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On certain aspects of estimation of areal rainfall

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सार — हाल ही में सभी जल स्रोतों के अन्वेषणों में संजाल अभिकल्पना ने महत्वपूर्ण स्थान प्राप्त कर लिया है। यद्यपि संजाल आयोजना के लिये बहुत सी तकनीकी उपलब्ध हैं, फिर भी इच्छित परिशुद्धता वाले प्रेक्षणों के संग्रह में बहुत सी कठिनाइयों का सामना करना पड़ता है। जल मौसमी चर का आकाशीय परिवर्तन एक महत्वपूर्ण तत्व है जिस पर पर्याप्त विचार करना होगा। इसके अतिरिक्त, सैद्धान्तिक विवेचना की आवश्यकतानुसार प्रेक्षणों के संग्रह में भी विभिन्न व्यावहारिक कठिनाइयां आती हैं। इन समस्याओं में पर्वतीय जलग्रहण क्षेत्र और वृद्धि कर देते हैं।

प्रस्तुत लेख में, कगन तकनीक (1966) का उपयोग करके वर्षण संजाल अभिकल्पना के अध्ययन का प्रयास किया गया है। यह लेख सैद्धान्तिक विवेचना से उद्भूत इच्छित संजाल के कार्यान्वयन में आने वाली विभिन्न व्यावहारिक क्षेत्रीय समस्याओं को भी उभारता है। महाराष्ट्र में तापी नदी की सहायक "पूर्णा जलग्रहण" का भी अध्ययन किया गया है।

ABSTRACT. Network design has of late assumed very great importance in all water resources investigations. Though many techniques are available for planning network for different hydrological parameters yet many difficulties are experienced in the collection of observations with desired accuracy. Spatial variation of the hydrological variable is an important factor, which has to be duly considered. In addition to this, there are various practical difficulties in collecting the observations as dictated by theoretical considerations. The mountaineous catchments further add to these problems.

In the present paper, an attempt has been made to study the precipitation network design using Kagan's technique (1966). It also highlights the various practical field problems involved in implementing the desired network from theoretical considerations. A case study has been done for 'Purna catchment', a tributary of river *Tapi* in Maharashtra.

1. Introduction

In designing a regional network one has to install a dense network of raingauges to adequately understand the variability of rainfall in time and space. The wide range of areal and temporal variability of rainfall makes the problem of network design more complicated for obtaining a universally satisfactory procedure. However, the optimum number of raingauges can only be determined after adequate sampling of areal and temporal variability of precipitation within the area. The results of the studies with high density of raingauge network can provide guidance on the gauge density for a representative sample needed to measure the precipitation with a desired level of accuracy.

1.2. The assessment of areal precipitation for operational purpose especially in the field of hydrological forecasting plays a crucial role in the accuracy of final forecast. Precipitation network design for a realistic estimation of areal rainfall can be carried out by using the principle of sampling. In the optimum design of such a network the only limitation would be economy but for operational purpose it becomes necessary to obtain the data with least possible delay and at the same time to be representative in character. Regression technique can be used to set up an operational network. The principle used here is that the areal average rainfall of a catchment can be represented by rainfall of a few representative stations by a regression equation of the form :

$$P_a = C + \sum_{i=1}^{n} A_i X_i$$

where $X_1, X_2, \ldots\ldots\ldots\ldots X_n$ are the rainfall recorded at '*n*' raingauge sites. $A_1, A_2, \ldots\ldots\ldots\ldots A_n$ are the regression coefficients and C is the intercept. It can be ensured that the rainfall of selected raingauges have the maximum Multiple Correlation Coefficient (MCC) with the areal rainfall and addition of a few more stations do not improve the MCC. Alternate key networks and the corresponding MCC can be worked out for different combination of raingauges. The advantage of such a study would be to obtain areal rainfall without waiting for receipts of data from all the stations.

