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A harmonic analysis of 100 mb zonal winds in the tropics

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ABSTRACT. Monthly-mean 100 mb zonal wind observations from stations in, or on the fringe of, the tropics have been subjected to Fourier analysis.

Annual and semi-annual harmonics are evident at many stations, and superpositions of these two harmonics
reproduce observations quite well at these stations. Dates corresponding to greatest rates of change of zonal-wind
ve to be monsoonal on the basis of rainfall and surface-wind reversals.

1. Introduction

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It is well-known that reversals of direction of the upper-tropospheric flow over the tropics of the eastern hemisphere are associated with the onset and retreat of summer monsoon circulations; the westerlies which prevail for much of the year are replaced by easterlies for about four months each summer.

The abruptness of the reversals prevents such flow-patterns from being described correctly as harmonic oscillations. Instead, it is more accurate to consider what Hantel (1971), discussing the annual march of temperature over southern Asia, termed 'a flipflop-like oscillation between relatively steady states, with discontinuous jumps from one state to another'. Even so, it is not precise to use the description 'discontinuous' for upper-wind patterns. Reversals are rapid, as Fig. 1 shows, but they are not sufficiently precipitous or smooth for dates corresponding to transitions to be determined exactly by visual Accordingly, some inspection of daily data. statistical procedure must be adopted to establish average dates.

Flow patterns of this kind are, as Hantel pointed out, 'characterised by sizeable contributions of higher harmonics'. However, as he was advised (by H. Van Loon) that harmonics higher than the semi-annual do not contribute significantly to the total variance of temperature. Hantel felt justified in describing the zonally-averaged annual patterns of temperature between 60° and 120°E in terms of superpositions of annual and semi-annual harmonics.

2. Outline of investigation

The present study, a harmonic analysis of monthly-mean 100-mb zonal-winds at 45 stations in, or on the fringe of, the tropics (see Fig. 2 and Appendix), showed that there is in addition to the anticipated, and usually considerable, annual harmonic, a conspicuous semi-annual harmonic present in the data of many stations (Table 1). Indeed, at some stations in the Southern Hemisphere the amplitude of the semi-annual harmonic exceeds that of the annual, a fact which has been discussed at length by Van Loon and Jenne (1969, 1970). Harmonics higher than the semi-annual do not contribute greatly to the total variance of zonal-wind at most stations and superpositions of annual and semi-annual harmonics reproduce 100-mb zonal-wind observations tolerably well in general (Fig. 3). However, at some stations harmonics lower than the annual are evident, while at Cocos Island the amplitude of the biennial harmonic exceeds that of the annual.

It was hoped initially that this study would provide insight into inter-annual variations of monsoon intensity. In particular, it was thought possible that some influence from the quasibiennial oscillation might be apparent, and

Wind directions at 100 mb over Masirah Island (20°30'N, 58°40'E), at 00 and 12 GMT between 15 March [1370 and 28 February 1971, illustrating the nature of the transitions between winter and summer flows Fig. 1.

Fig. 2. Location of stations used in this study

Fig. 3. Comparison between 100-mb zonal-wind observations (continuous lines) and superpositions of annual and semi-annual harmonics (dotted lines) at Cocos Island, Majuro and Dakar

TABLE 1

Proportion of total variance due to major harmonics, in per cent

*At these stations there are other harmonics contributing more than 10 per cent of the total variance, thus :

it was for this reason that 100-mb was chosen, being a level both typical of upper tropospheric wind patterns and perhaps likely to yield some evidence of the oscillation. This hypothesis was discarded when evidence for a significant near biennial harmonic appeared in the data of only two stations, Cocos Island and Canton Island, both some distance from recognised monsoonal centres of action.

Nevertheless some interesting consistencies and inconsistencies in the temporal and spatial characteristics of reversals and of annual and semiannual harmonics emerged from the study, and it is these which provide the raison d'être of this paper.

3. Data

The 100-mb wind observations were extracted from the United States Department of Commerce publication Monthly Climatic Data for the World and for each station expressed as a Fourier series in the form:

$$
u = \frac{1}{2} a_0 + \sum_{n=0}^{N} c_n \sin\left(\frac{2\pi n t}{T} + \delta_n\right) \tag{1}
$$

where.

 $u =$ zonal velocity, westerlies positive,

 $t =$ time in months,

 $T =$ length of record analysed, in months, c_n^2 = sum of the squares of the usual Fourier coefficients a_n and b_n

 δ_n = the phase relationship, tan⁻¹ (a_n/b_n)

The number of observations available for analysis varies from station to station (see Appendix), so a test was performed upon the observations from Bahrein to determine the minimum length of record which yields reliably the amplitudes and phases of the major components, the annual and semi-annual harmonics. These amplitudes and phases were computed for $T=$ 12 (12) 132 (T was analysed in steps of 12 months from 12 to 132, *i.e.*, the amplitudes and phases
were computed for $T = 12$, $T = 24$, $T = 36$ etc) and it was decided thereby that stations for which only 36 consecutive observations were available would be acceptable (see Fig. 4).

