Mausam, (1986), 37, 1, 39-44

551.524.78 : 551.558.1

Surface tension driven convection subjected to rotation and non-uniform temperature gradient*

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सार — इस आलेख में द्रव की क्षैतिज सतह पर पृष्ठ तनाव द्वारा चालित मारंगोनी संवहन के शुरु होने पर समान घूर्णन तथा असमान तापमान प्रवणता के प्रभावों का अध्ययन किया गया है। यह अध्ययन दोनों परिसीमक पृष्ठों को मुक्त तथा रूद्धोष्म मानते हुए अत्यणु आयाम विश्लषण के आधीन है। एइजनमान, जिन्हें बाद में आंकिक रूप से अभिकलित किया जाता है, प्राप्त करने के लिये एकल-पद विस्तार के साथ एकल-पद गेलर्किन तकनीक का उपयोग किया जाता है। उस स्थिति पर विशष ध्यान दिया जा रहा है जहां विशिष्ठ मारंगोनी संख्या समान तापमान प्रवणता की संख्या से अधिक या कम है। आंकिक परिणाम कुछ विशेष मामलों में ही निकाले जाते हैं तथा विभिन्न आधारभुत तापमान पृष्ठ चित्रों के अस्थाईकारी प्रभावों तथा कोरियॉलिस बल के स्थाईकारी प्रभावों के विषय में कुछ सामान्य निष्कर्ष प्रस्तुत किए जाते हैं। ऐसा पाया गया था कि सर्वाधिक अस्थाईकारी तापमान प्रवणता केवल एक ही है जिसके लिये तापमान प्रवणता गहराई का एक पद फलन है तथा सर्वाधिक स्थाईकारी तापमान प्रवणता एक उल्टा परवलय है। कोरियॉलिस बल का मारंगोनी संवाहन के आरंभ पर निरोधी प्रभाव होता है।

ABSTRACT. The effects of uniform rotation and non-uniform temperature gradient on the onset of Marangoni convection driven by surface tension in a horizontal layer of fluid is studied subjected to infinitesimal amplitude analysis assuming both the bounding surfaces free and adiabatic. A single-term Galerkin technique with single term expansion is used to obtain the eigenvalues which are then computed numerically. Attention is focused on the situation where the critical Marangoni number is greatest or least than that for the uniform temperature gradient. Numerical results are obtained for special cases and some general conclusions about the destabilizing effects of various basic temperature profiles and the stabilizing effect of Coriolis force are presented. It was found that the most destabilizing temperature gradient is one for which the temperature gradient is a step function of the depth and the most stabilizing temperature gradient is inverted parabola and the Coriolis force has inhibiting effect on the onset of Marangoni convection.

1. Introduction

The determination of the criterion for the onset of convection induced by surface tension has considerable interest in meteorology. Convection driven by surface tension gradients is inevitable in meteorology because such configurations often involve free surfaces. The experiments on the Apollo 14 and 17 flights (reported by Grodzka & Bannister 1972, Bannister *et al.* 1973) have shown that convection can still be induced by surface tension effects, even if buoyancy forces are absent. The neglect of such a convection may cause considerable errors in setting up experiments in space and in the interpretation of their results. Hence, for meteorological studies in space, it is of importance to evaluate the Critical Marangoni number below which convection cannot occur and to suggest the mechanism to accelerate or suppress convection.

