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Cyclone eye

AIR CMDE S. LAKSHMINARAYANAN
Air HQ, New Delhi

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सार — क्षैतिज-गति की गतिकी समीकरण से ऊर्ध्वाधर गति के लिए व्यंजक निकाला गया है और उसे प्रमेय रूप दिया गया है । इसका उपयोग करके तीव्र चक्रवात में नेत्रिका का अस्तित्व सिद्ध किया गया है ।

ABSTRACT. From the dynamical equation for horizontal motion, an expression for vertical motion is derived and cast in the form of a theorem. Using it, the eye is shown to occur when the cyclone is very intense.

1. Introduction

1.1. The innermost area of an intense tropical cyclone is free of cloud, surrounding which huge cloud formations with torrential rain are seen. This cloud free area in the interior of an intense cyclone is picturesquely called the 'cyclone eye'. Any cloud formation requires vertical upward motion. Absence of cloud in general cannot, however, be interpreted to mean existence of vertical downward motion, since cloud formation requires not only upward vertical motion but also sufficient amount of moisture. In the context of cyclone, the airmass is highly moist. Hence, it is realistic to assume that the cyclone eye is characterised by downward vertical motion surrounding which vertical motion is upward and that the eye boundary is the zero isopleth of vertical motion.

1.2. Unlike horizontal motion, atmospheric vertical motion is not generally determined with the help of instruments. Instead certain equations are used. Equation of continuity and omega equation are popular among meteorologists in the context of study of atmospheric vertical motion. A critical study shows that the present approaches are not adequate in explaining the eye and a *de novo* attack on the problem is desirable. Hence the objectives of this paper are (*i*) to enunciate a theorem on vertical motion from the equation of motion for horizontal wind vector and (*ii*) to show that the eye forms when the cyclone is very intense.

2. Vertical motion

The equation of motion for horizontal wind vector $\mathbf{V} = V(\cos\Psi\,\mathbf{i} + \sin\Psi\,\mathbf{j})$ is

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}.\nabla)\,\mathbf{V} + K\mathbf{V} + (\mathbf{k}\times\mathbf{V})\,f + w\left(\frac{\partial \mathbf{V}}{\partial z} + \mathbf{i}\,f_1\right) = -\frac{1}{\rho}\nabla p \qquad (1)$$

where V is speed and Ψ is direction, K coefficient of friction, $f = 2\,\Omega \sin\Phi$ coriolis parameter, $f_1 = 2\,\Omega\cos\Phi$, w vertical motion, p pressure, ρ density, Φ latitude and Ω angular velocity of earth, **i, j, k** point east, north and upwards. To study vertical motion associated with a given horizontal motion, it is advantageous to get an expression explicitly in term of w.

2.1. *Theorem*

Given that the atmospheric motion equation satisfies Eqn. (1), *i.e.*,

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}.\nabla)\mathbf{V} + K\mathbf{V} + (\mathbf{k}\times\mathbf{V})f + w\left(\frac{\partial \mathbf{V}}{\partial z} + \mathbf{i}\,f_1\right) = -\frac{1}{\rho}\nabla p$$

the vertical motion w is given by

$$w = \frac{-\sqrt{\left(f + V\frac{\partial \Psi}{\partial s} + \frac{\partial \Psi}{\partial t}\right)^2 + \left(K + \frac{\partial V}{\partial s} + \frac{\partial \log V}{\partial t}\right)^2}\,\cos(\theta_{is} + \theta_1)}{\sqrt{\left(\frac{\partial \Psi}{\partial z}\right)^2 + \left(\frac{\partial \log V}{\partial z}\right)^2}\,\cos(\theta_2 + \theta_{is}) + \frac{f_1}{V}\cos(\theta_{is} - \Psi)} \qquad (2)$$

where,

$$\tan\ \theta_{is} = \frac{[(\mathbf{k}\times\nabla p)\times\mathbf{V}]\,.\mathbf{k}}{[(\mathbf{k}\times\nabla p)\,.\,\mathbf{V}]} = \frac{\left(-\dfrac{\partial p}{\partial s}\right)}{\left(-\dfrac{\partial p}{\partial n}\right)}$$

