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A parametric wave prediction model based on time delay concept

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सार — इस शोधपत्र में मार्च 1986 में भारत के पश्चिमी तट के परे एकत्र किए गए तरंग आंकड़ों और टाइम सिरीज पवन के विश्लेषण पर आधारित अल्प से सामान्य सागर की स्थिति के लिए महत्वपूर्ण तरंग ऊंचाई (H_s) और शून्य अप-क्रासिंग अवधि(T_z)की प्रागुक्ति के लिए प्राचलिक पवन-समुद्र संबंधों को प्रस्तुत किया गया है। समुद्र विज्ञानीय अनुसन्धान पोत "गवेषणी" से डेटावेल तरंग आरोही प्लव के प्रस्तर से तरंग का माप लिया गया। प्रेक्षण, पवन गति 0 से 11.5 मी./से. तक समुद्र स्थिति की बढ़ती और क्षीण होती अवस्थाओं का क्रम दर्शाते है। प्रस्तावित प्राचलिक निदर्श की मुख्य विशेषता पवन अवधि सीमा के स्थान पर समय विलम्ब धारणा की शुरूआत है। पवन गति और तरंग ऊंचाई के मध्य 6 घण्टों की समय पश्यता (टाइम लैग) देखी गई है और उसे ही इस निदर्श में समाविष्ट किया गया है जिससे कि 6 घण्टेवार सिनॉप्टिक समय अन्तरालों में समुद्रों का पूर्वानुमान दिया जा सकता है। निदर्श की तुलना की गई है और H_s और T_z के प्रागुक्त मूल्य रिकार्ड किए गए तरंग प्रेक्षण के साथ काफी मिलते हैं। परिणाम यह दर्शाते है कि वर्तमान पद्धति प्रेक्षित H_s के लिए 0.12 मी. के मूल माध्य वर्ग अन्तर के साथ सार्थक तरंग ऊंचाई पूर्वानुमान 0.6 से 2.3 मी. तक के परास में किया जा सकता है।

ABSTRACT. Parametric wind-sea relationships for prediction of significant wave height (H_s) and zero-upcrossing period (T_z) for slight to moderate sea states have been presented in this paper based on the analysis of time series wind and wave data collected off west coast of India in March 1986. Wave measurements are made by deploying Datawell wave rider buoy from an oceanographic research vessel *Gaveshani*. Observations revealed a series of growing and decaying phases of sea state with wind speeds ranging from 0 to 11.5 m/s. The characteristic feature of the proposed parametric model is the introduction of 'Time-delay' concept in place of wind duration limit. A time-lag of 6 hr is noticed between wind speed and wave height and same has been incorporated in this model which enables forecasting seas at 6-hourly synoptic time intervals. Model comparison is made and the predicted values of H_s and T_z closely agree with the recorded wave observations. Results show that the present method yields significant wave height prediction with an r.m.s. difference of 0.12 m for the observed H_s ranging from 0.6 to 2.3 m.

1. Introduction

Knowledge about sea surface waves is very essential for many off-shore engineering works and as such structural design considerations require long-term wave statistics in shallow as well as in deep sea areas. The required information can be obtained by making wave measurements continuously over a period of 1 or 2 years at place(s) of interest or alternatively one can adopt wave prediction methods which in turn provide hindcast wave parameters from climatological wind inputs. Since the former is neither practicable always nor economical, the wave prediction models are put to use on many occasions for establishing design wave conditions. Several investigations have been made in the past on wave prediction aspects and currently available literature on this subject can broadly be divided into two types, namely, significant wave methods and spectral methods. As some examples of significant wave approach we cite the investigations of Sverdrup, Munk and Bretschneider (or in short SMB method); details of which are shown in the *Shore Protection Manual*, Vol I (1977) [Wilson (1955) and Darbyshire and Draper (1963)]. For wave spectrum method contributions of Pierson *et al.* (1955) and Hasselmann *et al.* (1976) may be cited. For better appreciation of the problem the works of Earle (1979) and Cardone and Ross (1979) may be referred to which give current state of art of wave modelling and many other relevant particulars.