In small scale fluid mechanics, the fact that interfacial regions between fluid phases play in driving as well as impeding convection was observed for the first time by Block (1956). Pearson (1958) gave a detailed mathematical analysis for the onset of convection driven by surface tension gradients. Later, Sternling & Scriven (1959) and Scriven & Sternling (1960, 1964) have examined the onset of steady cellular convection driven by surface tension gradients as an extension of Pearson's (1958) stability analysis. Nield (1975) has examined the onset of transient convective instability driven by surface tension using the Galerkin method. But, the effect of Coriolis force on the surface tension driven convection has not been given much attention. Sarma (1979) has investigated the problem of thermo-capillary stability of a thin liquid layer heated uniformly from below and subjected to a rotation about the transverse axis. He has illustrated the vital role of the different boundary conditions and the destabilizing character of the long-wave disturbances at the fluid-fluid interface using a neutral stability curve based on analytical solutions of the pertinent eigenvalue problem. These results pertain to a basic uniform temperature

*Paper was presented in the symposium "Indo-French School on recent advances in Computer Techniques in Meteorology, Biomechanics and Applied Systems" held at IIT, New Delhi, 4-13 February 1980.

gradient. In meteorological problems, however, it is difficult to maintain a basic uniform temperature gradient. There is usually sudden heating or cooling giving rise to a non-uniform basic temperature gradient. Recently, Rudraiah (1982) has investigated the effect of non-uniform temperature gradient on surface tension driven convection with rotation about the vertical axis with the lower surface $z = 0$ in contact with a fixed rigid plane and the upper surface $z = d$ free. In meteorological problems it is well known that both the boundaries are free, where due to latent heat release there will be an increase in temperature at irregular intervals giving rise to a non-uniform temperature gradient. The case of increase in temperature at finite number of points along the basic state temperature profile would be of great physical interest in meteorological problems. Therefore, in this article we extend the analysis of Rudraiah (1982) to investigate the condition for the onset of convection driven by surface tension when both the boundaries are free and adiabatic. To achieve these objectives the plan of this paper is as follows.

The basic equations and the corresponding boundary conditions are discussed in section 2. A simple method to include the non-linear effects is also discussed in section 2. A condition for the onset of convection for various temperature profiles is derived in section 3 using a single term Galerkin expansion. Some important conclusions and the relevant physical interpretations are made in section 4.

2. Formulation of the problem

In this section, we consider the basic equations and the corresponding boundary conditions. For this, we consider an infinite homogeneous liquid layer of uniform thickness 'd' extending to infinity in the x-direction and rotating with a constant angular velocity $\vec{\Omega}$ about the z-axis which is transverse to the layer. The lower surface $z = 0$ and the upper surface $z = d$ are free. The only physical quantities that are assumed to vary within the fluid are the temperature, the surface tension, which is regarded as a function of temperature only, and the rate of heat loss from the surface, which is also a function of temperature only. The basic temperature profile is non-linear due to sudden heating (or cooling) at a boundary.

2.1. *Basic equations*

The basic equations of motion are :

$$\frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q}\cdot\nabla)\,\mathbf{q} = -\frac{1}{\rho}\,\nabla p + v\nabla^2\,\mathbf{q} - \frac{2}{\rho}\,\vec{\Omega}\times\mathbf{q} \tag{2.1}$$

$$\nabla\cdot\mathbf{q} = 0 \tag{2.2}$$

$$\frac{\partial\widetilde{T}}{\partial t} + (\mathbf{q}\cdot\nabla)\,\widetilde{T} = \kappa\nabla^2\,\widetilde{T} \tag{2.3}$$

where,

$\mathbf{q} = (u, v, w)$ is the velocity field,

$p =$ the total pressure,

$\widetilde{T} =$ the temperature,

$\rho =$ the density of the fluid,

$v = \mu/\rho$ is the kinematic viscosity of the fluid,

$\mu =$ the viscosity of the fluid,

$\kappa = k/\rho\, c_p$ is the thermal diffusivity,

$k =$ the thermal conductivity,

$\vec{\Omega} = (0,\,0,\,\Omega)$ is the uniform angular speed of the system,

$$\nabla \equiv \frac{\partial}{\partial x}\,\mathbf{i} + \frac{\partial}{\partial y}\,\mathbf{j} + \frac{\partial}{\partial z}\,\mathbf{k}\,,$$

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \tag{2.4}$$

$(\mathbf{i}, \mathbf{j}, \mathbf{k}) =$ unit vectors in the direction of the space variables (x, y, z).

