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Fractal dimensions of clouds around Madras

P. S. JAIN
Regional Meteorological Centre, Madras
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सार — लवज्वॉय के अनुसंधान का अनुकरण करते हुए उत्तर-पूर्व मानसून ऋतु (अक्टूबर-नवम्बर 1986) में मद्रास के आसपास रेडार निर्धारित मेघों की संरचना की जांचपड़ताल की गई।

मद्रास में स्थित उच्च शक्ति S-बैंड रेडार के द्वारा 200 कि. मी. के अर्द्धव्यास में 50 और 5350 वर्ग कि. मी. के क्षेत्र में छः सौ मेघ प्रतिध्वनियों के नमूने का पता लगाया गया और परिमाप और क्षेत्र के मध्य सम्बंध के लिए उनका विश्लेषण किया गया।

लघु P बनाम लघु A आलेख से परिमाप (P) और क्षेत्र (A) और मेघ परिमाण के फ्रक्टल विमाएँ (D) के मध्य सम्बंध की संगति स्थापित की गई और यह निष्कर्ष निकाला गया कि मेघ परिमाप फ्रैक्टल वक्र के समरूप हैं।

समाश्रयण रेखा से लघु P बनाम लघु A प्रकीर्ण आरेख के $D=1.30$ और $S=2.07$ माध्य मूल्य का मूल्यांकन किया गया। जहां सम्बंध $P \sim A^{D/2}$ के लिए अनुपातिक स्थिरता S है। मद्रास के आसपास उत्तर-पूर्व मानसून ऋतु में प्रत्येक मेघ के D मूल्य निकालने के लिए मेघों को पहचानने के लिए मूल्यांकन करने का सुझाव दिया गया। जिसमें प्रबल क्षैतिज/उर्द्धवाधर पवनें उनके साथ अंतः स्थापित है विशेषरूप से चक्रवातों के साथ संबंध मेघ क्षेत्र में, उनके संभव वास्तविक समय के लिए इन मूल्यों को आधारभूत मूल्यों के रूप में प्रयोग किए जाने का सुझाव है।

ABSTRACT. Following lead from Lovejoy's (1982) investigations the structure of radar determined clouds around Madras in northeast monsoon season (October-November 1986) was investigated.

Six hundred cloud echo samples having area between 50 and 5350 sq. km detected within 200 km radius by a high power S-band radar located at Madras were analysed for the relation between their perimeter and area.

Consistency of the relation between the perimeter (P) and area (A) through a log P *vs* log A plot and of fractal dimensions (D) of cloud perimeter, has been established and conclusions drawn that cloud perimeter conforms to fractal curve.

From the regression line of the log P *vs* log A scatter diagram the mean values of $D=1.30$ and $S=2.07$ were evaluated, where S is the constant of proportionality for the relation $P \sim A^{D/2}$. These values have been suggested to be used as base values for finding out D values for individual clouds in northeast monsoon season around Madras for their possible real time use in identification of clouds having strong horizontal/vertical winds embedded within them, especially in cloud field associated with cyclones.

1. Introduction

Mandelbrot's (1981) theory on 'Fractals and the Geometry of Nature' generated good interest in the scientific community and several studies investigating geometrical shapes of radar and satellite determined clouds and rain areas, have since been made by research workers. Lovejoy (1982) studies area-perimeter relationship of radar and satellite determined cloud and rain areas extending from 1 to 1.2×10^6 sq. km area, and suggested that area-perimeter of cloud fit well in a relation $P \sim A^{D/2}$, where P is the perimeter in km, A the area in sq.km and D the fractal dimensions of cloud perimeter. From the observed consistency of area-perimeter relationship he drew the conclusions that cloud and rain areas are fractals since they do not exhibit any characteristic horizontal length scale between 1 and 1000 km.

In this study, structure of radar determined clouds around Madras have been analysed with a view to ascertain whether these conform to fractals.

Since fractal is relatively a new concept and, perhaps, this being the first of such studies attempted in our country, the introduction to 'fractals' is given in some details, in the following paragraph.

1.1. *What are fractals?*

Before Mandelbrot's theory on 'fractals' the irregular and fragmented faces of nature such as mountains, rivers, coast line, clouds etc could not be incorporated in theory by classical or Euclidean geometry. Mandelbrot coined a word 'fractal' to describe these irregular and apparently complex natural shapes.

Mandelbrot, with only three parameters fed to computer could generate models which had a striking resemblance to natural mountainous landscape and cloud patterns, thereby suggesting that these grossly irregular complex facets of nature must have a basic geometric simplicity.