The formulae or nomograms presented in significant as well as spectral methods of wave forecasting are based on field data evidences and hence, these methods are known as 'semi-empirical' or 'semi-theoretical'. Atleast for deep water conditions the wind-wave relationships established are expected to be same but in practice the empirical equations used in different wave models are found to yield varying results thus leading to some discrepancies. These discrepancies normally arise due to parametric constants and/or coefficients that are used in various models which are in turn dependent on the nature and quantum of data used. On the other hand, the available wave forecasting relationships are based on non-dimensional ratios involving wind speed (U); fetch (x) and wind duration (t), *viz.*, gx/U^2 and gt/U: where g is acceleration due to gravity. Estimation of x and t parameters often lead to some difficulties in real time applications and with the present state of knowledge, it is not always possible to quantify these two parameters to the required accuracy. The situation is worse when one

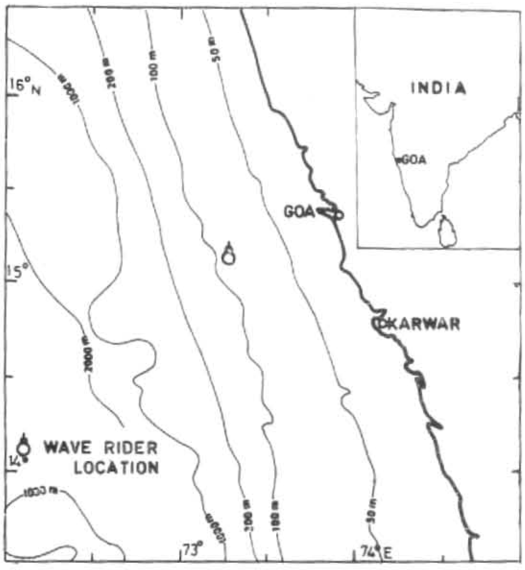


Fig. 1. Map showing location of waverider buoy observations

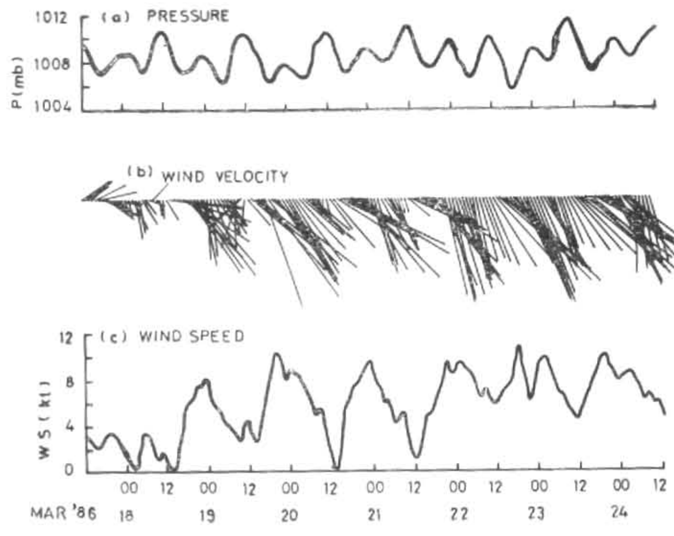


Fig. 2. Synoptic variability of met. parameters (a) atmospheric pressure, (b) stick plot depicting wind velocity and (c) wind speed

encounters fluctuating or varying winds on synoptic time scales in low wind speed regime during moderate sea state conditions. We, therefore, made some attempts in this study to introduce 'time-delay' concept, which is normally employed for communication engineering studies, in place of the usual 'duration limit' criteria. 'Time-delay' feature is rather justified for ocean waves since growth of wind waves does not take place instantaneously and certain time delay occurs between wind and wave evolution process. Utilising a week-long time-series data on wind and waves collected at sea, a set of empirical formulae have been derived for prediction of significant height (H_s) and zero-upcrossing period (T_z). A comparison of hindcast and actual wave observations is also presented in this paper.

2. Data source

Wave data utilised in this study are obtained with the help of 'Datawell' waverider buoy. Buoy mooring and retrieval operations are carried out from an oceanographic research vessel 'RV *Gaveshani*'. Real time wave data are recorded on board ship through telemetry and the ship is positioned in the close vicinity of buoy location. Marine meteorological data including wind velocities are gathered from standard equipment available on board ship. The location for waverider is shown in Fig. 1. The coordinates of buoy mooring position are 15° 08.6′ N and 073°16.0′ E. Water depth at this point is around 80 m. 1-hourly time series data are obtained between 17 and 24 March 1986.