2.2. *Boundary conditions*

The boundary conditions on velocity are obtained from mass balance, the no-slip condition and the stress principle of Cauchy. Since the layer is bounded by free surfaces, the boundary conditions on velocity can be obtained by equating the change of surface-traction due to the temperature variation across the surface to the shear stress experienced by the liquid at the free surface (Pearson 1958). By balancing the surface tension gradient with shear stress at the free surface, we have

$$\tau_{xz} = \frac{\partial\sigma_s}{\partial x} = \mu\,\frac{\partial u}{\partial z} \text{ and } \tau_{yz} = \frac{\partial\sigma_s}{\partial y} = \mu\,\frac{\partial v}{\partial z}$$

where σ_s is the surface-tension, τ_{xz} and τ_{yz} are the shear stresses.

By proper differentiation and using (2.2), we get

$$\frac{\partial^2\sigma_s}{\partial x^2} + \frac{\partial^2\sigma_s}{\partial y^2} = \mu\,\frac{\partial^2 w}{\partial z^2} \tag{2.5}$$

Following Pearson (1958), we can assume that σ_s can be expanded as the first order in powers of the temperature variation at the surface, in the form

$$\sigma_s = \sigma_o - \sigma_T T \tag{2.6}$$

where, σ_o is the unperturbed value of σ_s and

$$-\sigma_T = \left(\frac{\partial\sigma_s}{\partial T}\right)_{T = T_s}$$

For most liquids σ_T is positive, for, as the temperature rises, the difference between the liquid and its vapour phase decreases. Hence, the suitable boundary conditions at the free surface in the presence of surface tension is

$$w = 0 \text{ and } \mu\,\frac{\partial^2 w}{\partial z^2} = \sigma_T\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{2.7}$$

which in the non-dimensional case takes the form

$$w = 0 \text{ and } \frac{\partial^2 w}{\partial z^2} = M_a\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{2.8}$$

where,

$M_a = \dfrac{\sigma_T\triangle Td}{\mu\kappa}$ is the Marangoni number.

In the absence of variation of surface-tension with temperature the boundary conditions on velocity at the free-surface are

$$w = \frac{\partial^2 w}{\partial z^2} = 0 \text{ at } z = 0 \text{ and } d \qquad (2.9)$$

In the study of convection, the thermal conditions applied at the upper and lower surfaces of the fluid are based on the supposition that these surfaces are in contact with the materials of infinite thermal conductivity and heat capacity. For, the temperature at the surface is not perturbed when the quiescent state breaks down. A more general thermal boundary condition is

$$T = \lambda \frac{\partial T}{\partial z} \qquad (2.10)$$

where λ is a constant depending on the thermal properties of the boundary and the liquid. The extreme cases $\lambda = 0$ and $\lambda^{-1} = 0$ are limiting approximations for temperature perturbations to a very good and bad conductor respectively. In practice, these are referred to as the isothermal and adiabatic cases. Since the meteorological problems involve free surfaces, the actual physical situation, *viz.*, the heat exchange between the surface and environment, suggests that the standard thermal boundary condition of fixed temperature (*i.e.*, isothermal) may be too restrictive. In that case, adiabatic boundary conditions

$$\frac{\partial T}{\partial z} = 0 \text{ at } z = 0 \text{ and } d \qquad (2.11)$$

are more realistic. This is considered in this paper.

