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A stochastic model of drought

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सार — इस लेख में विविध मौसम विज्ञानी उपमण्डलों तथा विभिन्न जलवायु क्षेत्रों मे होने वाली प्रतिदिन की वर्षा की श्रेणी पर प्रसम्भाव्य प्रक्रम के लागू होने पर विचार किया गया है। विभिन्न अवधियों में न्यूनतम वर्षा के सांख्यिकीय ढंग से रूपान्तरित बंटन द्वारा आकलित स्वसहसंबंधों के साथ न्यूनतम *m*-दिवस वर्षा ज्ञात करने के लिए स्वसमाश्रयण माडल AR (1) का उपयोग किया जाता है। यह मॉडल विभिन्न अवधियों में प्रेक्षित न्यूनतम वर्षा की संतोषप्रद व्याख्या करता है। तत्पश्चात, कोई कितनी अच्छी तरह सूखा क्षेत्रों का निरूपण कर सकता है यह समझने के लिये विभिन्न अवधियों में न्यूनतम वर्षा की प्रायिकताओं का आकलन किया जाता है।

ABSTRACT. The paper deals with an application of stochastic process to daily rainfall series of various meteorological sub-divisions and of different climatic regimes. An Auto-regressive AR(1) model is used to generate *m*-day minimum rainfall with the auto-correlations estimated through statistically transformed distribution of minimum rainfall of different durations. The model statisfactorily explains the observed minimum rainfall of different durations. Further, the probabilities of minimum rainfall of different durations are estimated to understand how best one can delineate the areas of droughts.

1. Introduction

Years 1965, 1972, 1979 are of large scale droughts in India and are related to large scale flow patterns in general circulation (Krishnamurty *et al.* 1981). However, droughts in India have generally been of a localised nature. Every year they occur in one part of the country or the other. WMO (1975) has enumerated definitions of droughts based on rainfall analysis and water budgeting technique. Much work on duration of weather has been undertaken on rainfall and drought (Palmer 1965; Feyerherm and Bark 1965). However, the present study based on stochastic process satisfactorily explains the observed minimum rainfall of various durations. Further, the probabilities of minimum rainfall of different durations are estimated to understand how best one can delineate the areas of droughts.

2. Method

A variable Y, $N(0, 1)$ is generated through a Markov chain as follows :

$$Y_0 = \eta_0$$

$$Y_i = \rho y_{i-1} + \sqrt{1-\rho^2}\,\eta_i, \quad i \geqslant 1 \tag{1}$$

where ρ is the auto-correlation between i^{th} and $(i-1)^{th}$ variable, η_i are random numbers with N (0, 1). Gringorten (1966) used Eqn. (1) to simulate probability distribution through Monte Carlo exercise. Fig. 1 is the outcome of such an exercise, *viz.*, it consists of a very large number of simulated time series, representing in length and other characteristics to a monthly series of hourly obervations. One single model is used for different meteorological variables and different stations by resorting to each specific time series on the basis of frequency distribution of hourly observations of the variables under consideration. With this, values of the meteorological variables were transformed into a statistical normal variate with mean zero and standard deviation unity N (0, 1). The duration of different time series was considered by allowing differences in serial correlation in both the actual and simulated series. The degree of dependence between successive observations largely determined the pattern of fluctuation within a time series and also the duration of persistence of high or low meteorological values. High values of correlation corresponds to a time series in which hour-to-hour variation was relatively small and the effect of random variation damped. A time series in which variations were more prominent yielded lower correlation coefficients. Under this circumstance, duration for the same value of a climatic variable would also be shorter, as illustrated for all levels of cumulative frequency P, as shown in Fig. 1.

3. Data and results

3.1. *Application of the model to tropical monsoon rainfall*

The data field consisted of 19 years (1955-1973) daily rainfall. The input to the model was the mean daily rainfall over a division covering monsoon period June to September. Fig. 2 gives the areal extent of divisions. The numbers in the figure represent meteorological divisions.