3. Results and discussion

3.1. For the month of March the climatological data pertaining to the study area, *i.e.*, off Goa, shows that the mean wind direction is NNW and average wind speed is of 3 m/s (Hasternath and Lamb 1979). The observed synoptic wind field differs from the mean monthly conditions as far as wind speed variation is concerned though wind direction appeared to be more or less steady, *i.e.*, NNW. Fig. 2 presents a week long hourly synoptic weather picture recorded at the wave observation site. It is clear that wind direction is mostly NNW and wind speed varied from 0 (calm) to 11.5 m/s. The variation in atmospheric pressure (mb) shows a semi-diurnal pattern (2 cycles/day). Characteristically wind speed also registered a daily oscillation (diurnal cycle) with low values recorded during noon time and maximum around mid-night. During initial phase of observation, *i.e.*, 17 18th noon, conspicuously low wind speeds are noticed compared to the rest of the period.

3.2. Fig. 3 gives one hourly time-series variation of analysed wave parameters. Analog strip chart wave records of each 20 minutes length have been processed to obtain H_s and T_z following Tucker-Draper method (Tucker 1963, Draper 1967). Apart from it, the height of the highest wave (crest to trough vertical distance), H_{max} and corresponding period TH_{max} from each record have been estimated. At the outset it might be noted that a remarkable agreement prevails between H_s and H_{max} as well as T_z and TH_{max}. Statistical correlation for H_s *vs.* H_{max} is found very strong and the coefficient of correlation value is 0.98. Since H_s is statistical mean of the highest one-third number of waves and H_{max} is occurrence of a single event within the 20 minutes period (record length) the strong correlation between these two parameters emphasises the stationary random nature of wind waves (Longuet-Higgins 1952). Wave data used in this investigation mainly comprises sea states 2 and 3, *i.e.*, slight to moderate sea conditions. Analysis of data shows variation in H_s between 0.6 and 2.3 m and T_z from 3.7 to 6.2 s. From Fig. 3 it is seen that wave heights declined and periods increased during the first day of observations (17 18th noon). A steady growth of wave period is

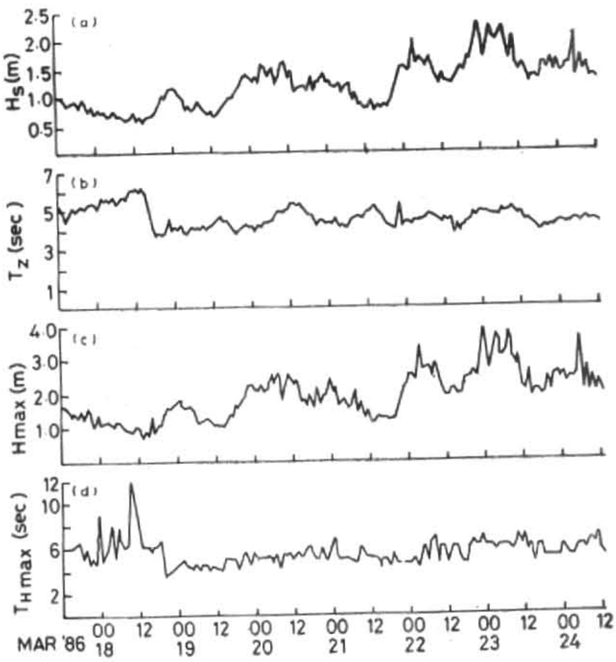


Fig. 3. One-hourly time-series of (a) Sig. wave height, (b) zero-crossing period, (c) maximum height wave and (d) period corresponding to the maximum wave

observed in the mean zero-upcrossing period, T_z, rather than for TH_{max}. The wind speeds are also very low during this period. The average wind speed is around 2 m/s for this period. Wind speed of this order could raise wave height (H_s) of about 0.2 m with a period (T_z) of 1.5 s or so. These are the maximum possible values of wind seas in a fully grown sea condition. On the contrary the observed H_s values are more than 0.5 m (up to 1.0 m) and T_z more than 5.0 s (up to 6.2 s) Therefore, it signifies 'swell' domination in the initial one-day observation period.

