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Application of Markov chain model in determining drought proneness

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सार — वर्षा वंटन का, विशेषकर उसके आर्द्र और शुष्क दौरों का, कृषि उत्पादन पर बहुत प्रबल प्रभाव पड़ता है। महाराष्ट्र की शुष्क कृषि पट्टी पर दक्षिण-पश्चिम मानसून के दौरान शुष्क और आर्द्र सप्ताहों के अनुक्रमों के मूल्यांकन हेतु प्रथम कोटि का द्विचरणी मार्कोव श्रृंखला मॉडल उपयोग में लाया गया है।

सूखे की प्रवृत्ति की श्रेणी मालूम करने के लिये इस मॉडल के प्राचलों पर आधारित एक सूचकांक विकसित किया गया है। इस सूचकांक का अनुसरण करते हुए महाराष्ट्र की शुष्क कृषि पट्टी को सूखे की प्रवृत्ति की विभिन्न श्रेणियों के अनुसार पांच क्षेत्रों में बांटा गया है।

ABSTRACT. Rainfall distribution especially run of wet and dry spells will have dominating effect on the crop yield. Two state Markov chain model of order one has been used to evaluate sequences of dry and wet weeks during southwest monsoon period over dry farming tract of Maharashtra.

An index based on parameters of this model has been developed to bring out the degree of drought proneness. Following this index dry farming tract of Maharashtra has been delineated into five zones of different degree of drought proneness.

1. Introduction

There is no universally agreed definition of drought although the word is associated with rainfall deficiencies over more or less prolonged period. There may be several kinds of drought such as atmospheric, hydrological and agricultural droughts. But in the context of agriculture, it is mainly concerned with inadequacy of rainfall to offset evaporation and transpiration which occurs in that season. In the low rainfall areas especially in tropics, the importance of rainfall overrides that of all other climatic factors which determine yield. In the dryland area of Maharashtra [which is defined as the area bounded by 40 cm and 100 cm annual isohyetes (Swaminathan 1970)] the rainfall season is mainly southwest monsoon from June to September. Hence, the yield in this area will depend on variation of rainfall in this season.

The new varieties of crops do not require more water than the old varieties but they require timely supply of adequate water. Fertiliser applications also may not increase water requirements of the crop very much but result in more production with a given quantity of water. The minimum amount of water required is mainly available from the distribution of the amount of rainfall which is important for good yield. Different units of time period are used for rainfall analysis. For agriculture, week may be nearer to the optimum length of time. Let the week with rainfall more than a minimum amount (threshold value) be considered as a wet week. If the number of wet weeks in a given period is more, the rainfall distribution will be good, even if the total seasonal rainfall is less. Thus the expected number of wet weeks can decide the crop potential of an area. Probability of sequences of wet weeks can tell us the adequacy of water and probability of sequences of dry weeks can tell us the recurrence of the risk of crop failure. The Markov chain-model can give the basic probable representation for the spells' distribution and goes further in making it possible to derive several other properties of rainfall occurrence patterns. This has been successfully used by many authors (Basu 1971, Gabriel and Neumann 1962, Victor & Sastry 1979), for the occurrence of daily rainfall sequences. Weekly rainfall sequences also can be well represented by the Markov chain model. Attempt has been made in this paper to use Markov chain model to evaluate sequences of wet weeks during southwest monsoon period over dry farming tract of Maharashtra so that it could be used for bringing out drought-proneness.

2. Data

For 12 districts that are located entirely in the dry farming tract of Maharashtra, daily rainfall data of (a) one station of each taluka for 7 districts and (b) one station in other five districts where talukawise data of long duration are not available have been used. Selected stations have been used for four districts partially covered by dry-farming tract. In all, data from 84 stations, during southwest monsoon period, from 23rd to 41st standard weeks have been used for this study. Daily data for all the available years ranging from 55 to 70 years are converted into weekly totals. The weeks are then classified as wet or dry weeks according to the rainfall amount exceeding or equal to the threshold and less than it.