4. Analysis

At all but Cocos Island of the stations listed in Table 1 the annual and semi-annual harmonics each make greater contributions to the total variance than any other harmonics. For these stations, and including Cocos Island, despite its large biennial harmonic, Eq. (1) was reduced to:

Fig. 4. Amplitudes and phases of annual and semi-annual harmonics at Bahrein for $T=12(12)$ 132

$$
u = \frac{1}{2} a_{\mathsf{u}} + A \sin \frac{2\pi}{12} (t + \phi) + B \sin \frac{4\pi}{12} (t + \psi) \tag{2}
$$

where, A and B, ϕ and ψ , are the amplitudes and phases of the annual and semi-annual harmonics respectively. Hence, the second derivative was obtained:

$$
\frac{d^2u}{dt^2} = -\left(\frac{2\pi}{12}\right)^2 \left[A \sin \frac{2\pi}{12} (t+\phi) + 4B \sin \frac{4\pi}{12} (t+\psi)\right]
$$
\n(3)

Then, for each station, the values of t for which $d^2u/dt^2=0$ were computed, so identifying dates corresponding to greatest rates of change of zonal-wind speed in Eq. (2). Observations vielded four such values of t at all Southern Hemisphere stations and at many of the Northern Hemisphere stations, but only two such values of t at the remainder of the Northern Hemisphere stations (where the second harmonic was small).

At Abidjan and Singapore several harmonics are conspicuous, and it is clear that this would have been the case also at the East African equatorial stations Entebbe and Nairobi had there been sufficient consecutive observations to allow quantitative analysis. At each of the other stations not listed in Table 1, there is a considerable annual harmonic but no significant semi-annual harmonic.

5. The first and second derivatives

Dates corresponding to $d^2u/dt^2=0$, associated

values of du/dt and dates during northern summer when $du/dt=0$ are listed in Table 2. The principal features are, briefly :

(i) All stations display a solution of $d^2u/dt^2=0$ within the period 15 September to 15 October. Only at Lima is the value of du/dt not positive for this solution, and at several Asian and North African stations du/dt exceeds 10 m/sec/ month. Large values of du/dt over southern Asia and North Africa at this time are not unexpected, for they correspond to the cessation of summer monsoon activity and the associated reversal of upper winds from easterly to westerly.

Strüning and Flohn (1969) have drawn attention to the extraordinary width of upper tropospheric easterlies during northern summer and have mentioned that they some times extend from 27°N to 12°S above the Indian Ocean and East Africa. These upper easterlies over the South Indian Ocean and adjacent lands weaken or reverse to light westerly when the monsoon of southern Asia retreats, the suggestion from the solutions of $d^2u/dt^2=0$ being that the transition occurs a few days earlier in the Southern Hemisphere than in the Northern.

 (ii) The reversal of upper winds from westerly to easterly which accompanies the onset of summer monsoon activity over southern Asia and North Africa is revealed by solutions of $d^2u/dt^2=0$ in late May or early June over these areas, and associated values of du/dt are again mostly large, exceeding -10 m/sec/month at several stations.

HARMONIC ANALYSIS OF 100 MB ZONAL WINDS

TABLE 2

Dates corresponding to $d^2u/dt^2=0$ associated values of du/dt and dates during northern summer when $du/dt=0$

185

TARLE 3

Dates corresponding to $\sin^2 \frac{\pi}{12}(t+\phi) = +1$ and $\sin \frac{4\pi}{12}(t+\phi) = -1$

The strongest easterlies, given by $du/dt=0$, occur generally in the last week of July over these areas.

Over the North Pacific Ocean and in the Southern Hemisphere solutions of $d^2u/dt^2=0$ are significantly later than over southern Asia and North Africa.

(iii) All the Southern Hemisphere stations display a solution of $d^2u/dt^2=0$ between December 9 and 30, corresponding to the commencement of summer circulations. Many Northern Hemisphere stations show a secondary solution of $d^2u/dt^2=0$ at about the same time, but at the other stations, mostly in southern Asia, d^2u/dt^2 exhibits only a minimum in winter and does not reach zero.

(iv) The other singularity in Southern Hemisphere upper flow-patterns is indicated by solutions of $\frac{d^2u}{dt^2} = 0$ between mid-March and mid-April, corresponding to the weakening of summer circulations. Only over the western North Pacific Ocean do solutions of $d^2u/dt^2=0$ coincide approximately with those in the Southern Hemisphere, solutions at other Northern Hemisphere stations seeming to possess no clear relationship to flowpatterns elsewhere.