2.3. *Simplification of non-linear forces*

The equations of motion (2.1) to (2.4) are highly non-linear and hence the determination of solutions either analytically or numerically is very complicated. To understand the physical insight with reasonable mathematics, usually some assumptions are made. One of the assumptions is that the maximum temperature fluctuations from the mean must be small. In terms of the dimensionless parameters this amounts to saying that the deviation of the Critical Marangoni number M_{ac} from the Marangoni number is small (*i.e.*, $M_a - M_{ac} < 1$). This assumption implies that the non-linear terms $(\mathbf{q}.\nabla)\,\mathbf{q}$ and $(\mathbf{q}\cdot\nabla)\,\widetilde{T}$ in quations (2.1) and (2.3) can be divided into terms which are finite when averaged over a horizontal plane and into terms of zero average. To achieve this, we let

$$T^* = T(z) + T(x, y, z, t) \qquad (2.12)$$

If the bar over a quantity denotes the average over a horizontal plane $\left(= \frac{1}{41m} \int_{-l}^{l} \int_{-m}^{m} \{ \quad \} \, dxdy \right)$, then we have

$$\overline{T^*} = T(z), \ \overline{T(x, y, z, t)} = 0 \qquad (2.13)$$

Substituting (2.12) into (2.3) and dividing by (ρc); we get

$$\frac{\partial T}{\partial t} - \kappa \nabla^2 T - \kappa \frac{\partial^2 T(z)}{\partial z^2} = \beta W - (\mathbf{q}\cdot\nabla) T \quad (2.14)$$

where, $w = \mathbf{q}.\mathbf{k}$, $\beta = -\frac{\partial T(z)}{\partial z}$, is the negative vertical gradient of mean temperature. Taking the horizontal plane average of (2.14), we get

$$-\kappa \frac{\partial^2 T(z)}{\partial z^2} = \frac{\partial}{\partial z} \left(\overline{w\, T(z)} \right) \qquad (2.15)$$

which on integration yields

$$\kappa\beta + \overline{wT} = H \qquad (2.16)$$

where H is the vertical heat flux in the fluid.

Thus, taking the vertical average $\left(= \frac{1}{d} \int_0^d \{ \quad \} \, dz \right)$ of (2.16) we get

$$\kappa\beta_m + (\overline{wT})_m = H \qquad (2.17)$$

where the suffix m denotes the vertical average. From (2.16) and (2.17), we get

$$\frac{\beta}{\beta_m} = 1 + \frac{1}{\kappa\beta_m} \left[(\overline{wT})_m - (\overline{wT}) \right]. \qquad (2.18)$$

Eqn. (2.14), using (2.15) and (2.18), becomes

$$\frac{\partial T}{\partial t} - \kappa \nabla^2 T - \beta_m W = \frac{\left((\overline{wT})_m - \overline{wT} \right) w}{\kappa} - h \qquad (2.19)$$

where $h = (\mathbf{q}.\nabla)\, T - \frac{\partial}{\partial z} (\overline{wT})$ is the zero-average heat convection term.

Eliminating pressure p in (2.1), we get

$$\frac{\partial}{\partial t} (\nabla^2 w) + 2\,\Omega \frac{\partial \zeta}{\partial z} = \upsilon \nabla^4 w + L \qquad (2.20)$$

where,

$$L = \frac{\partial}{\partial z} \left[\frac{\partial}{\partial x} (\mathbf{q}.\nabla)\, u + \frac{\partial}{\partial z} (\mathbf{q}.\nabla)\, v \right] - \nabla_1^2\, \mathbf{q}.\nabla w$$

$$\nabla \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

and

$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$, is the vertical compenent of vorticity. From the first two equations of (2.1), we can get an equation relating w and ζ in the form

$$\frac{\partial \zeta}{\partial t} - \upsilon \nabla^2 \zeta - 2\,\Omega \frac{\partial w}{\partial z} = -Z \qquad (2.21)$$

where,

$$Z = \frac{\partial}{\partial x} (\mathbf{q}.\nabla)\, v - \frac{\partial}{\partial y} (\mathbf{q}.\nabla)\, u$$

is a zero-average non-linear term.

