procedure becomes relatively more reliable and economical in terms of computer time.

2. Phase velocity across the array

The basic parameter in the source location is the pair of time lags along the lines of equi-spaced (2.5 km) seismometers of the L-shaped medium-aperture (about 25 km × 25 km) Gauribidanur Array (GBA). Obviously, the closer to the true values the computed lag pair is, the higher is the accuracy of epicentral location for teleseismic events. A lag is governed by the component, along the arm, of the velocity of propagation in the plane of the array as well as on the interelement spacing *D*. The horizontal velocity, different from the actual velocity with which the wave travels in the earth's interior, is the apparent phase velocity V_a we are referring to.

An expression for V_a in terms of the two lags T_R and T_B is given by (Arora 1967, 1971) :

$$V_a = V_R \ V_B \ (V_R^2 + V_B^2)^{-1/2} = D \ (T_R^2 + T_B^2)^{-1/2} \qquad (1)$$

where V_R and V_B are the component velocities along the *R*- and the *B*-arm respectively. From the point of view of velocity and azimuthal discrimination of signals (Birtill and Whiteway 1965) the reliability factor tends to diminish as the incident wave approaches to coincide with any one of the arms, a zero lag being associated with infinite apparent speed.

In the case of digitized seismic data, the expression for V_a reduces to (Ram Datt *et al.* 1969) :

TABLE 1

Distance-velocity function based on Herrin's (1968) surface-focus *P*-travel-times

$\triangle$	V_a	$\triangle$	V_a	$\triangle$	V_a	$\triangle$	V_a
20·0	9·8494	20·5	10·0625	21·0	10·2849	21·5	10·5113
22·0	10·7312	22·5	10·9431	23·0	11·1520	23·5	11·3565
24·0	11·5518	24·5	11·7335	25·0	11·8953	25·5	12·0347
26·0	12·1538	26·5	12·2532	27·0	12·3339	27·5	12·3958
28·0	12·4367	28·5	12·4602	29·0	12·4743	29·5	12·4875
30·0	12·5087	30·5	12·5402	31·0	12·5752	31·5	12·6113
32·0	12·6460	32·5	12·6792	33·0	12·7148	33·5	12·7535
34·0	12·7938	34·5	12·8358	35·0	12·8787	35·5	12·9227
36·0	12·9690	36·5	13·0179	37·0	13·0693	37·5	13·1230
38·0	13·1776	38·5	13·2320	39·0	13·2861	39·5	13·3396
40·0	13·3931	40·5	13·4472	41·0	13·5021	41·5	13·5582
42·0	13·6148	42·5	13·6722	43·0	13·7310	43·5	13·7908
44·0	13·8519	44·5	13·9140	45·0	13·9768	45·5	14·0405
46·0	14·1060	46·5	14·1736	47·0	14·2434	47·5	14·3149
48·0	14·3867	48·5	14·4584	49·0	14·5294	49·5	14·5996
50·0	14·6695	50·5	14·7395	51·0	14·8094	51·5	14·8794
52·0	14·9498	52·5	15·0207	53·0	15·0926	53·5	15·1659
54·0	15·2410	54·5	15·3167	55·0	15·3929	55·5	15·4697
56·0	15·5481	56·5	15·6283	57·0	15·7106	57·5	15·7941
58·0	15·8780	58·5	15·9612	59·0	16·0438	59·5	16·1257
60·0	16·2061	60·5	16·2849	61·0	16·3626	61·5	16·4395
62·0	16·5159	62·5	16·5935	63·0	16·6737	63·5	16·7566
64·0	16·8416	64·5	16·9285	65·0	17·0166	65·5	17·1056
66·0	17·1950	66·6	17·2832	67·0	17·3729	67·5	17·4629
68·0	17·5564	68·5	17·6542	69·0	17·7551	69·5	17·8618
70·0	17·9779	70·5	18·1029	71·0	18·2314	71·5	18·3611
72·0	18·4912	72·5	18·6185	73·0	18·7412	73·5	18·8594
74·0	18·9756	74·5	19·0912	75·0	19·2063	75·5	19·3231
76·0	19·4434	76·5	19·5693	77·0	19·7039	77·5	19·8456
78·0	19·9890	78·5	20·1331	79·0	20·2803	79·5	20·4297
80·0	20·5783	80·5	20·7264	81·0	20·8766	81·5	21·0298
82·0	21·1893	82·5	21·3590	83·0	21·5457	83·5	21·7462
84·0	21·9506	84·5	22·1513	85·0	22·3445	85·5	22·5232
86·0	22·6854	86·5	22·8336	87·0	22·9746	87·5	23·1107
88·0	23·2397	88·5	23·3613	89·0	23·4747	89·5	23·5772
90·0	23·6676	90·5	23·7474	91·0	23·8202	91·5	23·8888
92·0	23·9551	92·5	24·0198	93·0	24·0802	93·5	24·1345
94·0	24·1823	94·5	24·2234	95·0	24·2577	95·5	24·2869
96·0	24·3108	96·5	24·3304	97·0	24·3459	97·5	24·3566
98·0	24·3608	98·5	24·3614	99·0	24·3619	99·5	24·3619
100·0	24·3619						