Cumulative frequency distributions of the mean daily rainfall for the four monsoon months for the 24 divisions were formed. Due to want of space diagram for one division, *i.e.*, for west Rajasthan is shown in Fig. 3. Further, from the mean daily rainfall, *m*-day

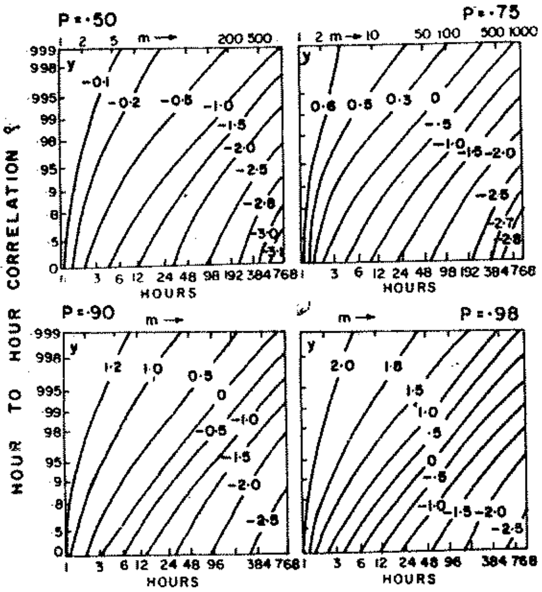

Fig. 1. The *P* percentile of the *m*-hour minima of *Y*, *N*(0, 1) or (1—*P*) percentile of the *m*-hour maxima of —*Y*, *N*(0, 1) (*after* Gringorten)

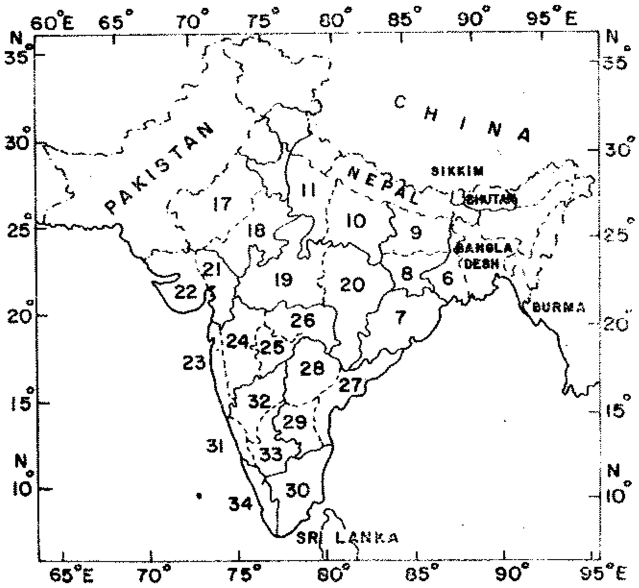

Fig. 2. Numbered sub-divisions show the areal coverage for the application of the stochastic model over India

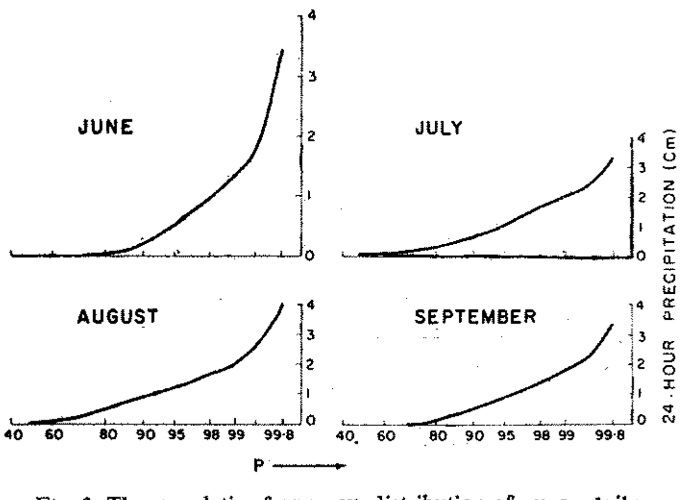

Fig. 3. The cumulative frequency distribution of mean daily precipitation amount over Rajasthan (west) based on 19 years (1955-73) of records