3.3. *Sea and swell composition*

Before we attempt to correlate wind and wave phenomena at any particular site it becomes essential first of all to differentiate sea and swell characteristics of measured wave data. It is necessary because swells have to be eliminated for detailed studies on wind wave mechanisms. To accomplish this task first we studied the variation of spectral width parameter (ϵ) derived through Tucker's formula :

$$\epsilon^2 = 1 - (N_z/N_c)^2 \tag{1}$$

where, N_z and N_c are the number of zero-up crossings and number of crests respectively for each wave record. This analysis showed ϵ values largely clustered between 0.8 and 0.9 which infers wideband spectral characteristics or predominant nature of the locally generated waves. Theoretically for a perfect swell, the number of zero-upcrossings will be equal to number of crests and ϵ becomes zero. But from ϵ distribution alone one cannot conclude the exact nature of prevailing sea state. We believe this parameter gives only a qualitative assessment of sea state composition and for better results alternate approaches have to be sought. We, therefore, chose to examine two non-dimensional wave parameters, namely, wave steepness (the ratio between H_s and wave length, L) and wave age (the ratio of wave speed, C to wind speed, U). For computation of L we used linear wave equation since water depth (d) at the observation site is sufficiently large ($d > L/2$) and in any case waves do not feel bottom.

3.4. The relationship between wave age (C/U) and wave steepness (H_s/L) is shown in Fig. 4. It may be seen that 'young waves' are steeper than 'old waves'. The thick curve in this figure pertains to the relation obtained by Sverdrup and Munk (1947). Thompson *et al.* (1984) in a recent investigation provide a classifiction for ocean waves like sea, young swell, mature swell and old swell depending on the significant wave steepness (H_s/L). According to this classification, those waves of which H_s/L ratio exceeds 0.025 can be considered as local 'seas' and waves falling within the steepness range of 0.01 to 0.025 may be treated as young swells. Majority of data used for this study fall under the first category (seas) with a few exceptions of young swells. From wave age point of view it is normally regarded $C/U \approx 1.37$ as the transition regime for wind waves and swells. Thus, we eliminated swells based on above criteria and incidentally the portion of waves that are eliminated in this study happened to be the observations taken between 17th and 18th.

3.5. *Time-delay approach*

Time histories of any two sets of time-series records can be tested to know their general dependence of values of one set of data on the other through cross-correlation function, R_{xy} (Bendat and Peirsol 1971). If $X(t)$ and $Y(t)$ are the given pair of time-series then R_{xy} can be obtained by :

$$\overline{R}_{xy}(\tau) = \text{Lim} \ \frac{1}{T} \int_0^T X(t)\, Y(t+\tau)\, dt \tag{2}$$

where, T is the total duration of record, τ is the time-lag and dt is sampling time interval. In the above equation summation (Σ) can be used for discrete time-series data sets. Values of R_{xy} for different time-lags can be evaluated by taking the average product of $X(t)$ and $Y(t)$. $\overline{R}_{xy}$ is always a real valued function possessing positive or negative value. Unlike the auto-correlation function, the cross-correlation function does not necessarily have a maximum value for $\overline{R}_{xy}$ at $\tau = 0$. Values of cross-correlation function ($\overline{R}_{xy}$) for wind speed and wave height are then computed following the above procedure by varying time-lag (τ) at hourly intervals between 0 and 48 hours. Results are shown in Fig. 5. The primary peak of R_{xy} is associated with 6-hour time-lag. A secondary peak is also seen around 28-hr time lag but this is less significant compared to the first one. Probably the secondary peak might have resulted due to the strong diurnal signal and the intra-diurnal variations persisting in the observed wind field. As we extend further the τ values beyond 48 hours the curve decays exponentially. Therefore, it appears the maximum correlation between wind and waves does not occur at the same time ($\tau=0$) and apparently wave process lags behind wind by about 6 hours. It is rather difficult to comment on the criticality of this factor at this juncture.

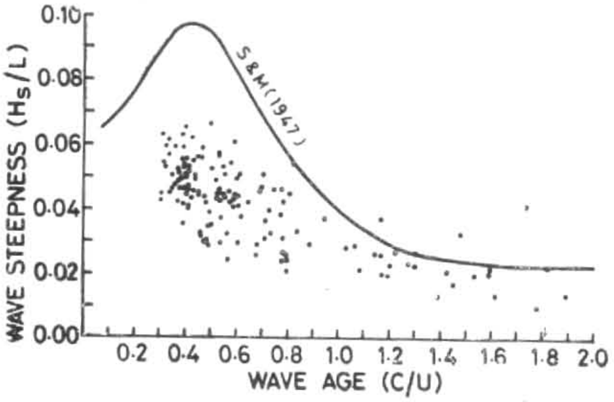


Fig. 4. Correlation between wave age and wave steepness. Curve shown is the relationship obtained by Sverdrup and Munk (1947). Points plotted indicate present observations