$\triangle$ = Epicentral distance (degrees) V_a = Apparent phase velocity (km/sec)

$$V_a = \frac{D}{\delta t}(N_R^2 + N_B^2)^{-1/2} \qquad (2)$$

where δt is the sampling interval, N_R and N_B are integer numbers proportional to the corresponding lags and satisfy the relations :

$$T_R = N_R\ \delta t\ ;\ T_B = N_B\ \delta t \qquad (3)$$

It follows therefore that in the digital approach while the resolution in V_a depends upon the sampling rate, the precision in V_a estimate depends upon the accuracy with which N_R and N_B are determined and hence upon the signal correlation across the array. Improvements in the lag estimates can be achieved by using an interpolation method so as to obtain an epicentre from the continuous distribution of points on the globe. In the GBA on-line system, once the arrival of a seismic signal is detected, control is transferred to a background program which estimates the lag pair and computes the epicentral distance together with the probable error in the computed distance.

3. Distance deduction : Polynomial fitting

We seek to obtain an nth order polynomial of the form :

$$\triangle = A_0 + A_1 V_a + \ldots\ldots\ldots + A_n V_a^n \quad (4)$$

where $\triangle$ is the epicentral distance, and $A_0, A_1 \ldots \ldots A_n$ are $n+1$ real coefficients. The set of a total of N, $N=161$, data points $[\triangle_i, (V_a)_i]$ in the range $20° \leqslant \triangle \leqslant 100°$ are derived (Arora and Krishnan 1970) from the Herrin's surface-focus P travel-times at $\frac{1}{2}°$ interval and are shown in Table 1. The summed error square function S of reproduction of $\triangle$ is given by :

$$S = \sum_{i=1}^{N} [A_0 + A_1 (V_a)_i + \ldots + A_n (V_a^n)_i - \triangle_i]^2 \quad (5)$$

For minimising S, we take partial d.c.'s w.r.t. each coefficient and equate each of them to zero. Thus

$$\left.\begin{aligned} & A_0 N + A_1 \sum_i (V_a)_i + \ldots + A_n \sum_i (V_a^n)_i = \sum_i \triangle_i \\ & A_0 \sum_i (V_a)_i + A_1 \sum_i (V_a^2)_i + \ldots \\ & \qquad + A_n \sum_i (V_a^{n+1})_i = \sum_i (V_a)_i \ \triangle_i \\ & \ldots\ldots\ldots\ldots \\ & A_0 \sum_i (V_a^n)_i + A_1 \sum_i (V_a^{n+1})_i + \ldots \\ & \qquad + A_n \sum_i (V_a^{2n})_i = \sum_i (V_a^n)_i \ \triangle_i \end{aligned}\right\} \quad (6)$$