The local non-linear stability is usually investigated using the solutions of the form

$$w = \epsilon w_0 + \epsilon^2 w_1 + \epsilon^3 w_2 + \cdots\cdots,$$
$$v = \epsilon v_0 + \epsilon^2 v_1 + \epsilon^3 v_2 + \cdots\cdots,$$
$$u = \epsilon u_0 + \epsilon^2 u_1 + \epsilon^3 u_2 + \cdots\cdots,$$
$$T = \epsilon T_0 + \epsilon^2 T_1 + \epsilon^3 T_2 + \cdots\cdots,$$
$$M_a = M_{a_0} + \epsilon M_{a_1} + \epsilon^2 M_{a_2} \cdots\cdots,$$

where ϵ is a constant parameter statisfying the suitable boundary conditions. The first term in each of (2.22) corresponds to the linear stability analysis which is studied in the next section.

3. Condition for the onset of surface tension driven convection

The condition for the onset of convection can be determined using the linear stability analysis. This is connected with the solutions of the first-order equations in (2.22) where the amplitude varies exponentially with time. In other words, for first order solutions to be complete, it is necessary that the parameter ϵ in (2.22) must be proportional to the amplitude of the disturbance and this amplitude must be infinitesimal. Neglecting the non-linear terms in (2.19)-(2.21) and making the equations dimensionless using d as the length scale, d^2/κ as the time scale, κ/d as the velocity scale and $\frac{\mu\kappa}{\sigma_T d}$ as the temperature scale, we get

$$(\partial/\partial t - \nabla^2) T - M_a^{\frac{1}{2}} a f(z) w = 0 \quad (3.1)$$

$$(\sigma^{-1} \partial/\partial t - \nabla^2) \zeta - \tau \frac{\partial w}{\partial z} = 0 \quad (3.2)$$

$$(\sigma^{-1} \partial/\partial t - \nabla^2)^2 \nabla^2 w + \tau^2 \frac{\partial^2 w}{\partial z^2} = 0 \quad (3.3)$$

where,

$\tau^2 = 4\Omega^2 d^4/\nu^2$ is the Taylor number.

The dimensional temperature gradient $f(z)$ must satisfy

$$\int_0^1 f(z)\, dz = 1.$$

The scales for w and T have been chosen such that M_a appears symmetrically in (3.1) and in the boundary conditions (*see* below) rather than in just the energy equation or the boundary condtion. This choice enables us to establish a variational principle for the present set of equations and as Finlayson (1972) shows, this leads to the conclusion that the eigenvalue M_a is stationary in the Galerkin method which we shall apply below.

We now apply the Galerkin method as described by Rudraiah (1982). It is shown that the consideration of even a single term in the expansions of w and T would give an accurate estimate for the critical value of M_a in certain cases. In other words, we set $w = Aw_1$ and $T = BT_1$ where w_1 and T_1 are suitably chosen trial functions and A and B are arbitrary constants. We note that the presence of rotation sets up overstable motions only for small values of the Prandtl number σ (*see* Veronis 1966, Rudraiah & Rohini 1975, Rudraiah *et al.* 1982, Rudraiah and Friedrich 1981). For other values of Prandtl numbers, however, overstable motion is not possible and the principle of exchange of stabilities is valid, *i.e.*, marginal stability is valid. The present analysis deals with the marginal stability. The marginal stability solution is the one for which the time derivatives in the differential equations (3.1) to (3.3) are zero.

Assuming the solutions for w and T in the form

$$f(z)\, e^{i(lx+my)}$$

equations (3.1) and (3.3) take the form

$$(D^2 - a^2)^3 w + \tau^2 D^2 w = 0, \quad (3.4)$$

$$(D^2 - a^2) T + a M_a^{\frac{1}{2}} f(z) w = 0 \quad (3.5)$$

where,

$$D = \frac{d}{dz} \text{ and } a^2 = l^2 + m^2.$$

The boundary conditions for free surfaces with temperature dependent surface tension, each subject to a constant heat flux, are