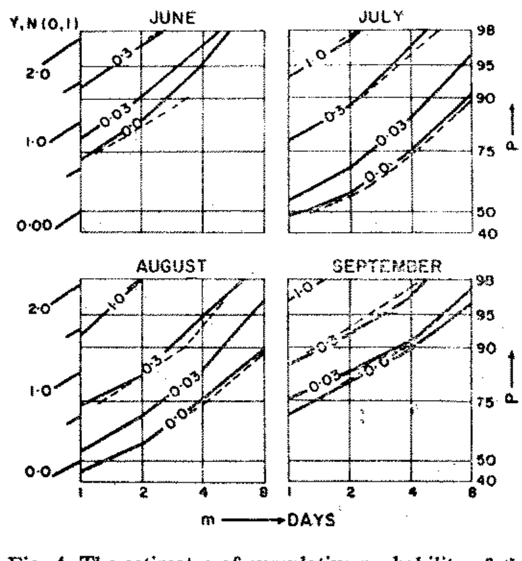

Fig. 4. The estimates of cumulative probability of the *m*-day minima of daily mean precipitation over Rajasthan (west) for the 4 monsoon months. The solid lines are direct estimates based on 19 years of data. The broken lines are the estimates from the model

TABLE 1

Percentage days with no measurable precipitation and probability of measurable precipitation in consecutive days taken at random over the Indian sub-divisions for the four monsoon months — June to September

Sub-division	Month	Percentage days with no measurable pptn.	Prob. of some measurable pptn. in consecutive days selected at random in 2 days	4 days	8 days	Sub-division	Month	Percentage days with no measurable pptn.	Prob. of some measurable pptn. in consecutive days selected at random in 2 days	4 days	8 days
Kerala (34)	Jun	5	91	84	74	Orissa (7)	Jun	21	68	54	32
	Jul	5	93	87	78		Jul	4	93	86	75
	Aug	14	80	70	55		Aug	4	93	87	75
	Sep	23	66	53	34		Sep	8	87	67	60
Konkan (23)	Do.	17	76	68	54	Coastal A.P. (27)	Do.	38	46	28	9
		1	97	95	91			20	67	48	26
		2	96	93	88			21	65	47	28
		18	72	62	44			24	61	42	23
Coastal Mysore (31)	Do.	8	89	84	75	Tamilnadu (30)	Do.	52	28	10	2
		1	98	97	95			38	44	22	6
		4	92	87	77			32	52	33	15
		28	64	50	34			37	46	25	6
Gujarat (21)	Do.	56	34	20	7	Rayalaseema (29)	Do.	52	18	10	0.7
		22	70	58	40			38	46	22	9
		24	68	57	40			42	42	26	10
		50	38	26	10			46	40	24	8
Saurashtra-Kutch (22)	Do.	63	27	15	6	Telangana (28)	Do.	38	44	24	7
		30	60	48	30			19	71	56	34
		38	50	36	22			20	68	50	30
		58	30	16	5			35	50	30	10
Rajasthan (West) (17)	Do.	80	10	3	0	Interior Mysore (North) (32)	Do.	32	56	32	12
		56	30	15	4			9	85	75	61
		55	30	15	4			15	76	63	45
		76	16	10	3			32	56	39	20
Rajasthan (East) (18)	Do.	60	26	15	5	Interior Mysore (South) (33)	Do.	36	43	22	8
		27	63	50	30			18	71	56	38
		22	69	57	40			24	64	48	29
		49	40	26	11			34	52	35	20
Uttar Pradesh (West) (11)	Do.	43	46	31	13	Madhya Maharashtra (24)	Do.	36	50	34	16
		14	81	71	54			15	80	68	51
		9	87	78	64			14	78	68	50
		35	55	40	22			28	60	42	22
Uttar Pradesh (East) (10)	Do.	48	40	23	3	Marathwada (25)	Do.	48	36	20	8
		19	73	60	42			30	60	40	20
		9	85	75	60			30	54	34	20
		32	58	43	21			42	44	26	10
Madhya Pradesh (West) (19)	Do.	33	56	42	26	Bihar Plateau (8)	Do.	30	55	40	20
		6	91	85	75			8	82	75	58
		7	89	84	74			9	85	74	58
		26	66	54	38			23	64	46	24
Madhya Pradesh (East) (20)	Do.	21	70	59	42	Bihar Plains (9)	Do.	34	54	35	15
		3	94	90	84			14	78	66	45
		3	95	91	85			10	81	70	52
		16	76	63	46			27	60	42	20
Gang. West Bengal (6)	Do.	23	68	46	28	Vidarbha (26)	Do.	34	54	37	20
		8	86	75	60			12	82	74	60
		6	89	79	62			10	83	72	54
		19	70	53	30			27	62	44	20