In addition to the above, other technique which is based on estimating cross correlation between the recorded rainfall at different sites and also the mean distances between the sites can be employed in the determination of network design. In the present study the same has been applied for catchment of Purna, a atributary of river *Tapi*.

The atmospheric factors such as wind and ice deposits have considerable effect on the actual rainfall. Due to these factors, the recorded precipitation may be an overestimation or underestimation of the actual rainfall. Precipitation measurements are equally affected due to terrain-configuration and vegetation-cover. In actual practice, for raingauge network design in a mountaineous region, one has to duly consider both these factors simultaneously. Precipitation correlation coefficients and location indicators computed on the basis of a representative raingauge station characterise the precipitation structure. The correctness of the precipitation structure is justified by comparing the precipitation correlation coefficients between the representative raingauge station and the location indicator during the monsoon or non-monsoonal periods. In precipitation network design one has to take into consideration the exposure conditions and its accessibility. The density of the network in a hilly region is greater than in the plains because of abrupt changes in physiography with altitude. The base network should be distributed in such a way so as to provide a more or less even cover of the conditions of precipitation structure in every topographic zone ranging from foot hills upto the top of the mountain.

2. Methodology adopted

A network design for hydrological purposes would mean an organised system of observational raingauge stations for collection of systematic information of the hydrological variable. Such a network can be designed to get various objective, for example we can design a network of precipitation gauges to measure well suited estimation of areal rainfall under all conditions with a specified accuracy. The density of observations in the catchment will naturally vary with the diversity of the heterogeneity of the terrain and spatial variation of rainfall should be capable of being detected with the accuracy that is sufficient for the purpose under consideration.

When the mean rainfall is computed by simple arithmatic mean method, the optimum number of raingauges is obtained by the following equation (Qubeck 1965):

$$N = \left(\frac{c_v}{P} \right)^2 \tag{1}$$

where 'N' is optimum number of raingauges 'c_v' is the coefficient of rainfall variability values computed from the existing network and 'P' is desired degrees of percentage error in the estimation of basin rainfall. The drawbacks in applying the above expression for precipitation network design is that it gives an over estimate of raingauges which cannot be implemented at the cost of economy and yet the hydrologist is faced with another problem of deciding the location of additional raingauges which are randomly distributed.

These drawbacks are, however, removed in the cross correlation technique used in the present study. The spatial variability of rainfall can be expressed as :

$$\rho\,(d) = \rho\,(0)\, e^{-d/d_0} \tag{2}$$

where $\rho\,(d)$ is the correlation function of the distance between stations and the forms of the function depends on characteristics of the catchment area as well as accounts for the type of precipitation. $\rho(0)$ is the correlation corresponding to zero distance and has been computed graphically from the plot of mean distance against mean correlation coefficient, d_0 is the distance at which correlation is reduced to $1/e$ times the correlation corresponding to zero distance. Often the microclimatic irregularities and random error in the measurement of precipitation make $\rho(0)$ different from unity and variance of these random errors is given as :

$$\sigma_1^2 = \Big[1 - \rho\,(0) \Big]\, \sigma_n^2 \tag{3}$$

where σ_n is the variance of precipitation time series at a fixed point. The equation $\rho(0)$ and d_0 provide the basis for assessing the accuracy of a network.

2.1. *Criteria adopted in the estimation of accuracy of the areal rainfall*

Kagan (1966) expressed variance of the errors in the estimation of areal rainfall over the area 's' surrounding a particular raingauge as :

$$V = \sigma_n^2 \Big[1 - \rho\,(0) \Big] + 0{\cdot}23\, \sigma_n^2 \sqrt{s} \Big/ d_0$$