$$w = D^2 w + M_a^{\frac{1}{2}} aT = DT = 0 \text{ at } z = 0, 1 \quad (3.6)$$

Multiplication of (3.4) by w and (3.5) by T and integration of the resulting equation by parts with respect to z (from 0 to 1) yields, after making use of the boundary conditions, the following :

$$aM_a^{\frac{1}{2}} \left\{ D^3 w(1) T(1) - D^3 w(0) T(0) \right\} - \left\{ D^4 w(1) T(1) - D^4 w(0) T(0) \right\} \times 3a^3 M_a^{\frac{1}{2}} \left\{ D w(1) T(1) - D w(0) T(0) \right\} = - \left\langle (D^3 w)^2 + 3a^2 (D^2 w)^2 + (3a^4 + \tau^2)(D w)^2 + a^6 w^2 \right\rangle \quad (3.7)$$

$$M_a^{\frac{1}{2}} a \left\langle f(z)\, wT \right\rangle = \left\langle (DT)^2 + a^2 T^2 \right\rangle \quad (3.8)$$

where the angle bracket $< >$ denotes integration with respect to z from 0 to 1.

Substituting $w = Aw_1$ and $T = BT_1$ into (3.7) and (3.8), eliminating A and B and dropping the suffixes, we get

$$M_a = \left\{ \left[\left\langle (D^3 w)^2 + 3a^2 (D^2 w)^2 + (3a^4 + \tau^2) \times (Dw)^2 + a^6 w^2 \right\rangle \left(D w(1)\, D^4 w(1) - D w(0)\, D^2 w(0) \right) \right] \left\langle (DT)^2 + a^2 T^2 \right\rangle \right\} \Big/ \left[a^2 \left\{ D^3 w(1) T(1) - D^3 w(0) T(0) \right\} \times \left\langle f(z)\, wT \right\rangle + 3a^2 D w(1) T(1) - D w(0) T(0) \right] \quad (3.9)$$

We select the trial functions as

$$w = z^2 (1 - z^2) \text{ and } T = 1 \quad (3.10)$$

so that they satisfy all the boundary conditions except the one given by $D^2w + M_a^{\frac{1}{2}}aT = 0$ at $z = 0$, 1 and a residual from this equation is included in a residual from the differential equation. The term on the left hand side of (3.7) represents this residual.

Substituting (3.10) into (3.9) we get

$$M_a = \frac{4x^3 + 198x^2 + 7938x + 66\tau^2 + 37800}{945\,(x+4)\left\langle f(z)\,(z^2 - z^4)\right\rangle} \qquad (3.11)$$

where, $x = a^2$.

For any given $f(z)$, M_a attains its minimum when $a_c^2 = x_c$, where x_c satisfies the equation

$$8x^3 + 246x^2 + 1584x - 66\tau^2 - 6048 = 0 \qquad (3.12)$$

This cubic equation for wave number which determines the nature of the cells has one real root and two complex roots. The real root is the critical wave number which increases with increase in τ^2. Therefore, the effect of Coriolis force is to contract the cells.

3.1. *Onset of convection for various temperature profiles*

Case 1 — Uniform temperature gradient

For uniform temperature gradient that is for the linear basic temperature profile $f(z) = 1$, Eqn. (3.11) takes the form

$$M_a = \frac{(2x^3 + 99x^2 + 3969x + 33\tau^2 + 18900)}{63\,(x+4)} \qquad (3.13)$$

The critical wave number and the corresponding Marangoni number denoted by $(M_{ac})_1$ vary with the Taylor number as shown in the table of section IV.