$$\tan\ \theta_1 = \frac{\left[\mathbf{V}\times\left\{\dfrac{\partial\mathbf{V}}{\partial t}+(\mathbf{V}.\nabla)\,\mathbf{V}+K\,\mathbf{V}+(\mathbf{k}\times\mathbf{V})\,f\right\}\right].\mathbf{k}}{\left[\mathbf{V}\cdot\left\{\dfrac{\partial\mathbf{V}}{\partial t}+(\mathbf{V}.\nabla)\,\mathbf{V}+K\,\mathbf{V}+(\mathbf{k}\times\mathbf{V})\,f\right\}\right]} = \frac{\left(f+V\dfrac{\partial\Psi}{\partial s}+\dfrac{\partial\Psi}{\partial t}\right)}{\left(K+\dfrac{\partial V}{\partial s}+\dfrac{\partial\log V}{\partial t}\right)}$$

$$\tan\ \theta_2 = \frac{\left[\mathbf{V}\times\dfrac{\partial\mathbf{V}}{\partial z}\right].\mathbf{k}}{\left[\mathbf{V}\,.\,\dfrac{\partial\mathbf{V}}{\partial z}\right]} = \frac{\left(\dfrac{\partial\Psi}{\partial z}\right)}{\left(\dfrac{\partial\log V}{\partial z}\right)}$$

$\dfrac{\partial}{\partial s}$ = partial differentiation along streamline

$\dfrac{\partial}{\partial n}$ = partial differentiation along normal to streamline

2.2. *Proof*

We split the terms along the streamline (s) and normal (n) to it.

$$\frac{\partial\mathbf{V}}{\partial t} = \frac{\partial\log V}{\partial t}\mathbf{V} + \frac{\partial\Psi}{\partial t}(\mathbf{k}\times\mathbf{V})$$

$$(\mathbf{V}.\nabla)\,\mathbf{V} = V\frac{\partial\log V}{\partial s}\mathbf{V} + V\frac{\partial\Psi}{\partial s}\,(\mathbf{k}\times\mathbf{V})$$

$$w\frac{\partial\mathbf{V}}{\partial z} = w\frac{\partial\log V}{\partial z}\mathbf{V} + w\frac{\partial\Psi}{\partial z}(\mathbf{k}\times\mathbf{V})$$

$$w\,f_1\,\mathbf{i} = \frac{w\,f_1}{V}\cos\Psi\,\mathbf{V} - \frac{w\,f_1}{V}\sin\Psi\,(\mathbf{k}\times\mathbf{V})$$

$$-\frac{1}{\rho}\nabla p = -\frac{1}{\rho}\frac{\partial p}{\partial s}\frac{\mathbf{V}}{V} - \frac{1}{\rho}\frac{\partial p}{\partial n}\frac{(\mathbf{k}\times\mathbf{V})}{V}$$

Hence,

$$\left(\frac{\partial\log V}{\partial t}+\frac{\partial V}{\partial s}+K\right)+w\,V\left(\frac{\partial\log V}{\partial z}+\frac{f_1}{V}\cos\Psi\right) = -\frac{1}{\rho}\frac{\partial p}{\partial s} \quad \text{(1a)}$$

$$V\left(\frac{\partial\Psi}{\partial t}+V\frac{\partial\Psi}{\partial s}+f\right)+w\,V\left(\frac{\partial\Psi}{\partial z}-\frac{f_1}{V}\sin\Psi\right) = -\frac{1}{\rho}\frac{\partial p}{\partial n} \quad \text{(1b)}$$

Multiplying (1a) by $\left(-\dfrac{\partial p}{\partial n}\right)$ and (1b) by $\left(-\dfrac{\partial p}{\partial s}\right)$ and subtracting,

$$\sqrt{\left(f+V\frac{\partial\Psi}{\partial s}+\frac{\partial\Psi}{\partial t}\right)^2+\left(K+\frac{\partial V}{\partial s}+\frac{\partial\log V}{\partial t}\right)^2}\ \cos(\theta_{is}+\theta_1) =$$

$$= -w\left[\sqrt{\left(\frac{\partial\Psi}{\partial z}\right)^2+\left(\frac{\partial\log V}{\partial z}\right)^2}\cos(\theta_{is}+\theta_2)+\frac{f_1}{V}\cos(\theta_{is}-\Psi)\right]$$

Note :

(*i*) All angles, *viz.*, Ψ, θ_{is}, θ_1 and θ_2 are reckoned positive counterclockwise.