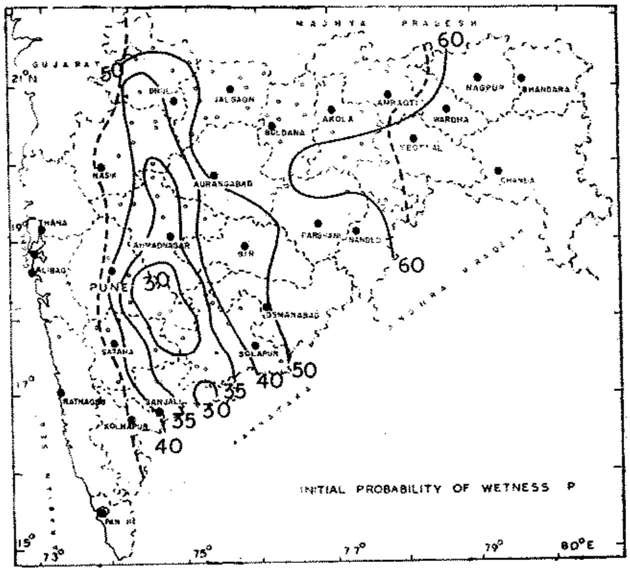

Fig. 1. Initial probability of wetness P

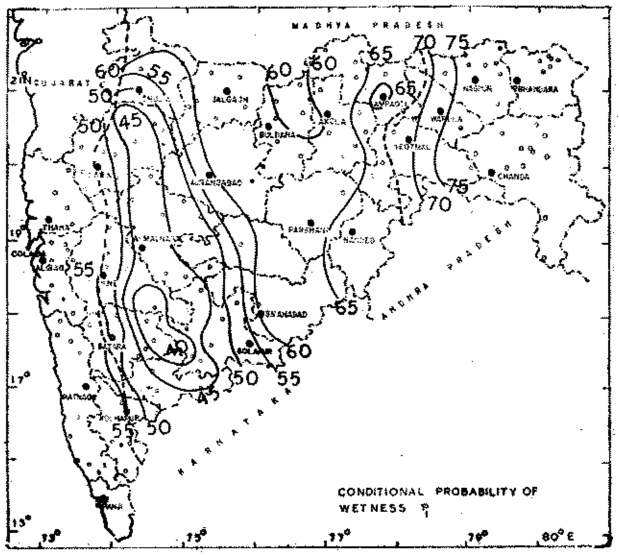

Fig. 2. Conditional probability of wetness P_1

TABLE 1

Station	Initial prob. of wetness P	Contd. prob. $W/W = P_1$	Contd prob. $W/D = P_0$	Exp. No. of wet weeks X_W	Prob. of getting wet weeks at least 6	8	10	12	Prob. of >3 consecutive dry weeks	Index of drought proneness DI
Sangli	.3793	.4784	.3219	9.1	.8869	.6664	.3632	.1271	.3118	26.8
Jath	.3312	.4389	.2794	8.3	.8212	.5557	.2578	.0749	.3742	20.2
Vita	.3734	.4414	.3342	8.4	.8389	.5636	.2546	.0233	.2951	24.7
Satara	.5699	.6569	.4576	12.5	.9949	.9616	.8340	.5753	.1596	69.0
Pusesawli	.3747	.4809	.3126	9.1	.8888	.6700	.3707	.0537	.3248	26.2
Mhaswad	.2755	.3689	.2413	7.0	.6628	.6591	.1056	.0183	.4367	13.4
Dahiwadi	.2938	.3989	.2514	7.6	.7357	.5596	.1635	.0367	.4195	15.7
Pune	.4609	.5453	.3922	10.4	.9572	.8264	.6591	.2578	.2245	41.4
Baramati	.2935	.3812	.2578	7.2	.6985	.3745	.1251	.0233	.4088	15.1
Indapur	.3238	.4278	.2759	8.1	.8023	.4801	.2296	.0618	.3797	19.1
Dhond	.3189	.4152	.2765	7.9	.7794	.4840	.1977	.0485	.3787	18.3
Nasik	.4659	.5535	.3914	10.5	.9616	.9319	.5793	.2810	.2254	42.4
Sinner	.4219	.4940	.3702	9.4	.9147	.7123	.4052	.1446	.2498	33.1
Malegaon	.3802	.4421	.3422	8.4	.8413	.5675	.2514	.0668	.2846	25.5
Dhule	.4569	.5200	.4037	9.9	.9441	.7794	.3121	.1922	.2120	39.8
Sakri	.3731	.4703	.3159	8.9	.8770	.6443	.3372	.1151	.3201	25.7
Sholapur	.4541	.5066	.4105	9.6	.9345	.7517	.4404	.1611	.2049	39.0
Sangola	.3341	.4044	.2996	7.7	.7611	.4483	.1635	.0344	.3436	19.3
Madha	.3971	.4471	.3654	8.5	.8554	.5832	.2611	.0681	.2556	28.0
Ahmednagar	.3887	.4702	.3390	7.4	.7122	.4013	.1401	.0287	.2889	27.7
Mirajgaon	.3718	.4352	.3351	7.1	.6736	.3446	.1038	.0170	.2939	24.3
Jalgaon	.5295	.6478	.3967	10.1	.9345	.7673	.5080	.2451	.2196	56.9
Buldhana	.5704	.6395	.4794	10.8	.9719	.8686	.6293	.3228	.1411	70.1
Akola	.5248	.6106	.4306	10.0	.9357	.7738	.4960	.2177	.1846	56.3
Amravati	.5789	.6610	.4682	11.0	.9719	.8749	.6480	.3520	.1504	71.9
Osmanabad	.5493	.6149	.4689	10.4	.9619	.8315	.5714	.2678	.1498	63.6
Bhir	.4537	.5308	.3897	8.6	.8531	.5987	.4801	.0885	.2272	39.5
Aurangabad	.5271	.6106	.4337	10.0	.9382	.6064	.5000	.3974	.1816	56.8
Parbhani	.5469	.6092	.4724	10.4	.9608	.8315	.5636	.2578	.1469	63.1
Nanded	.6049	.6925	.4733	11.5	.9803	.9049	.7126	.4247	.1461	79.5