Of the three parameters, one related to length, second one, the most important, denoted by 'fractal dimensions' (D) decided the irregularity or the smoothness of the pattern. The third parameter called the 'random seed' or 'chance' is best thought of as one of the possible alternative or conceivable outputs from computer which must have a definite number of possible outputs.

In drawing the mountains or cloud patterns the computer programming is so looped that it draws again and again the same task on a smaller scale. Model of a snowflake generated by computer by looping 4 replicas of a triangle reduced in the ratio 1/3 are shown in Fig. 1.

The non-random snowflake as shown in Fig. 1 is in it's first stages of construction which proceeds to an infinite number of steps before becoming a true fractal. The result is known as 'Kotch Curve' with fractal dimension' 1.26 (Mandelbrot 1981).

Other important characteristics ascribed to fractals can be described in following ways.

1.2. *Self similarity*

The structure of fractals is self similar at all scales of time and length — a property referred to as 'scaling. This implies that if cloud perimeter conforms to fractal curve, clouds must exhibit a property that their structure in some sense is same at all scales, with exception of scale factor.

Since fractal dimension as stated above, is a measure of smoothness or roughness of a structure, by stretching the analogy to cloud structure, smoothness or roughness of cloud structure can be visualised by a measure of how much perimeter is needed to enclose an area. If a long perimeter is needed the cloud is obvioulsy rough and in that case $D \rightarrow 2$ and if small perimeter is needed to enclose the same area, in that case $D \rightarrow 1$, and cloud will be smoother. Consistent value of D will signify that clouds are equally smooth or rough within horizontal length and areal extent of cloud samples investigated. This fact (consistency of D values) will also suggest self similarity in clouds at the scales investigated which is an essential requisite for a structure to be fractal.

1.3. *Notion of dimensions*

In the classical geometry dimensions are defined as number of co-ordinates to define a point on a curve or a plane. For self similar curves in Euclidean geometry such as straight line or a circle it can easily be seen that the ratio $\log N/\log (1/r)$ where, N is the number of self similar segments and r is the reduction ratio, represents the dimensions of the curve of plane, which is 1 for straight line or circle and 2 for interior of square.

If we consider the same ratio in case of fractals like snowflakes we get $\log 4/ \log 1/(1/3) = 1.26$ which is a fraction. It may be stated that fractals are the structures having dimensions between the standard values.

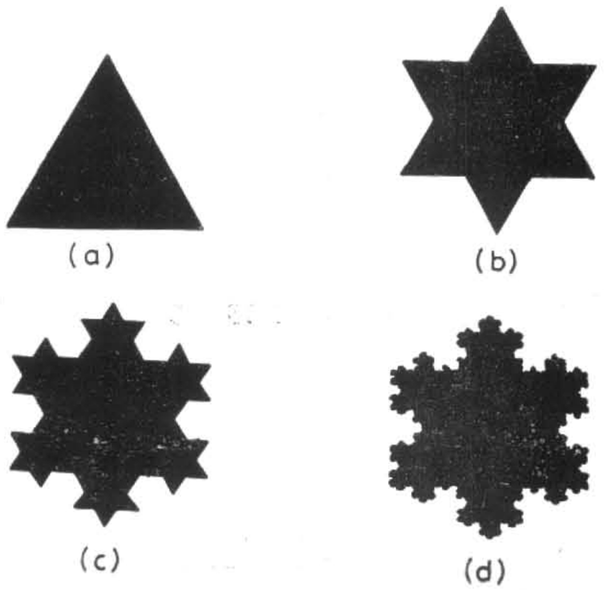

Fig. 1. First four stages in the modelling of snowflake

1.4. *Scope of the present study*

The horizontal projections of cloud and rain areas as detected by radar and observed on Plan Position Indicator [PPI] scope, exhibit characteristic shapes of clouds.

Following objectives were set for this study :

(*a*) To compute the area and perimeter of cloud samples within 200 km of Madras.

(*b*) To expose consistency of fractal dimensions by statistical methods.

(*c*) To evaluate constant for the relation $P \sim A^{D/2}$, so as to make use of this constant for calculation of fractal dimensions of individual cloud on a real time basis.

(*d*) To present spread of computed D values for cloud samples selected for study.