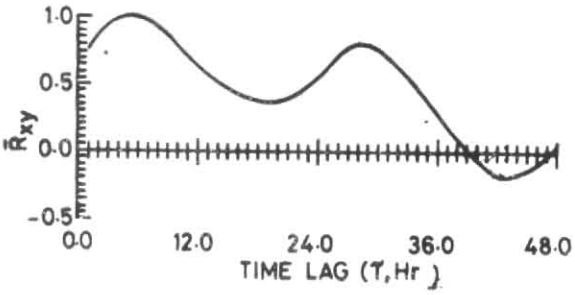


Fig. 5. Cross-correlation ($\overline{R_{xy}}$) between wind speed and wave height

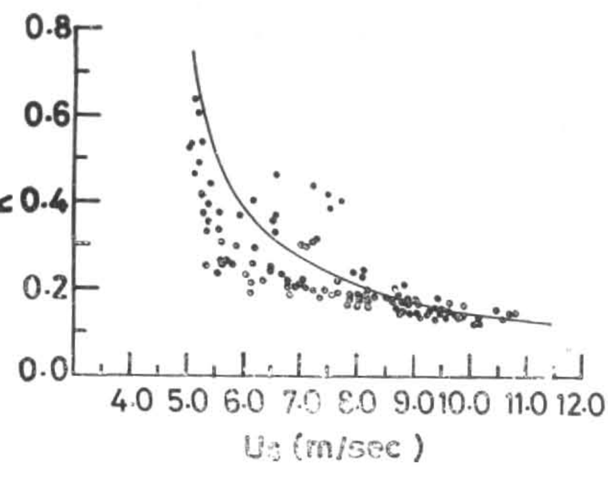


Fig. 6. Variation in K (Eqn. 3) with respect to wind speed (U_6) recorded 6 hr prior to wave observation

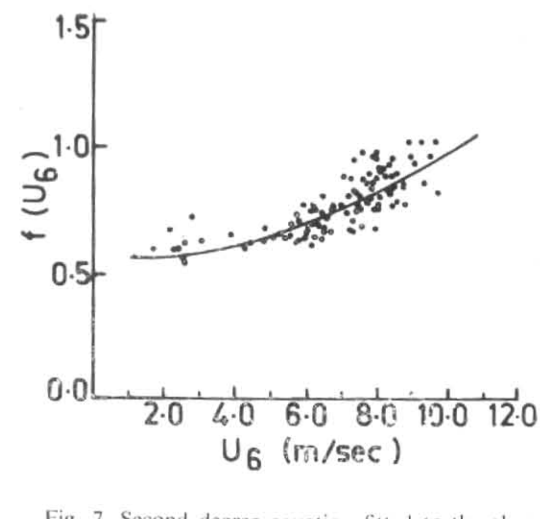


Fig. 7. Second degree equation fitted to the observed variability of U_6

This is mainly due to lack of further observational evidences with us. Another interesting feature which we found is that the correlation function does not show significant changes in the neighbourhood of 6-hr lag. It differs only marginally (variation in second decimal) for departure times up to 2 hours (6 ± 2). Thus the time-delay function exhibits certain amount of tolerance on either side of its peak. But further departure times beyond 2 hours could result in considerable change in R_{xy}.

3.6. *Model formulation and comparison with observed data*

We follow a parametric (significant wave) approach for wave prediction in this study. We prefer this technique mainly due to its simplicity and also it requires less computational effort compared to spectral method. Having obtained a time lag of 6 hours between wind and wave height (H_s); we make an assumption that the wave field at any given instant of time is a function of wind speed recorded at the time of wave observation (U_0) and the wind speed measured 6 hours prior to it (U_6). On the other hand wave height H_s, can be correlated with wind speed U, using a simple relation like :

$$H_s = KU^2/g \tag{3}$$

where g is acceleration due to gravity and K is a non-dimensional constant.