3. Method

It is assumed that the probability of wetness of any week depends only on whether the previous week was wet or dry. Given the event on previous week, the probability of wetness is assumed independent of events of further preceding weeks. Such a probability model is referred to as Markov chain of order one. Parameters of this are two conditional probabilities P_0 & P_1. P_1 is the probability that a week is wet given that the previous week is wet and is denoted by :

$$P_1 = P\ (W/W) \tag{1}$$

P_0 is the probability that a week is wet given that the previous week is dry and is denoted by :

$$P_0 = P\ (W/D) \tag{2}$$

Given the Markov chain and estimates of its two basic probabilities, one can derive various properties of rainfall occurrence patterns where,

$$d = P_1 - P_0 \tag{3}$$

$$\text{and} \quad P = P_0/(1-d) \tag{4}$$

P is absolute probability of a week being wet and d measures the sense and magnitude of the first order effect.

It is seen from Eqns. (1), (2) and (4), if any two of the P, P_0, P_1 are known, the third can be estimated. The maximum likelihood estimates of P_1 & P_0 are the appropriate relative frequencies (Anderson and Goodman 1957). It is further assumed that P_0 & P_1 do not change from year to year.

A wet spell of length k is defined as sequence of k wet weeks preceded and followed by dry weeks. Dry spells are defined correspondingly. A wet spell of length k is thus equivalent, to recurrence time of $k+1$ weeks for dry weather. Correspondingly for dry spells it is equivalent to a recurrence of wet weeks. The distribution of the spells by lengths are found to be geometric (Gabriel & Neumann 1962) with probability of wet spell of length k being :

$$P_{(W=k)} = (1 - P_1)\ P_1^{(k-1)} \tag{5}$$

& probability of wet sequences with length greater than k is :

$$P_{(W>k)} = P_1^K \tag{6}$$

Probability of a dry spell of length m being :

$$P_{(D=m)} = P_0\ (1 - P_0)^{(m-1)} \tag{7}$$

& dry sequences greater than m being :

$$P_{(D>m)} = (1 - P_0)^m \tag{8}$$

The distribution of the number of wet weeks (X_w) among n-week period for large n tends to normality with mean and variance :

$$E\ (X_w) = n \times P \tag{9}$$

$$\text{Var}\ (X_w) = n \times P \times (1-P) \times \frac{1+d}{1-d} \tag{10}$$

where d & P are defined by Eqns. (3) & (4).

3.1. *Index of drought proneness*

P gives the initial probability of wetness. Wet sequences are given by P_1 and dry sequences are given by $1-P_0$. Hence the product of P & P_1 can indicate the probability of wetness and its continuation. This product divided by $1-P_0$ which ultimately gives the dry sequences, can give a measure of drought proneness.

Hence an index of drought proneness is defined as :

$$\text{DI} = \frac{P \times P_1}{1 - P_0} \times 100 \tag{11}$$

The extent of the drought proneness is given below :

Criteria	*Degree of drought proneness*
DI $\leqslant$ 20	Chronic
20 $<$ DI $\leqslant$ 35	Severe
35 $<$ DI $\leqslant$ 50	Moderate
50 $<$ DI $\leqslant$ 70	Mild
70 $<$ DI	Occasional

4. Results and discussion

The choice of the threshold value for Markov chain model is very much important especially when it is used for agricultural purposes. A small amount of water in dry region may be very much useful whereas the same amount may be insignificant in humid region. The requirement of water by crop depends on many factors. Literature available on this subject supports that crop yield especially in dryland tract, will not be affected adversely if plants get about 30%-70% of potential evapotranspiration (PE) depending on the growing stage of the crop. As weekly PE of the region under study is 30 mm to 35 mm, it will be realistic to take 18 mm as the threshold value for the period 23-41 weeks over the dry farming tract of Maharashtra.