6. Phases of annual and semi-annual harmonics

The dates upon which $\sin[(2\pi/12) (t+\phi)]=+1$ are shown in Table 3, together with the dates upon which $\sin [(4\pi/12) (t+\psi)]= -1$.

At most Northern Hemisphere stations the dates corresponding to sin $[(2\pi/12) (t+\phi)]=+1$ occur in late January or early February, which is consistent with the accepted view that at this time of year the tropospheric temperature contrast between low and high northern latitudes is most marked, and hence upper tropospheric westerly momentum should then be greatest (or easterly momentum least). Similarly, at most Southern Hemisphere stations the dates occur near midwinter, but it will be noticed that the difference in the values of t for which sin $[(2\pi/12)(t+\phi)]=1$ between Northern and Southern Hemispheres is not six months.

Over the western Pacific Ocean the above explanation does not appear to apply, for dates do not conform to the general tendency for sin $[(2\pi/12) (t+\phi)]=+1$ to occur in mid-winter; this inconsistency deserves investigation.

The semi-annual harmonic exhibits only small differences of phase between localities, and periods during which sin $[(4\pi/12)$ $(t+\psi)]$ is negative coincide well with periods of monsoon activity over 2π (4) $4x - 11$... 4π ...

TABLE 4

Values of A and 4B in m/sec, dates when $du/dt = 0$ and the values of du/dt in m/sec / month when

Africa and northern southern Asia, North Australia.

Inspection of Table 3 shows that at several stations the dates upon which $\sin \left[\left(2\pi/12 \right) (\bar{t} + \phi) \right] =$ +1 and sin $[(4\pi/12) (t+\psi)]=-1$ occur within a few days of each other. For a coincidence Eq. (3) reduces to:

$$
\frac{d^2u}{dt^2} = -\left(\frac{2\pi}{12}\right)^2 \left[A - 4B\right]
$$
\n
$$
\frac{du}{dt} = \frac{2\pi}{12} \left[A \cos \frac{2\pi}{12} (t + \phi) + 2B \cos \frac{4\pi}{12} (t + \psi)\right] = 0
$$
\n(4)

There is, therefore, a turning point of the function $u=f(t)$, and for $u=\text{maximum}, d^2u/dt^2<0$ and $A > 4B$.

Stations for which Eq. (4) is a good approximation are listed in Table 4, together with (i) values of A and $4B$, (ii) dates during winter when du/dt $=0$, and *(iii)* the values of du/dt corresponding to the dates upon which $\sin [(2\pi/12)(t+\phi)] = +1$ and $\sin\left[\left(4\pi/12\right)\left(t+\psi\right)\right] = -1$. It can be seen that for all stations yielding only two solutions of $d^2u/dt^2 = 0$ during the year (see Table 2) the condition $A > 4B$ is satisfied, whereas for all stations yielding four solutions $A \leq 4B$.

Where $A > 4B$ there is one maximum value of the function $u=f(t)$ and the winter part of the graph of $u=f(t)$ resembles Fig. 5(a), where, $A<4B$, however, the graph of $u=f(t)$ resembles either

Fig. 5. The function $u=f(t)$, i.e., Eq. (2), plotted for winter months

Fig. 5(b) or Fig. 5(c). At the stations where Fig. 5(c) is appropriate a [maximum, positive, value of the function $d^2u/dt^2 = f(t)$ accompanies the near coincidence of dates upon which $\sin\left[\left(2\pi/12\right)\right]$ $(t + \phi)$] = + 1 and sin $[(4 \pi/12) (t + \psi)] = -1$ but, as Table 4 shows, du/dt is not simultaneously zero. It should be noted that values of ϕ and ψ were calculated to the nearest ± 0.1 month; although both dates given in Table 3 for Port Sudan are 30 January there is, in fact, a difference of about one day between them. This explains why the value of du/dt given for this station in Table 4 is not zero. For stations at which A < 4B, dates during winter when d^2u/dt^2 is maximum are listed in Table 5, together with the associated value of d^2u/dt^2 ,

For stations at which $A \leq 4B$, the date during winter when du^2/dt^2 is maximum and the associated value of du^2/dt^2

Evidently the influence of the semi-annual harmonic can be sufficient to so counteract the annual harmonic that westerly momentum is in fact greatest (or easterly momentum least) not in mid-winter but in early or late winter.

7. Concluding remarks

The purpose of this paper has been to report the results of the analyses, rather than to discuss their dynamical implications. It is clear that upper easterlies associated with summer monsoon circulations influence 100-mb wind patterns well beyond those areas generally considered by geographers to be monsoonal, on the basis of rainfall activity and surface-wind reversals.

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REFERENCES

APPENDIX

Key to stations whose location is shown in Fig. 2 and, in brackets, the number of observations analysed (months)

