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A Markov chain model for the probability of drought incidence in India

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सार — सूखे वाले वर्षों/बिना सूखे वाले वर्षों की प्रतिबंधित प्रायिकताओं के निर्धारण के लिये 1875 से 1974 तक की अवधि में ऋतुनिष्ट वर्षा विचलनों पर मार्कोव श्रृंखला विश्लेषण लागू किया गया है । अति आर्द्र, आर्द्र, अर्द्ध आर्द्र (शुष्क और आर्द्र), अर्द्ध-शुष्क और शुष्क आदि प्रत्येक जलवायु क्षेत्र से एक मौसम विज्ञानी उपमण्डल चुनकर यह कार्य किया गया है । एक बिना सूखे के वर्ष के बाद एक सूखे वाले वर्ष तथा उल्टे क्रम में जलवायु वृतों को अपेक्षित अवधि अभिकलित करके उस पर चर्चा की गई है ।

ABSTRACT. Seasonal rainfall departures for the period 1875-1974 have been subjected by Markov Chain analysis to determine the conditional probabilities of drought/non-drought years. This has been done by selecting one meteorological sub-division, from each of the climatic type, *viz.*, per-humid, humid, sub-humid (dry and moist), semi-arid and arid regions. Expected length of climatic cycles, that is, drought spell followed by a non-drought spell and *vice versa*, have also been computed and discussed.

1. Introduction

In the wake of disastrous drought that occurred during 1965-66 and 1971-73 in many parts of Africa and Asia, there has been an upsurge of interests in systematic study of drought. Drought is experienced with greater or less frequency in all climatic regimes. These marginal areas often called semi-arid are transitional zone between the truly arid and the moisture regime with more reliable precipitation. Whereas in extratropical region these marginal areas lie at the edges of a large sub-tropical high pressure cells, in tropics they are located within the monsoon circulation system. Distortion of monsoon pattern leads frequently to anomalous rainfall distribution with disastrous effects on the economy.

Foreshadowing drought is a rather complex problem as it involves identification of appropriate parameters which directly or indirectly govern the precipitation processes. Attempts have been made to forecast drought by a number of workers, but efforts in this direction have met with mixed success (Rao *et al.* 1973, Banerjee *et al.* 1976, Sen *et al.* 1978, Chowdhury *et al.* 1979, 1981 and Varma 1980 etc). An attempt has been made in this paper to study drought by Markov chain method. Bhargava *et al.* (1973) had earlier applied this technique to daily rainfall over Raipur district.

2. Basic data

For each of the climatic type, *viz.*, per-humid, humid, sub-humid (dry & moist) semi-arid and arid regions, one meteorological sub-division of the country has been selected in this study. Seasonal rainfall departures (June-September) for a hundred year period (1875-1974) have been examined. For any sub-division an year has been classified as a drought year when the seasonal rainfall is less than 75% of the normal. An year is treated as a non-drought year when the rainfall is greater than or equal to 75% of the normal. This mode of classification yield a sequence of drought and non-drought years which can be regarded as a two-state Markov chain.

3. Computational procedure

Let $F(D)$ and $F(N)$ be the respective frequency of drought and non-drought years and $F(D/N)$ represents frequency of a drought year provided previous year is a non-drought year. $F(D/D)$, $F(N/D)$ and $F(N/N)$ have similar meanings. From this, the following four conditional probabilities are possible:

Drought and non-drought years	*Frequency*	*Probability*
(1) An year of drought succeeding a non-drought year	$F(D/N)$	$P(D/N)$
(2) A drought year succeeding a drought year	$F(D/D)$	$P(D/D)$
(3) Non-drought year succeeding an year of drought	$F(N/D)$	$P(N/D)$
(4) Non-drought year succeeding a non-drought year	$F(N/N)$	$P(N/N)$

These probabilities can be computed as below :

$$P(D/N) = \frac{F(D/N)}{F(D/N) + F(D/D)} = \frac{F(D/N)}{F(D)} \quad (1)$$

$$P(N/N) = 1 - P(D/N) \quad (2)$$

$$P(N/D) = \frac{F(N/D)}{F(N/D) + F(N/N)} = \frac{F(N/D)}{F(N)} \quad (3)$$

$$P(D/D) = 1 - P(N/D) \quad (4)$$

The geometrical distribution model enables to examine the number of drought/non-drought spells and climatic cycles. These spells are the years of drought (failure) or non-drought (success). Hence, the number of failures (or successes) before the first success (or failure) which define the geometrical distributions gives the expected number of years as follows :

$$E(D) = \frac{1}{P(N/D)} \quad (5)$$

$$E(N) = \frac{1}{1 - P(N/N)} \quad (6)$$

The expected length of the climatic cycle $E(C)$, *i.e.*, a drought spell followed by non-drought spell or *vice versa* is given by :

$$E(C) = E(D) + E(N) \quad (7)$$

Distribution of number of success in a sequence of dependent Bernoulli trials is a characteristic feature of two state Markov chain. If D_n is the number of drought years then D_n is assymptotically normally distributed for large value of n. Moreover, since there is only one series for D_n the assymptotic formulae would be still valid. The mean and variance is, therefore, given by:

$$E(D_n) \simeq \frac{nP(D/N)}{1 - P(D/D) + P(D/N)} \quad (8)$$

$$V(D_n) \simeq [nP(D/N)\{1 - P(D/D)\}\{1 + P(D/D) - P(D/N)\}] / [1 - P(D/D) + P(D/N)] \quad (9)$$