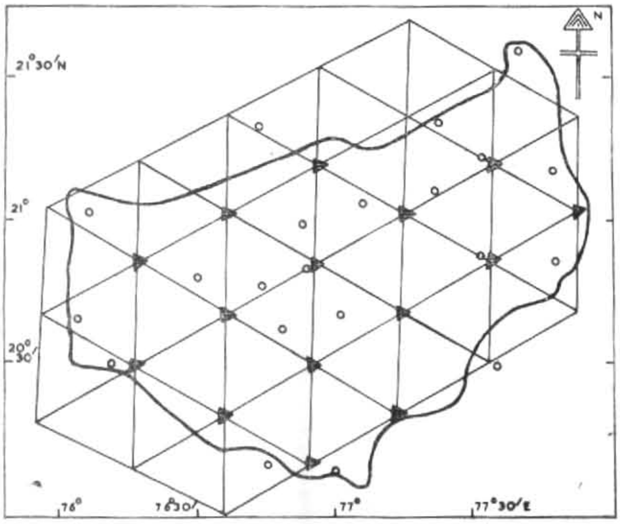

Fig. 1. Purna catchment

For total area S

$$V_n = \frac{\sigma_n^2}{n}\left[1 - \rho(0) + 0{\cdot}23\sqrt{s}\Big/ d_0\sqrt{n}\right]$$

where, $S = ns$

The relative root mean square error is then,

$$Z_1 = \frac{\sqrt{V_n}}{\bar{h}} = \frac{c_v}{n}\sqrt{1 - \rho(0) + \frac{0{\cdot}23\sqrt{S}}{d_0\sqrt{n}}} \tag{4}$$

where $c_v = \frac{\sigma_n}{\bar{h}}$ and $\bar{h}$ is the average precipitation over the area S.

The above equation can be used to obtain Z_1, for a given value of n, *i.e.*, the number of raingauges provided that $\rho(0)$ and d_0 are known. The permissible value of Z_1, *i.e.*, the desired percentage error in the estimation of areal rainfall can be stated and correspondingly one can obtain the optimum number of raingauges. If these raingauges are assumed to be uniformly distributed on the appexes of the triangular grid then the spacing 'L' between the gauges can be determined by using the expression :

$$L = 1{\cdot}07\left(\frac{S}{n}\right)^{\frac{1}{2}} \tag{5}$$

2.2. *Criteria adopted for the accuracy of spatial interpolation*

The relative error in the linear interpolation of data at the midpoint of the two raingauge stations separated by a distance 'd' as given by Drozdov and Seplevsky (1946) is :

$$Z_2 = c_v\sqrt{\frac{1}{2} + \rho(0) - 2\rho\left(\frac{d}{2}\right) + \frac{1}{2}\rho(d)}$$

The maximum error which occurs during the interpolation at the centroid of a triangle, at the appex of which raingauges are situated can be written as under:

$$Z_3 = c_v\sqrt{\frac{1}{3} + \rho(0) - 2\rho\left(\frac{d}{\sqrt{3}}\right) - \frac{2}{3}\rho(d)}$$

$$= c_v\sqrt{\frac{1}{3}\left[1 - \rho(0)\right] + 0{\cdot}49\frac{[\rho(0)]}{d_0}L}$$

$$= c_v\sqrt{\frac{1}{3}\left[1 - \rho(0) + 0.52\frac{\rho(0)}{d_0}\sqrt{\frac{S}{n}}\right.}$$

2.3. In actual practice the correlation ρ_{ij} are classified into intervals on the basis of the distance between the existing raingauge sites. The average distance and the average correlation for the stations falling within each interval is then plotted on a semilog graph paper. From the plotted values, the value of ρ_0 is obtained