Case 2 — Piecewise basic temperature profile for heating from below

When the layer of liquid is heated from below at a constant rate, we know (Nield 1975) that the non-uniform basic temperature gradient $f(z)$ is not only non-negative but also decreases monotonically. Thus, we are interested in knowing which temperature profile gives the least M_{ac} subject to $f(z) \geqslant 0$. Recently, in the absence of Coriolis force, Nield (1975) demonstrated that the piecewise linear profile with $f(z)$ given by

$$f(z) = \begin{cases} 1/\epsilon \text{ for } 0 \leqslant z < \epsilon \\ 0 \quad \text{for } \epsilon < z \leqslant 1 \end{cases} \qquad (3.14)$$

is the appropriate one. Even in the presence of Coriolis force, we can demonstrate that this piecewise linear profile given by (3.14) with ϵ suitably chosen, is the appropriate one, at least for disturbances of small wave numbers.

Thus, for the bottom heating piecewise linear profile, substituting (3.14) into (3.11), we get

$$M_a = \frac{15\,(4x^3 + 198x^2 + 7938x + 66\tau^2 + 37800)}{945\,(x+4)\,(5\epsilon^2 - 3\epsilon^4)}\,. \qquad (3.15)$$

Then the Critical Marangoni number is given by

$$M_{ac} = \frac{2\triangle}{\text{Max}\,(5\epsilon^2 - 3\epsilon^4)} \qquad (3.16)$$

where, $\triangle = (M_{ac})_1$ for linear profile discussed above. But

$$\text{Max}\,(5\epsilon^2 - 3\epsilon^4) = 2.08333333.$$

Thus, as ϵ increases from 0 to 1, M_{ac} decreases from $+\infty$ to a minimum value of

$$(M_{ac})_2 = 0.96\triangle \qquad (3.17)$$

at $\epsilon \doteq 0.913$ and then increases to $\triangle$ at $\epsilon = 1$.

Case 3 — Piecewise basic temperature profile for cooling from above

When the layer of liquid is cooled from above at a constant rate, the temperature gradient is not only non-negative but also monotonically decreasing. In this case the piecewise linear profile is

$$f(z) = \begin{cases} 0 & 0 \leqslant z < 1 - \epsilon \\ \epsilon^{-1} & 1 - \epsilon < z \leqslant 1 \end{cases} \qquad (3.18)$$

Substituting this in (3.11), we get

$$M_a = \frac{2(2x^3 + 99x^2 + 3969x + 33\tau^2 + 18900)}{63\,(x+4)\,(-3\epsilon^4 + 15\epsilon^3 - 25\epsilon^2 + 15\epsilon)} \qquad (3.19)$$

Then the Critical Marangoni number is given by

$$(M_{ac})_3 = \frac{2\triangle}{\text{Max}(-3\epsilon^4 + 15\epsilon^3 - 25\epsilon^2 + 15\epsilon)}\,. \qquad (3.20)$$

But

$$\text{Max}\,(-3\epsilon^4 + 15\epsilon^3 - 25\epsilon^2 + 15\epsilon) = 2.939637 \text{ at } \epsilon = 0.482723$$

Then, $$(M_{ac})_3 = \triangle/1.47 \qquad (3.21)$$

As expected from the physical grounds, we see that cooling from above is more effective than heating from below in causing instability.

Case 4 — Parabolic basic temperature profile

In the absence of rotation Debler & Wolf (1970) have considered the problem with a parabolic disturbance in which the basic temperature gradient is zero at the lower boundary, for which $f(z) = 2z$. Even in the presence of Coriolis force, the parabolic basic temperature distribution leads to $f(z) = 2z$. In this case (3.11) takes the form :

$$(M_{ac})_4 = \triangle/1.25 \qquad (3.22)$$

Case 5 — Inverted parabolic temperature profile

For inverted parabolic profile $f(z) = 2(1 - z)$ (*see* Nield 1975, p. 448) Eqn. (3.11) takes the form :

$$(M_{ac})_5 = 1.333\,\triangle$$

As expected on physical grounds this is less destabilizing.