(*ii*) Isobars are the same as the $(\mathbf{k} \times \nabla p)$ vectorlines. θ_{is} is the angle between the isobar/$(\mathbf{k} \times \nabla p)$ vectorline and streamline as illustrated.

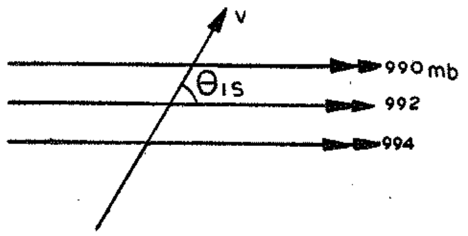

Fig. 1

Single barb indicates streamline and double-barbed lines indicate $(\mathbf{k} \times \nabla p)$ lines which are the same as isobars.

θ_{is}, the angle between $(\mathbf{k} \times \nabla p)$ and $\mathbf{V}$ is reckoned positive counterclockwise.

$(0 < \theta_{is} < 180) \Rightarrow$ cross-isobaric flow into low pressure

$(180 < \theta_{is} < 360) \Rightarrow$ cross-isobaric flow into high pressure

$(\theta_{is} = 0$ or $\theta_{is}=180) \Rightarrow$ isobaric flow, *i.e.*, isobars, and streamlines are parallel.

(*iii*) $(\theta_{is} + \theta_1)$ is the angle between isobar and the vector $\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}.\nabla)\,\mathbf{V} + K\,\mathbf{V} + (\mathbf{k} \times \mathbf{V})\,f$.

Likewise $(\theta_{is} + \theta_2)$ is the angle between isobar and $\frac{\partial \mathbf{V}}{\partial z}$

We reckon these angles in two stages, *i.e.*, from isobar to the streamline and streamline to the respective vector line for purposes of analytical clarity.

(*iv*) The theorem transforms the equation of motion for horizontal wind vector from a vector form into a scalar form explicitly in term of w. Hence, wherever the equation of motion holds good, the theorem also holds good. If horizontal motion and its derivatives, *viz.*, V, $\frac{\partial \Psi}{\partial t}$, $\frac{\partial \Psi}{\partial s}$, $\frac{\partial \Psi}{\partial z}$, $\frac{\partial V}{\partial t}$, $\frac{\partial V}{\partial s}$ and $\frac{\partial V}{\partial z}$ are given along with the configuration of isobars, coefficient of friction K and coriolis parameter f; then w can be evaluated using the expression given in the theorem.

2.2. The vertical shear magnitude

$$\left|\frac{\partial \mathbf{V}}{\partial z}\right| = V\sqrt{\left(\frac{\partial \Psi}{\partial z}\right)^2 + \left(\frac{\partial \log V}{\partial z}\right)^2}$$

is generally in excess of one mps per kilometre, *i.e.*, greater than 10^{-3} sec^{-1}. Since the highest value of f_1 is 1.64×10^{-4} sec^{-1}, the term containing f_1 in the denominator can be neglected unless high order of accuracy in evaluation of w is desired. We omit this term and take the approximate expression as :

2.3. The boundary of the cyclone eye is taken as closed zero w isopleth inside/on/outside which vertical motion is downward/zero/upward as illustrated in Fig. 2.

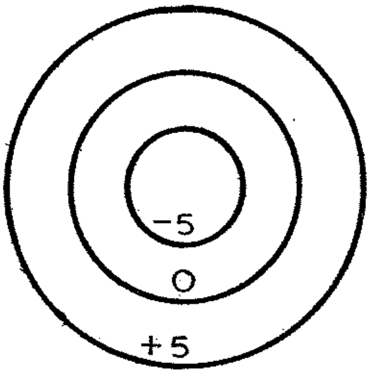

Fig. 2

We investigate the possibility of occurrence of closed zero w isopleth in the context of spiralling wind fields as in cyclones. This will amount to study of occurrence of zero of numerator (N) of (3).