minimum, m varying from 1 to 8 days were computed. The same for west Rajasthan is shown in Fig. 4.

3.2. *Fitness of the model to rainfall*

Kamte *et al.* (1982) computed the average values of day to day correlation ($\bar{\rho}$) for west Rajasthan for the monsoon months. These values were referred to the model charts Fig. 1 at the key y values. These gave estimates of m-day minima, as plotted and joined by broken lines in Fig. 4. A similar procedure was followed for other divisions. The solid and broken lines almost coincided showing that the model is applicable to m-day minimum precipitation of various climatic regimes. As such no separate statistical test of goodness of fit was applied. Moreover, other research workers have concluded that higher order models do not contribute substantially to the explained variance of rainfall series over that explained by AR(1) process. Due to want of space the figures for other divisions are not presented.

3.3. *Probability of measurable precipitation*

Taking 2.5 mm as measurable precipitation, Fig. 4 gives no measurable precipitation in a day or the probability of measurable precipitation on consecutive days taken at random. For example in Fig. 4 for the Rajasthan (west) for the month of June, there is no measurable precipitation on 80% of days, or the probabilities of measurable precipitation in 2, 4 and 8 consecutive days taken at random are 10, 3 and 0 % respectively. A similar interpretation is given to the rest of months. Table 1 gives the spatial and temporal distribution of percentage of days with no measurable precipitation during summer monsoon over India based on a threshold value of 2.5 mm as measurable precipitation.

During the month of June, Gujarat, Saurashtra, Kutch, Rajasthan (east & west), Tamilnadu and Rayalaseema show more than 50% of days with no measurable precipitation.

For the months of July and August, all sub-divisions continue to show low percentage of days with no precipitation except for the sub-division of Rajasthan (west) which continue to show more than 50 % of days with no measurable precipitation.

During the month of September except for the regions of Gujarat, Saurashtra, Kutch and Rajasthan (east & west), rest of the divisions shows less than 50% days with no measurable precipitation.

Table 1 also shows the probability of measurable precipitation on 2, 4 and 8 consecutive days taken at random for the 24 sub-divisions over India. The main feature is that the probability of measurable precipitation on 8 consecutive days taken at random is less than 50% during all the monsoon months for Gujarat, Saurashtra, Kutch, Rajasthan (east & west), Coastal A.P., Tamilnadu, Telangana, Interior Mysore (S) and Marathwada sub-divisions.

4. Conclusions

(*i*) A stochastic model satisfactorily explains the observed minimum rainfall distribution.

(*ii*) Measurable precipitation (2.5 mm) probability clearly delineates the areas which are sensitive to droughts and areas which may seldom be affected by droughts.

(*iii*) The values in Table 1 reflect the continuous and discontinuous behaviour of the southwest monsoon over India.

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