2. Data

Madras (13° 04′ 57″ N, 80° 17′ 30″E) is situated in the southern part of India within tropical zone. October and November months, in and around Madras region, are most active weather months. Widespread cloud activity is experienced in this region, during, these months, and it is classified as northeast monsoon season here. The area around Madras is also most prone to cyclones in this season.

Routine radar observational data, mostly taken at 3 hourly intervals in October and November 1986 by a high power, S band radar (Mitsubishi, Japan Make, RC-32E Model, peak transmitted power 500 kw, wave length 10 cm, beam width 2° conical, M.D.S. better than —110 dbm) located at Madras, is utilised for this study. Echoes from clouds as seen on PPI scope are referred to as radar determined clouds or cloud samples in the text.

TABLE 1

Areawise distribution of cloud samples

Area (sq. km)	No. of samples	Area (sq. km)	No. of samples
50-100	104	1025-1500	26
125-200	153	1525-2000	9
225-300	93	2025-2500	9
325-400	64	2525-3000	5
425-500	40	3025-3500	1
525-600	30	3525-4000	6
625-700	26	4025-4500	—
725-800	13	4525-5000	1
825-900	9	5025-5500	2
925 1000	9		
Total			600

From the photographs of the PPI scope of the radar, boundaries of the echoes from cloud patches were traced on grided polar diagrams. From these tracings, clouds which could be identified as single contiguous patches, were selected for the study. The perimeter of the cloud patches was determined with resolution of 5 km and area was estimated by counting the number of grid squares (5 km $\times$ 5 km) covered by individual cloud patches.

The study was limited to cloud samples within 200 km of radar range referred as area around Madras, since approximation/errors are introduced in the observation of cloud dimensions due to beam width and earth curvature, at larger ranges.

3. Data analysis

Scatter diagram drawn for all the six hundred cloud samples is given in Fig. 2. In this log P *vs* log A plot, the repeats of cloud samples having same perimeter for a particular area, are indicated (in the scatter diagram) by printing a number of such cases instead of a dot marked for individual cloud sample.

Least squares fit line for x-axis (log P) on y-axis (log A), *i.e.*, for estimating log P values from log A has been drawn in the scatter diagram. Equation for this regression line (of the form $x = my + c$) is worked out as shown below :

$$x = .6513\ y + .3169 \qquad (1)$$

From this equation and the relation $P = S\ A^{D/2}$ where, S is the constant of proportionality, we can easily deduce the mean value of fractal dimension (D) of perimeter. $D = 2 \times$ slope of the line (1) :

$$D = 2 \times\ .6513 = 1.3026 \qquad (2)$$

Similarly log S is given by 'c'

i.e., log S = .3169

or $S = 2.0744 \qquad (3)$

Correlation coefficient (r) is also computed as $r =$ 9569 which shows significant correlation between the two variables log P and log A . Fractal dimensions of individual cloud samples have been computed, using the relation $P = 2.0744\ A^{D/2}$. Frequency distribution of class intervals of fractal dimensions has been shown through histogram in Fig. 3. It can be seen that range of D values are lying between 1.13 and 1.52.

Areawise population of cloud samples with area computed with a resolution of 25 sq. km is shown in Table 1.

4. Discussions

It may be seen from Table 1 that out of 600 cloud samples selected for study, most of the samples (90.17%) were of the size less than 1000 sq. km. Only 9.83% of samples were larger than 1000 sq. km. Area of the largest sample was 5350 sq. km. However, the consistency of relation between perimeter and area of cloud samples, for the entire range of cloud area is quite evident from the log P *vs* log A plot given in Fig. 2. There is no indication of any preferred or characteristic horizontal length scale. The mean value of fractal dimensions for cloud perimeter determined from the regression line of the scatter diagram comes to $D = 1.30$, which is in good agreement with the value $D = 1.35$ arrived at by Lovejoy (1982), for tropical Atlantic.

Correlation coefficient $r = 0.9569$ also points to the significance of relationship between the two variables (log P and log A). It will be seen from the histogram in Fig. 3. that 92.83% of cloud samples exhibited $D = 1.30 \pm .1$ and within this range 49.83% of total samples were having $D = 1.30 \pm .03$.

Above figures speak well for the consistency of the fractal dimensions of cloud perimeter observed around Madras.

Consistency of fractal dimensions, close agreement between the mean D value ($D = 1.30$) computed in this study and with D ($= 1.35$) reported by Lovejoy (1982) and excellent correlation exposed by log A *vs* log P plot, all these factors are indicative of some fundamental and common process connected with the development of clouds. This leads to turbulence which is responsible for the generation of clouds.