3.7. In Fig. 6, we show the variation of K for the present data set. For this purpose the observed significant wave height, H_s, and wind speed, U_6, are substituted in Eqn. (3) to derive values for K. We attribute the variation in K solely to the growing/decaying nature of the prevailing sea state. For fully developed sea K takes a maximum value of 0.283 (Kraus 1972). It is seen that K exponentially increases for low wind speed whereas it decreases below 0.283 on the higher wind speed side. $K < 0.283$ can be explained due to the non-fully grown nature of sea state. But during low wind velocities, especially for wind speeds 5 to 7 m/s, the predicted wave heights could be under-estimated even by using fully developed wave formula (Eqn. 3). The present model specially focusses attention on this aspect and anomalies of predicted and observed wave heights have been minimised following component and sensitivity analysis techniques. We write the significant wave height (H_s) as a product of two functions $f(U_0)$ and $f(U_6)$:

$$H_s = f(U_0).\, f(U_6) \tag{4}$$

Further we found that $f(U_0)$ and $f(U_6)$ can be best approximated as :

$$f(U_0) = U_0/(U_0^{1/2} + C) \tag{5}$$

$$f(U_6) = A + BU_6^2 \tag{6}$$

By combining Eqns. (5) and (6) and substituting in Eqn. (4) we get :

$$H_s = U_0(A + BU_6^2)/(U_0^{1/2} + C) \tag{7}$$

where, $A = 0.56$, $B = 0.0047$ and $C = 1.5$. In Eqn. (7), A and B are the arbitrary parameters which are obtained through least squares approximation. The functional relationship of $f(U_6)$ with U_6 is shown in Fig. 7 which exhibits a parabolic trend. The constant C is a non-linearity parameter and its value has been estimated by trial and error following numerical procedures.

3.8. Observed as well as predicted significant wave heights derived through Eqn. (7) have been compared and the results of one-hourly time-series between 18 and 24 March 1986 are shown in Fig. 8(a). Actual measured data are represented by thick curve and predicted values are shown with dashed curve. Overall very good agreement has been noticed and the predicted H_s differs only marginally with an r. m. s. error of 0.12 m for the range of significant wave heights recorded at the site.

3.9. In the case of wave period, the observed variation in T_z is small after excluding the initial one day data, *i.e.*, swells. The variation of T_z, essentially consisted of mean and random fluctuating component. Preliminary studies have indicated a positive correlation of T_z with the product of U_0 and U_6 when the high frequency variations are eliminated. Subsequently it is found that the fluctuating or high frequency component of T_z shows a fairly good response with the quotient of U_6 and U_0. Thus the structural form of T_z may be written as :

$$T_z = f(U_0\, U_6) + f(U_6^*/U_0) \tag{8}$$

$$\text{where,} \quad U_6^* = U_6 + U_0^{1/4} \tag{9}$$

In fact the composition of U_6^* is derived through significant analysis technique after having evaluated all the relevant parameters on the right hand side of Eqn. (8). The function $f(U_0U_6)$ is approximated as :