3. Cyclone eye

3.1. To enable use of the expression for vertical motion (3), we discuss certain important characteristics of spiralling circulations generally associated with cyclone.

3.2. *Streamlines*

Confluenting streamlines spiralling inward terminate at a point called spiral centre where speed is zero. The curvature is counterclockwise in northern hemisphere and clockwise in southern hemispheric cyclones.

Hence $\left(f + V\frac{\partial \Psi}{\partial s}\right)$ is positive/negative at all points in northern/southern hemispheric cyclones. Further, $\partial\Psi/\partial n$ is negative at all points of cyclones of both hemispheres (*see* Figs. 3 and 4).

3.3. *Speed field*

Proceeding from the spiral centre in any direction, speed increases, reaches a maximum whereafter it decreases. The locus of points of maximum speed is a line enclosing the spiral centre called Ring of Maximum Wind (RMW). Speed profile along any direction is schematically illustrated in Fig. 5.

$$w = \frac{-\sqrt{\left(f + V\frac{\partial \Psi}{\partial s} + \frac{\partial \Psi}{\partial t}\right)^2 + \left(K + \frac{\partial V}{\partial s} + \frac{\partial \log V}{\partial t}\right)^2}\,\cos\,(\theta_{is} + \theta_1)}{\sqrt{\left(\frac{\partial \Psi}{\partial z}\right)^2 + \left(\frac{\partial \log V}{\partial z}\right)^2}\,\cos\,(\theta_{is}+\theta_2)} = \frac{N}{D} \qquad (3)$$

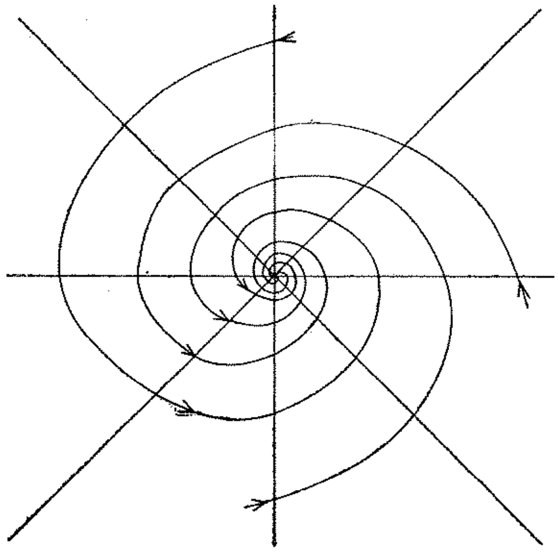

Fig. 3. Northern hemisphere cyclone

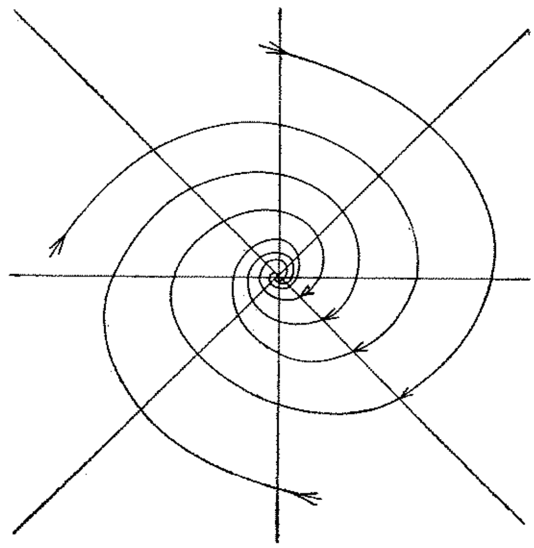

Fig. 4. Southern hemisphere cyclone

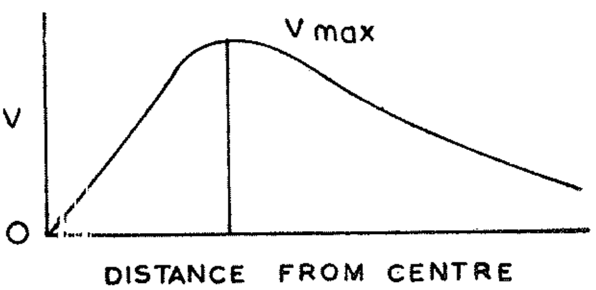

Fig. 5

$\partial V/\partial s$ is the rate of change of speed along the streamlines and is negative/zero/positive inside/on/outside RMW.