Thus, the study of fractal dimensions of clouds may help in understanding the complexities of turbulence which are not properly understood so far. An attempt to relate fractal dimensions D of clouds with theory of relative turbulent diffusion has been published by Hentschel and Procaccia (1984).

Our other areas of interest are those in which fractal dimensions are shown to have indicated tendency towards smooth dimensionality ($D \rightarrow 1$) for increasing strength of horizontal/vertical winds, within the clouds.

Observations made by Rys (1986) in this respect are interesting. Through his investigations of hail clouds he concluded that the stronger winds cause smoothening of contour shape. His observations are summarised in the following remarks :

"For small contours, the strong vertical winds which are known to be present near the centre of thunderstorms in severe convective storms are thought to be responsible for this smoothening whereas large cloud contours are sensibly smoothed only in presence of important horizontal wind strength".

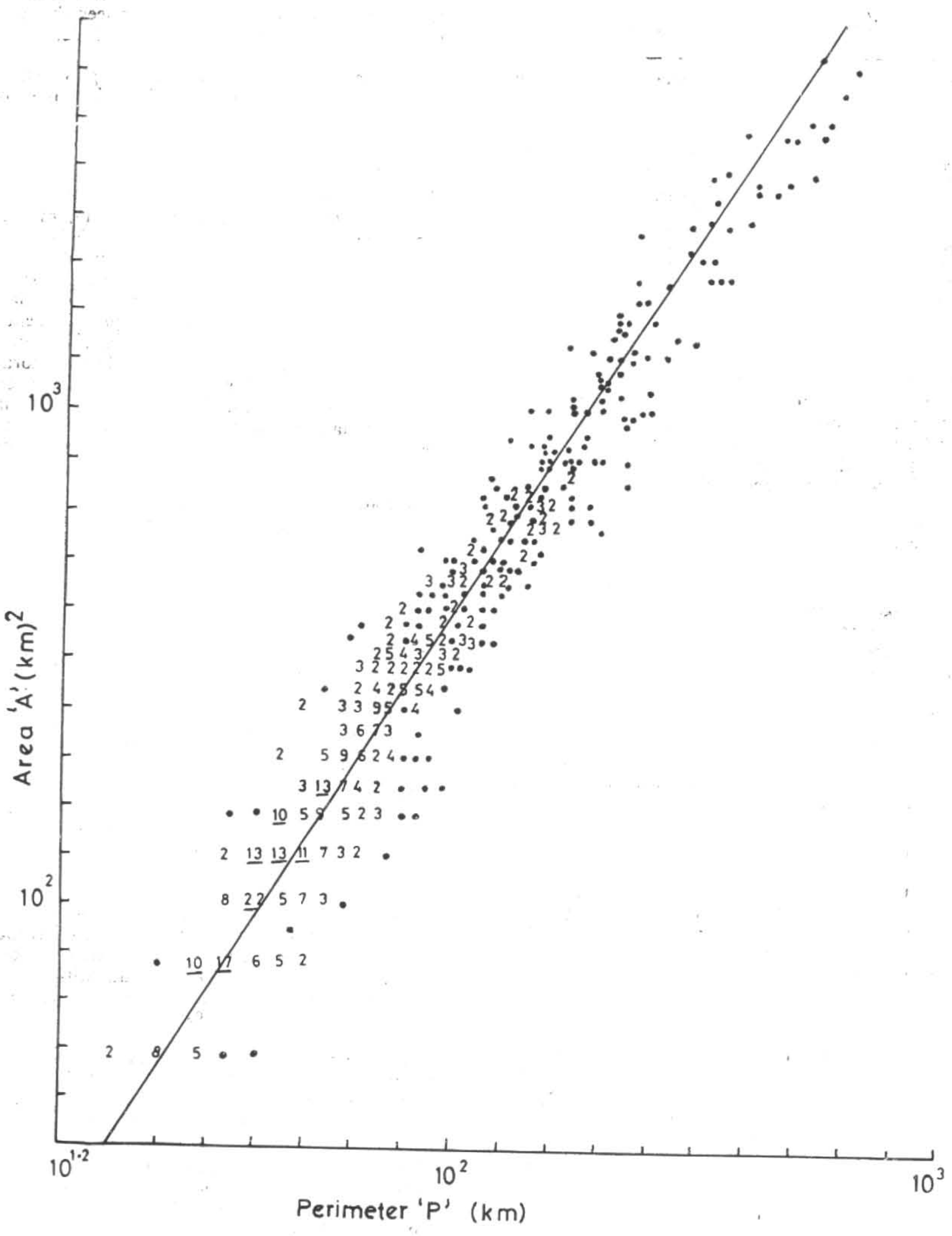

Fig. 2. Area-perimeter relationship for radar determined clouds. Numbers written in the plot indicate cases of cloud samples having same area and perimeter. Dot represents data for individual cloud samples. Two digit numbers are underlined.