$$f(U_0U_6) = a + b\,(U_0U_6)^x \tag{10}$$

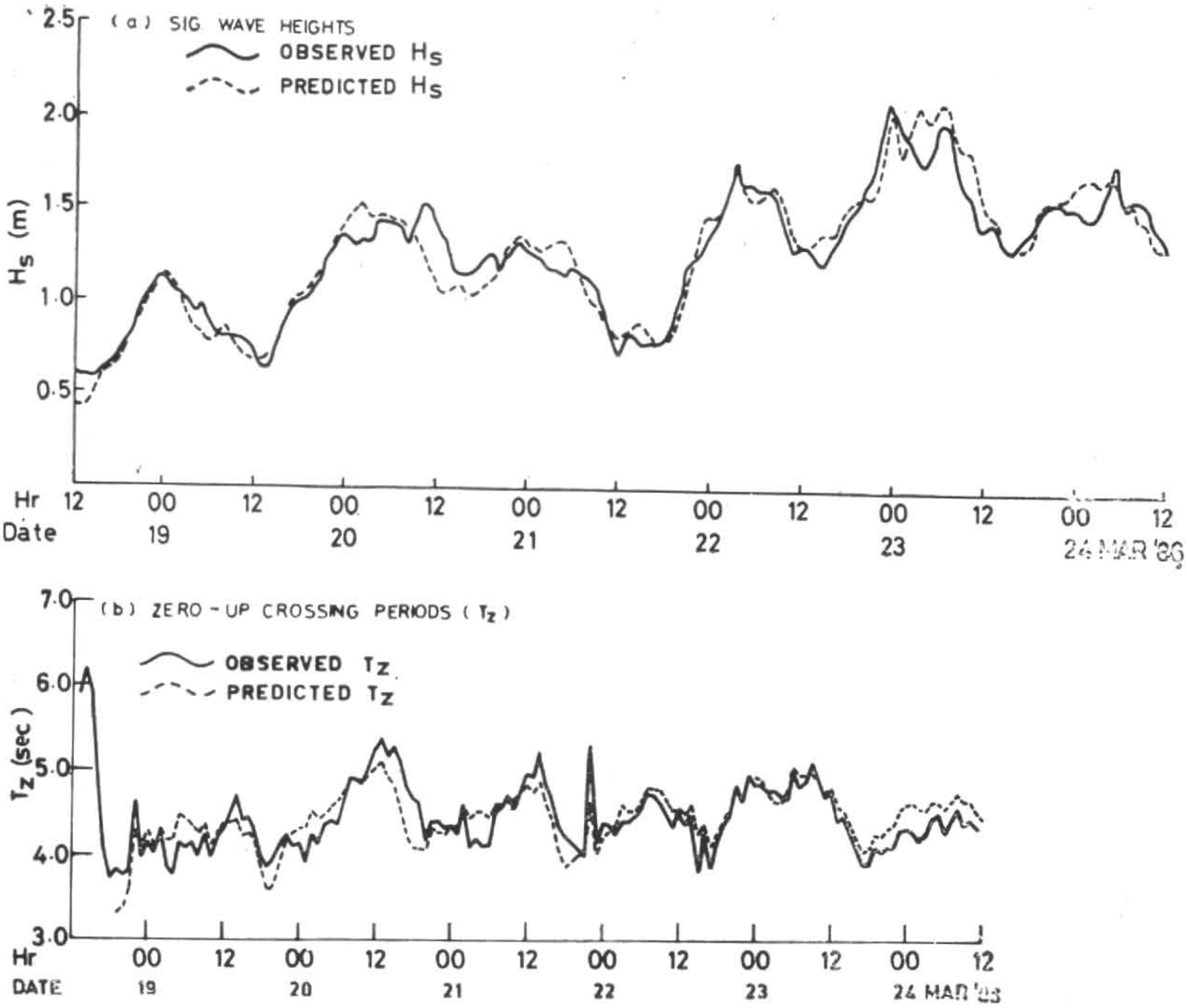
Non-linear least-square approximation is used to compute a and b parameters whereas power-law variation method is adopted to estimate numerical value for $x(=0.625)$. Variation of $f(U_0U_6)$ with U_0U_6 is shown in Fig. 9. This is the mean component of T_z which shows growth of wave period with the increase of wind speed.

3.10. The random or fluctuating component of T_z gives a logarithmic relation (*see* Fig. 10) which accounts for apparent period decrease during wave growth and increase in the decay phase. One probable reason for the random behaviour of wave period could be due to its slow response characteristic against wind speed variation or in other words we say that the hourly variation of wind speed might have not caused corresponding order of changes in T_z. Thus, the random component of T_z assumes importance for fluctuating winds associated with series of growth and decay of waves. Moreover the random component of T_z or $f(U_6^*/U_0)$ becomes zero for steady wind conditions, *i.e.*, $U_6 \simeq U_0$ and it takes positive values for decreasing and negative values for increasing wind speeds. We write this function as :

$$f(U_6^*/U_0) = \log_{10}(U_6^*/U_0)^{\lambda} \tag{11}$$

$$\text{where,} \quad \lambda = 2.25 + 0.0006 \log_{10}(U_6^*/U_0) \tag{12}$$

The second term of the right hand side of Eqn. (12) is quite small compared to the first term. Variation of λ with $\log_{10}(U_6^*/U_0)$ is shown in Fig. 11. From this



Figs. 8 (a&b). Comparison of (a) observed and predicted sig. wave heights and (b) observed and predicted zero-upcrossing periods (T_z)

figure it is obvious that λ may be taken as 2.25 after neglecting second term in Eqn. (12). It is at this stage U_6^* is estimated (Eqn. 9) following significant analysis method by varying a, b and λ independently such that the deviations are minimum between predicted and observed data. The term $U_0^{1/4}$ in Eqn. (9) is found important only when wind speed increases. Thus, by substituting Eqns. (10) and (11) in Eqn. (8), we get :

$$T_z = a + b\,(U_0\,U_6)^{5/8} + \log_{10}\left[\,(U_6 + U_0^{1/4})/U_0\,\right]^{9/4} \qquad (13)$$

where, $a = 3.7$ and $b = 0.102$. A comparison of observed and predicted values of T_z is given in Fig. 8(b). The agreement seems to be reasonably good and the increasing and decreasing trends in wave period are well represented by the prediction curve (dotted curve in Fig. 8 b). It may, however, be noted that on 18th the observed T_z values are quite high and discrepancy between predicted and observed data is strikingly significant. This, perhaps, might have arisen because of the influence of swells in the initial stage of observations.