Table 1 gives the values of P, P_1 & P_0 for the selected stations. Figs. 1 & 2 give the isolines based on P (initial probability) and P_1 (conditional probability) values respectively of all the stations. It is seen from the table that P_1 is always greater than P, *i.e.*, conditional probability is always higher than the initial probability which suggests that the effect of persistence is significant. Moreover, probability of wet week after a wet week is also higher than the probability of wet week after a dry week. Hence d which measures the sense and magnitude of the first order is always positive. Expected number of wet weeks are computed from Eqn. (9). Using Eqns. (9) & (10) and assuming distribution of wet weeks to be normal, the probabilities of getting more than 6 wet weeks, 8 wet weeks, 10 wet weeks and 12 wet weeks are computed. Experience shows that generally to raise a good crop,

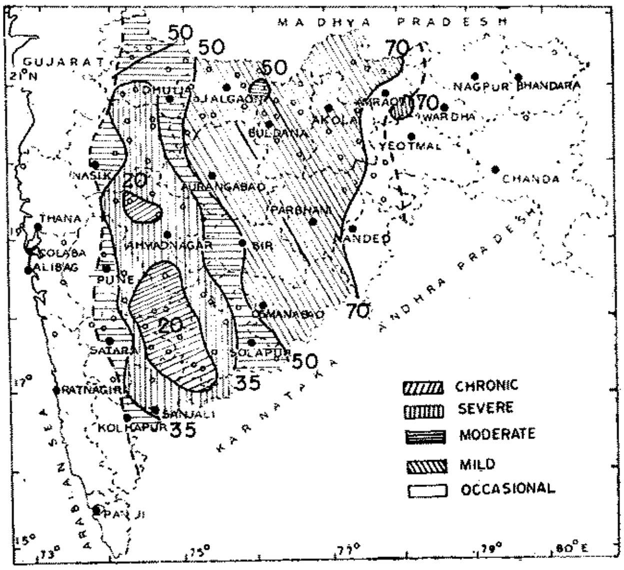

Fig.3. Drought proneness in dry farming tract of Maharashtra

10-12 wet weeks are necessary. If there is a continuous spell of at least 3 dry weeks in between the wet weeks, the crop cannot be sustained. Hence the probability of sequences of more than 3 dry weeks are computed from Eqn. (8). Demarcation of the areas with different degrees of drought proneness is given in Fig. 3.

4.1. *Chronic drought-prone area*

Drought index of the area is less than 20.

It consists of a part of Ahmednagar, Pune, Satara, Sangli and Sholapur districts. Once in three years, more than three consecutive dry weeks are expected in this area. Hence it is defined as a chronic drought-prone area. Expected number of wet weeks are 7-8. Rabi crops are grown generally in this area. Hence whereever soil is shallow, rabi crop is also risky. Moreover, early withdrawal of monsoon affects the crop severely. From Table 1 it is seen that more than 10 wet weeks are expected only once in four years. During this time some amount of ground water recharge may be there and crop prospects are high.

4.2. *Severely drought-prone area*

Drought prone index of this area is between 21 & 35.

This area consists of major parts of Nasik, Ahmednagar, Sangli and Sholapur districts, small strips of Satara and Pune and a part of Dhule, Aurangabad, Bhir and Osmanabad districts. Major part of this area is a rabi growing area. Drought risk is once in four years in this area. Though the expected number of wet weeks are 9-10, probability of sequences of more than three dry weeks is high. Therefore, this area is defined as severely drought prone. As it is expected to get more than 10 wet weeks once in 3 or 4 years, then some potential for ground water recharge is there and a good crop can be expected.

4.3. *Moderately drought-prone area*

Drought prone index of this area is between 36 & 50.

This area consists of parts of small strips of Sangli, Satara, Pune, Nasik, Dhule, Aurangabad, Bhir, Osmanabad and Sholapur districts. Drought risk in this area is once in five years. Expected number of wet weeks are 10. Kharif crops on shallow soil and rabi crops on deep soil are possible in this area. Number of wet weeks more than 10 are expected twice in five years. Hence a good crop may be expected during this period.

4.4. *Mild drought prone-area*

Drought prone index of this area is between 51 & 70.

Once in two years, more than 10 wet weeks are expected in this area. So a good potential of ground water recharge is expected. Good crop may be raised from this area at least once in two years.

4.5. *Occasional drought-prone area*

Drought prone index of this area is more than 70.

Once in seven years, the drought risk is there in this area. But more than 10 wet weeks can be expected twice in three years. Good potential of ground water recharge is also there. Hence this area is considered as occasionally drought-prone.

5. Conclusion

Markov chain model has been fitted to weekly rainfall totals to obtain sequences of dry and wet spells during the monsoon season. These sequences of dry and wet spells can be an aid to better understanding of drought-proneness. Different kinds of drought-proneness have been identified with the help of simple index. In chronic drought areas crop failure is very frequent. In severe drought-prone area a good crop may be raised in about 35% of the years. In moderately and mild drought prone area a good crop may be raised in about 40-50% and 50-55% of the years, and crop prospect is high in occasional drought prone area. Better results will emerge if atmospheric demand and soil characteristics of the region are superimposed on these results.

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