In the following paragraphs the conditional probabilities have been obtained and discussed for Kerala (per-humid climate), Himachal Pradesh (humid type), Orissa (moist sub-humid regime), Vidarbha (dry sub-humid category), Telangana (semi-arid) and west Rajasthan (arid).

TABLE 1

Transitional probabilities (per cent)

Climatic type	Subdivision	$P(D/D)$	$P(N/D)$	$P(N/N)$	$P(D/N)$
Per-humid	Kerala	22	78	92	8
Humid	Himachal Pradesh	13	87	92	8
Sub-humid (moist)	Orissa	0	100	96	4
Sub-humid (dry)	Vidarbha	8	92	89	11
Semi-arid	Telangana	20	80	80	20
Arid	West Rajasthan	13	87	74	26

4. Results and discussion

4.1. *Conditional probability*

The conditional probability obtained from the first order Markov chain for each of the sub-division mentioned above is given in Table 1. Since $P(D/N)$ and $P(D/D)$ are complementary to each other and as also $P(N/N)$ and $P(N/D)$, only the probabilities $P(N/N)$ and $P(D/D)$ have been described.

$P(D/D)$ is the probability of drought year succeeding a drought year, but the probability of a year being drought is mostly observed to be low. The probability of a drought year succeeding a drought year in a century will be the product of the two, which would indeed be very low. Surprisingly, in the per-humid and humid regimes this probability is substantially large. The probability $P(D/D)$ is, however, zero at Orissa; it means that drought does not have any chance to repeat itself in the succeeding year over this region.

In contrast, the humid region of Himachal Pradesh has 13% probability of recurrence. In the dry sub-humid region the probability is still more, however, in the arid region of west Rajasthan it is somewhat less. The probability $P(D/N)$ is complementary to $P(D/D)$ as such wherever $P(D/D)$ is low, $P(D/N)$, *i.e.*, a non-drought year being followed by drought year would be high.

The probability of an year of good monsoon repeating itself in the succeeding year, *i.e.*, $P(N/N)$ is

TABLE 2

Length of observed and expected spells of drought and non-drought and the climatic cycles (years)

S. No.	Sub-division	Expected years of		Observed drought spell (years)				Drought		Non-drought		Climatic cycle	
		Drought	Non-drought	1	2	3	4	Observed mean	Expected spell	Observed mean	Expected spell	Observed length	Expected length
1	Kerala	9.3	90.7	5	2			1.28	1.28	11.4	13.0	12.7	14.3
2	Himachal Pradesh	8.4	91.6	6	1			1.14	1.14	11.7	13.7	12.8	14.2
3	Orissa	3.9	96.1	5				1.00	1.00	19.4	23.8	20.4	24.8
4	Vidarbha	18.6	81.4	10	1			1.09	1.09	8.2	8.8	9.3	9.9
5	Telangana	20.0	80.0	13	2	1		1.25	1.25	5.0	5.0	6.3	6.3
6	West Rajasthan	23.0	77.0	18	3			1.14	1.14	3.5	3.8	4.6	4.9

quite large in all climatic regions of the country. By and large this probability is more than 80% and in per-humid and humid regions it exceeds even 90%. In other words, in these regions, if one year has good monsoon rain, it is almost certain that in the following year also the rainfall would be abundant, correspondingly the probability $P(N/D)$, is small.

4.2. *Drought spells and length of climatic cycles*

Employing these transitional probabilities, expected length of drought and non-drought spells can be generated as shown above. The observed and expected lengths of spells of drought and non-drought and the climatic cycles are given in Table 2. As may be seen the variations in the lengths of non-drought spells are large for all climatic types. It is also seen that mean expected length of drought spell is nearly unity. The closeness between the observed and the expected drought length is rather remarkable.

Considerable variations are observed in the expected lengths of non-drought spells which varies from 23.8 years for Orissa to 5.0 at Telangana. A noteworthy feature is that the expected lengths are greater than the observed lengths at all regions except Telangana where they are equal. Observed lengths of climatic cycles fluctuated from 20.4 years at Orissa to 4.6 years at west Rajasthan, whereas the expected lengths for these two regions were respectively 24.8 years and 4.9 years. It may, however, be mentioned that since the non-drought cycles do not confirm to the geometrical model, as such the length of the climatic cycle does not have much significance.

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