4. Summary and conclusions

Parametric wind sea formulae for significant wave height, H_s and zero-upcrossing period, T_z, have been

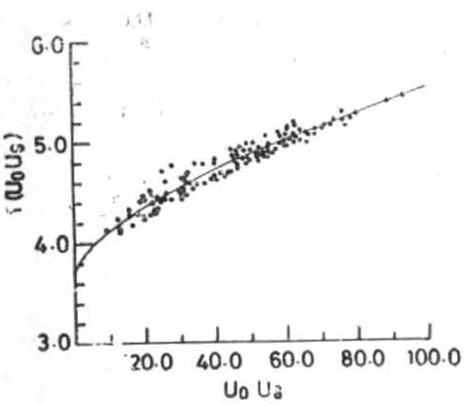


Fig. 9. Functional relationship assumed for the variation in wave period (T_z)—mean component

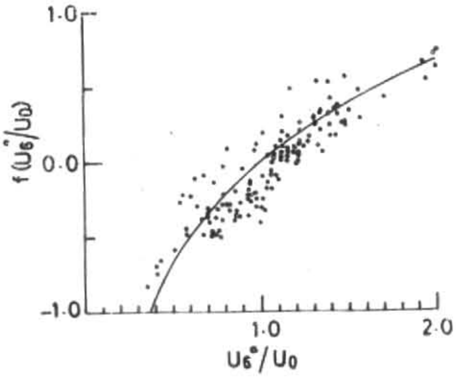


Fig. 10. Same as in Fig. 9 except that it is for random or fluctuating component

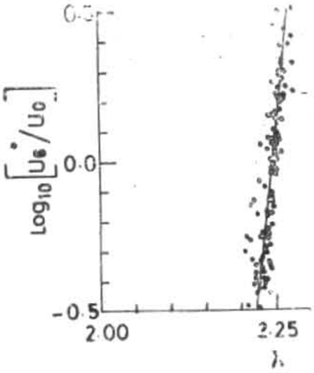


Fig. 11. Power law approximation (λ) used in T_z formulation

presented in this paper based on the analysis of time-series wind and wave data. The observations made at 1-hr synoptic time interval have indicated successive growth and decay phases of wind-sea evolution process. Sufficient care has been taken to exclude swells while formulating wind-wave relatonships. The duration limit normally used in wave forecasting is replaced with time-delay criteria. For this, our argument is that in moderate sea state conditions estimation of wind duration becomes rather difficult due to fluctuating nature of winds. In such circumstances time-delay approach is more convincing. At early stages of wave growth or in slight to moderate sea states it is found that the wave heights are under-estimated by using available formulae especially in low wind speed regime. A factor which is responsible for it could be our poor understanding of wave generation mechanism. With this background we made a few attempts in this study to correlate wind and wave phenomena following statistical and empirical methods. The cross-correlation ($\overline{R}_{xy}$) between wind speed and wave height has yielded a time-lag of about 6 hr. The prediction relationships for H_s and T_z are then derived using input of wind speed at the time of prediction (U_0) and wind speed prevailing 6 hr prior to it (U_6). Fetch and wind duration parameters are thus eliminated which simplifies the wave forecasting procedure. Generally for moderate sea states in open sea environment fetch becomes less significant compared to wind duration and latter is taken care through the introduction of time-delay in our model. The validity of 6 hr time lag for sea state beyond moderate sea conditions (rough, very rough etc) has to be studied further. As far as the limitations of this study are concerned, like many other wave prediction formulae available in literature, the present wave forecasting relations are also based on empirical considerations and the model parameters/arbitrary constants are derived using observations. Therefore, from theoretical point of view these prediction formulae do not have analytical value and are not valid from dimensional considerations. But in the absence of theoretical knowledge, the only alternative at present seems to be either empirical or semi-empirical approach for solving the wave prediction problem.

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