The set of Eqns. (6) can be solved by inverting the non-singular square matrix of order $n+1$

$$\begin{bmatrix} N & \sum_i (V_a)_i \ldots\ldots & \sum_i (V_a^n)_i \\ \sum_i (V_a)_i & \ldots\ldots\ldots & \\ \vdots & \ldots\ldots\ldots & \\ \sum_i (V_a^n)_i & \ldots\ldots & \sum_i (V_a^{2n})_i \end{bmatrix} \quad (7)$$

and pre-multiplying it with the column vector (8). The solution matrix is the vector of $n+1$ elements (9) :

$$\begin{Bmatrix} \sum_i \triangle_i \\ \sum_i (V_a)_i \triangle_i \\ \vdots \\ \sum_i (V_a^n)_i \triangle_i \end{Bmatrix} \quad (8) \qquad \begin{Bmatrix} A_0 \\ A_1 \\ \vdots \\ A_n \end{Bmatrix} \quad (9)$$

A computer routine SOLVER, which incorporates matrix inversion and matrix multiplication, has been written for the BESM-6 computer to perform the above job. Polynomials upto 12th order have been fitted through the entire range $20° \leqslant \triangle \leqslant 100°$ in powers of V_a as well as $1/V_a$ and the results showing the errors in computed $\triangle$ together with the standard deviation in this error are presented in Table 2.

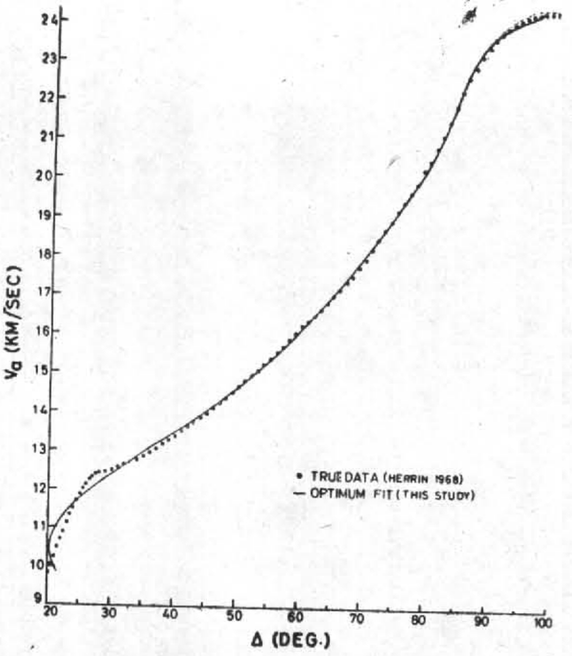

Fig. 1. Seventh order polynomial in positive powers of V_a fitted in the range $20° \leqslant \triangle \leqslant 100°$ through the Herrin's (1968) distance-velocity data

4. Discussion

As shown in Table 2, we have calculated departures from the true values in the reproduced values of $\triangle$ over three distinct ranges, *viz.*, $20° \leqslant \triangle \leqslant 28°$, $28° \leqslant \triangle \leqslant 88°$ and $88° \leqslant \triangle \leqslant 100°$, of the Herrin's velocity function using V_a as well as $1/V_a$ data. From these computations it appears that a seventh order polynomial (Fig. 1) in V_a is reasonably satisfactory for our purpose. In comparison, the optimum polynomial in $1/V_a$ goes to eighth order. Besides, the chain of coefficients in the V_a case is found to be highly convergent while that in the $1/V_a$ case is highly divergent (Table 3). Under the circumstances the seventh order polynominal in V_a becomes more tempting to use, particularly when a single polynomial is needed for use in small on-line computer systems to yield best possible results throughout the range.

Although the polynomial fitting ideally serves to give $\triangle$ to within $\frac{1}{2}°$ accuracy, the errors in basic parameter V_a generally creeps in due to following main reasons which may affect the source location.