TABLE 1

Mean distance v/s mean correlation

Distance class (km)	Mean distance (km)	No. of cases	Mean correlation Jun	Jul	Aug	Sep	Seasonal
0–20	18.30	7	0.8000	0.7900	0.8400	0.8400	0.7900
21 40	30.76	29	0.7062	0.6672	0.7668	0.7972	0.7413
41 60	51 03	35	0.6662	0.6091	0.7254	0.7542	0.7120
61–80	70.93	30	0.6380	0.5873	0.6750	0.7090	0.6950
81–100	91.90	29	0.6144	0.5510	0.6579	0.6300	0.6696
101–120	109.50	29	0.5832	0.5520	0.6031	0.5606	0.6224
121–140	131.28	25	0.5636	0.5380	0.6204	0.5464	0.6270
141–160	150.14	14	0.5260	0.4957	0.7370	0.4800	0.5600
161-180	170.77	9	0.5253	0.4877	0.6466	0.5610	0.5400
181–200	188.00	3	0.4466	0.5000	0.5533	0.4070	0.5000
201–220	203.00	1	0.4100	0.5200	0.5700	0.4300	0.5500

by extrapolating to zero distance and d_0 is obtained as the distance corresponding to $\rho(0)/e$, where, $e=2.7$.

3. Data used

Monthly rainfall data for the monsoon period from 1901-1950 was collected in respect of 21 raingauge stations lying within the Purna catchment.

4. Results and discussion

The existing raingauge network of the Purna catchment has been shown diagramatically in Fig. 1. Monthly rainfall data of 21 raingauge stations over a period of 50 years was used to obtain a correlation metrix. Mean distance between the raingauges and the corresponding mean correlation were computed for all the months and for the season as well. The results of these computations have been shown in Table 1. Mean distances were then plotted against mean correlations separately for each month and for the season on graph papers to extrapolate the corresponding estimates of the parameters $\rho(0)$ and d_0 and are shown in Table 2. As an illustration the computations of these parameters for the season, have been shown in Fig. 2. Having obtained these parameters, the relative errors of mean areal rainfall Z_1 and that of spatial interpolation Z_3 were calculated using relations (4) & (6). The uniform spacing of the raingauges has also been worked out and

TABLE 2

Computed parameters $\rho(0)$ and d_0

	$\rho(0)$	d_0 (km)	cv coefficient of variation
Jun	0.82	280.0	59.10
Jul	0.77	300.0	51.56
Aug	0.83	470.0	68.52
Sep	0.89	250.0	68.78
Seasonal	0.86	400.0	37.99

have been tabulated in Table 3. Table 3, which provides an estimate of the optimum number of raingauges together with their spacing can be used to set up an optional network design corresponding to any desired degree of percentage error in the estimation of basin rainfall. It is worth mentioning here that for an anticipated error of 7.65 per cent in the estimation of mean areal rainfall one may require 15 raingauges only and the proposed network design has been shown in Fig. 1. The installation of raingauges in accordance to the proposed network would be more economical, involve less man power for its maintenance without affecting the reliability of the areal estimate. The relative errors Z_1 & Z_3 for the season have also been plotted against the number of raingauges (Fig. 3.) It may be seen that the

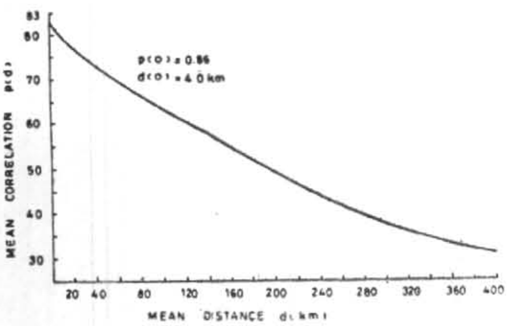

Fig. 2. Graphical representation for the computation of $\rho(0)$ and d_0

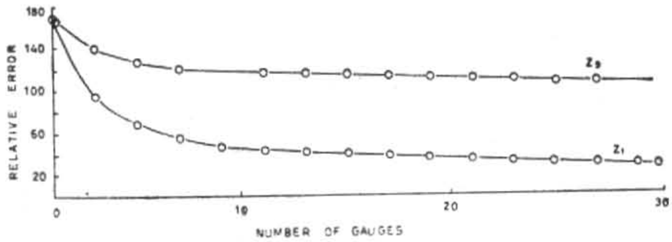

Fig. 3. Relative error of the mean areal rainfall and the spatial interpolation as a function of number of gauges