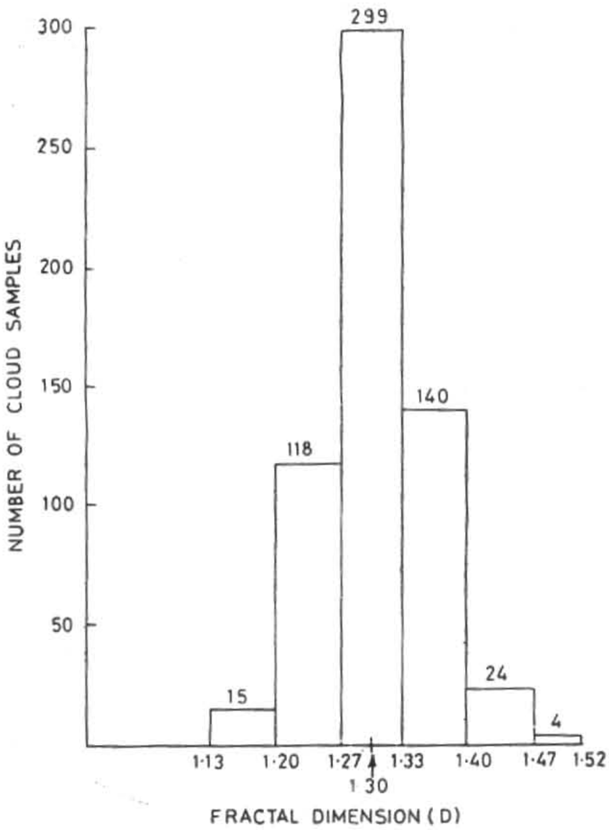

Fig. 3. Frequency distribution of fractal dimensions (D) values

As a direct inference from above, it may be possible to indentify clouds with strong horizontal/vertical wind embedded within them, by real time computation of fractal dimensions of clouds particularly those associated with cyclones. There comes the importance of evaluation of constant for the relation $P \sim A^{D/2}$ and for the average value of fractal dimensions for a particular season or in a typical synoptic situation to serve as a base value for comparison with the fractal dimensions of clouds estimated on a real time basis.

As an extension of this study, it is proposed to examine the past radar data of cyclones for fractal dimensions with values of required parameters arrived at in this study ($S = 2.07$ and $D = 1.30$).

Compared to $D = 1.30$ obtained in this study and results of Lovejoy (1982) with $D = 1.35$, higher value of $D = 1.5$ has been reported by Yano and Takeuchi (1987) as a result of their analysis of an infra-red satellite imagery of clouds over Inter Tropical Convergence Zone. The possible reason for obtaining higher values for D as compared to Lovejoy (1982) has been given by the authors as their having used different concept for calculations of fractal dimensions. Since Lovejoy's (1982) concept has been followed in this study, same explanation holds good for results of this study.

Orographic effect on fractal dimensions has been brought out by Skoda (1987). Substantial, though temporary, growth in D values has been found by him when clouds moved over to lee side, specially in situation with strong vertical shear.

In this study no appreciable difference in D values could be noticed in cloud over sea or land. Comparison was restricted to cases when clouds having equal area were observed at the same time of day or night over land and sea. However, these results are not conclusive, since there were only very few samples (20) which could be identified under above stipulated conditions.

Also, further studies are needed to be made to bring out the effect of transition phase, *i.e.*, for finding out possible changes in fractal dimensions of clouds while moving from sea to over land or *vice versa*.

5. Conclusions

As a result of this study fractal nature of the perimeter of clouds in northeast monsoon season, in a tropical zone around Madras, is established. There is no indication of any preferred or characteristic horizontal length scale in the cloud samples having area from 50 to 5350 sq. km.

Mean value of fractal dimension $D = 1.30$ in the northeast monsoon season is suggested to be considered as a base value and with the constant of proportionality derived as $S = 2.07$, the fractal dimensions of individual cloud can be computed from the relation $P = S\,A^{D/2}$ for real time use of fractal dimensions.

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