Case 6 — Step-function basic temperature profile

We consider the step-function profile in which the basic temperature drops suddenly by an amount $\triangle T$ at $z = \epsilon$, but is otherwise uniform, and is of the form

$$f(z) = \delta(z - \epsilon)$$

TABLE 1

Values of the Critical Marangoni and wave numbers for various values of Taylor number

τ^2	a_c	$(M_{ac})_1$	$(M_{ac})_2$	$(M_{ac})_3$	$(M_{ac})_4$	$(M_{ac})_5$	$(M_{ac})_6$
0	1.625	71.967	69.088	48.957	57.574	95.932	38.383
10^{-1}	1.626	71.974	69.095	48.962	57.579	95.942	38.386
10^0	1.632	72.045	69.163	49.010	57.636	96.036	38.424
10^1	1.689	72.743	69.833	49.485	58.194	96.966	38.796
10^2	2.111	78.893	75.737	53.699	63.115	105.165	42.076
10^3	3.513	116.290	111.638	79.109	93.032	155.014	62.021

where, ϵ is value of z at which $\overline{wT}$ has a maximum and δ denotes the Dirac delta function. In this case (3.11) takes the form

$$M_a = \frac{4x^3 + 198x^2 + 1984x + 66\tau^2 + 9450}{18790\,(x+4)\,(\epsilon^2 - \epsilon^4)} \, .$$

Then the Critical Marangoni number is

$$M_{ac} = \frac{2\triangle}{15\ \mathrm{Max}\,(\epsilon^2 - \epsilon^4)}$$

which has a minimum value

$$(M_{ac})_6 = \frac{\triangle}{1.875} \text{ attained at } \epsilon = 0.707 \, .$$

Thus, the most unstable basic temperature profile, for which $f(z) \geqslant 0$ everywhere, is the step-function profile for which the step occurs at the level at which w is maximum, since $\overline{T}$ is constant in our approximation.

4. Conclusions

The single-term Galerkin method provides a quick means for obtaining the above results in the presence of Coriolis force with different basic temperature profiles. The results (3.13) and (3.17) give the critical wave-numbers and the corresponding Marangoni numbers which vary with the Taylor number. These are numerically evaluated for different values of τ^2 and the results are tabulated (Table 1).

When the basic temperature gradient is uniform, the condition for the onset of convection driven by surface tension when one boundary is rigid and the other free was investigated by Sarma (1979) in the presence of Coriolis force. He has obtained exact analytical solutions which are mathematically cumbersome and the Critical Marangoni number for different values of τ^2 are obtained from them. Recently, Rudraiah (1982) discussed convection driven by surface tension with non-uniform temperature gradient in the presence of Coriolis force when one of the boundaries is rigid. However, in meteorological applications both boundaries, are usually free. Therefore, in this article the Critical Marangoni numbers are obtained when both boundaries are free using single term Galerkin expansion method and the results are shown in the Table 1. We found that $(M_{ac})_6 < (M_{ac})_3 < (M_{ac})_4 < (M_{ac})_2 < (M_{ac})_1 < (M_{ac})_5$. Thus, the most unstable basic temperature profile is the one for which the temperature gradient is the Dirac delta function. The physical reason is that at the level $z = \epsilon$, wT is a maximum. Since T is constant is our analysis w has to be maximum so that $Dw = 0$. Then from the equation of continuity (2.2) it follows that the horizontal component of velocity which is proportional to Dw is zero, *i.e.*, the velocity is vertical. Thus the entire convection will be transported upwards and hence the step-function profile is more unstable. The most stable one is inverted parabolic temperature profile.

Experimental work to confirm the present results is needed. We suggest that using a solution, such as sugar solution whose concentration acts as the diffusing quantity rather than heat, would be convenient to carry out the analysis, since the condition of constant mass flux could then be satisfied without any effort.

Acknowledgements

I am grateful to Prof. M.P. Singh of Indian Institute of Technology, New Delhi, for inviting me to give a lecture at the Indo-French School of Recent Advances in Computer Techniques in Meteorology etc. The financial support from the UGC under the DSA project is gratefully acknowledged.

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