TABLE 3

Relative error of mean areal precipitation & relative error of spatial interpolation as a function of number of raingauges

Relative error of mean areal rainfall $Z_1(\%)$					No of raingauges	Relative error of the spatial interpolation $Z_3(\%)$					
Jun	Jul	Aug	Sep	Seasonal	77	Jun	Jul	Aug	Sep	Seasonal (°C)	Spacing in km.
31.92	29.81	33.33	33.35	17.73	1	30.51	26.16	29.15	36.92	16.90	145.54
16.87	16.04	18.06	16.95	9.43	3	25.00	21.92	24.53	29.32	13.89	84.03
12.67	12.13	13.70	12.53	7.10	5	23.04	20.42	22.93	26.54	12.82	65.09
10.53	12.12	11.44	10.31	5.91	7	21.93	19.60	22.02	24.94	12.22	55.01
9.18	8.84	10.01	8.93	5.16	9	21.18	19.04	21.42	23.85	11.81	48.51
8.24	7.95	9.01	7.97	4.60	11	20.63	18.64	20.98	23.04	11.51	43.88
7.53	7.28	8.25	7.26	4.23	13	20.20	18.32	20.64	22.41	11.28	40.36
6.97	6.75	7.65	6.70	3.92	15	19.86	18.07	20.37	21.89	11.09	37.58
6.52	6.32	7.17	6.25	3.57	17	19.57	17.86	20.14	21.46	10.94	35.30
6.15	5.95	6.77	5.88	3.46	19	19.32	17.66	19.95	21.07	10.81	33.39
5.83	5.66	6.42	5.56	3.28	21	19.11	17.52	19.78	20.78	10.69	31.76
5.56	5.39	6.13	5.29	3.13	23	18.93	17.39	19.64	20.50	10.59	30.36
5.32	5.16	5.87	5.05	2.99	25	18.77	17.27	19.51	20.25	10.50	29.11
4.92	4.78	5.43	4.66	2.77	27	18.49	17.07	19.29	19.82	10.35	27.03
5.11	4.96	5.64	4.85	2.88	29	18.62	17.77	19.40	20.02	19.43	28.01
4.75	4.62	5.25	4.50	2.68	31	18.37	16.98	19.20	19.64	10.29	26.14

TABLE 4

Monthwise and seasonal estimated optimum number of raingauges for anticipated areal rainfall errors

	$Z_1(\%)$	Optimum No. of gauges
June	6.97	72
July	6.75	58
August	7.65	80
September	6.70	105
Season	3.92	94

accuracy of spatial interpolation is more stringent criterion than the accuracy with which areal precipitation over an area can be obtained.

The formula $N = (c_v/P)^2$ was used to obtain the optimum number of raingauges corresponding to an anticipated error of 7.65 per cent in the estimation of mean areal rainfall for each month and for the season. The results have been enlisted in Table 4. It may be seen from the table that in order to design network with an anticipated error of 7.65% in the estimation of areal rainfall for Purna catchment one may require 80 raingauges. The above formula gives an exorbitantly overestimate of the required number of raingauges as compared to the number of raingauges (only 15) obtained by using crossed correlation technique which takes into account the location of raingauges as well.

5. Conclusions

(*i*) The study reveals that for computing the mean areal rainfall over any catchment area, the evaluation of spatial correlation function which accounts for the coefficient of rainfall variability and the spacing of the raingauges as well gives a better estimate of the network design than that provided by using the formula :

$$(c_v/P)^2 = N$$

(*ii*) In this formula the additional raingauges are located randomly and a lot of subjectivity is involved in deciding their locations. This subjectivity automatically gets diluted by designing the network on the basis of cross-correlation technique.

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