(*i*) Surface-focus assumption, (*ii*) Limited directional response of the medium-aperture L-pattern of the array, (*iii*) Lateral inhomogenities in the receiver crust and the possible effects, though small, of a layered crust on the arrival angles and hence on the apparent velocity (Nuttli 1964, Hasegawa 1971, Brown and Enayatollah 1973).

The basic Fortran compiler of the TDC-12 with 12-bit word length permits usage of number

TABLE 2

Comparative estimate of departure from true values in computed △ using a single polynomial fit with (*i*) positive powers of V_a and (*ii*) negative powers of V_a

n = Order of polynomial; $\sigma(n)_{a,b}$ = True *minus* computed △ in the range a to b of △

n	With powers of V_a			With powers of $1/V_a$		
	$\sigma(n)_{20,28}$	$\sigma(n)_{28,88}$	$\sigma(n)_{88,100}$	$\sigma(n)_{20,28}$	$\sigma(n)_{28,88}$	$\sigma(n)_{88,100}$
1	—5·51±2·19	1·37± 3·50	—3·37±2·00	3·17± 8·01	—1·08±0·88	2·86±2·77
2	—0·41±4·62	—0.14± 1·68	0·66±2·72	—0·71± 4·36	0·03±1·74	0·01±2·53
3	—0·28±4·76	—0·14± 1·64	10.58±2·67	—0·98± 2·24	—0·17±1·39	1·22±2·78
4	—0·91±1·89	0·15± 0·96	—0·25±1·33	—0·07± 2·60	—0·11±0·84	0·39±2·40
5	—0·80±1·65	0·13± 0·90	—0·22±1·57	—0·57± 1·23	0·06±0·85	—0·04±2·02
6	—0·29±2·01	0·05± 0·53	—0·11±1·02	—0·59± 1·29	0·06±0·85	—0·06±2·00
*7	—0·47±1·55	0·05± 0·62	—0·01±0·87	—0·19± 1·77	0·04±0·52	—0·17±1·55
†8	—0·48±1·32	0·07± 0·62	—0·07±0·96	—0·28± 0·85	0·06±0·48	—0·17±1·18
9	—0·29±1·10	0·02± 0·42	0·05±0·73	—0·28± 0·85	0·06±0·48	—0·17±1·18
10	—0·25±1·16	0·03± 0·39	0·14±0·75	—0·19± 1·12	0·05±0·41	—0·13±1·07
11	—0·04±1·06	2·17± 2·42	12·90±1·27	1·26± 1·14	0·27±0·35	0·01±0·83
12	0·90±0·96	14·38±17·53	96·40±9·29	23·96±14·32	3·74±3·32	0·23±0·91

*Most optimum order of polynomial in powers of V_a † Most optimum order of polynomial in powers of $1/V_a$

TABLE 3

Values of coefficients pertaining to the optimum polynomial (*see* Table 2), in powers of V_a and $1/V_a$, beginning from 0th order term

Coefficient	Value: Base	Value: 10 exp.	Value: Base	Value: 10 exp
	Function of V_a		*Function of $1/V_a$*	
A_0	—4·65892	3	1·0046101	5
A_1	2·58129	3	—1·218039936	7
A_2	—5·7381	2	6·3818902880	8
A_3	6·738	1	—1·885209237566	10
A_4	—4·55	0	3·4339045615737	11
A_5	1·8	—1	—3·94987137035772	12
A_6	—3·8	—3	2·802280118826496	13
A_7	3·4	—5	—1·121374661839774	14
A_8	—	—	1·938414433474491	14

as large as $10^{\pm 616}$ (a real number is represented by a floating point configuration comprising a 12-bit signed exponent and 24-bit signed mantissa) so that the mantissa does not have resolution better than 1 in 10^8. Hence, to derive any advantage out of the above seven-degree polynomial, it is necessary to employ four computer words as the velocities are to be computed upto two decimal places. It has been verified that use of the hardware multiply/divide scheme in the existing GBA system consumes only 25 milliseconds to evaluate the polynomial expression which falls well within the real-time system operation.

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