3.4. *Curvature term*

Curvature and speed both increase as one approaches from outside the RMW. Inside the ring of maximum wind, speed decreases and is zero at the spiral centre. The speed is usually taken as linearly increasing with radial distance from spiral centre (Francis E. Fendall 1974) and beyond RMW to decrease as a function of distance.

$$V = \begin{cases} V_{\max} \dfrac{r}{\sigma} & (0 \leqslant r \leqslant \sigma) \\ V_{\max} \left(\dfrac{\sigma}{r}\right)^n & (\sigma \leqslant r \leqslant \infty;\ n \text{ is positive}) \end{cases}$$

where σ is the radius of ring of maximum wind (RMW). Taking radius of curvature as proportional to radial distance, *i.e.*, $\dfrac{\partial \Psi}{\partial s} = \dfrac{C}{r}$ where C is the constant of proportionality, we get inside the RMW :

$$V \frac{\partial \Psi}{\partial s} = C \frac{V_{\max}}{\sigma} \quad (0 \leqslant r \leqslant \sigma)$$

and outside the ring of maximum wind,

$$V \frac{\partial \Psi}{\partial s} = C\, V_{\max} \frac{\sigma^n}{r^{n+1}} \quad (\sigma \leqslant r \leqslant \infty)$$

Instead of linear function, if we take

$$V = V_{\max}\, e^{\frac{1}{2}} \left(\frac{r}{\sigma}\right) \exp\left(-\frac{r^2}{2\sigma^2}\right) \quad \text{then}$$

$$V \frac{\partial \Psi}{\partial s} = C \frac{V_{\max}}{\sigma} e^{\frac{1}{2}} \exp\left(-\frac{r^2}{2\sigma^2}\right)$$

In both the cases, $V(\partial \Psi/\partial s)$ increases as we proceed from outside towards the RMW. In the case of wind field which is linearly modelled inside RMW, $V(\partial \Psi/\partial s)$ is a constant and in the non-linear model it increases.

3.5. *Speed divergence/convergence*

Positive $\partial V/\partial s$ decreases as we proceed towards the RMW whereafter it changes sign and magnitude of speed convergence increases as we proceed into the interior. Taking V as $V(r, \theta)$ we get

$$\frac{\partial V}{\partial s} = \frac{\partial V}{\partial r}\frac{dr}{ds} + \frac{\partial V}{\partial \theta}\frac{d\theta}{ds}$$

Isopleths of speed enclose the spiral centre where speed is zero. These isopleths are nearly circular shaped. Taking that the speed isopleths are circles and positioning the polar coordinate origin at the spiral centre, we note

$$\frac{\partial V}{\partial \theta} = 0 \text{ and hence, } \frac{\partial V}{\partial s} = \frac{\partial V}{\partial r}\frac{dr}{ds}$$

Since radial distance decreases as we proceed on the streamline, we note that dr/ds is negative.

$$\frac{\partial V}{\partial s} = \frac{V_{\max}}{\sigma}\frac{dr}{ds} \quad 0 \leqslant r \leqslant \sigma \text{ (for linear model)}$$

$$\frac{\partial V}{\partial s} = \frac{V_{\max}\, e^{1/2}}{\sigma}\left(1 - \frac{r^2}{\sigma^2}\right)\exp\left(-\frac{r^2}{2\sigma^2}\right)\frac{dr}{ds}$$

(for non-linear model)

We note that $\dfrac{\partial V}{\partial s}$ is positive and decreases as we proceed towards RMW inside which $\frac{\partial V}{\partial s}$ is negative. $\left|\frac{\partial V}{\partial s}\right|$ is constant if we take the linear model but increases if we take the nonlinear model.

3.6. *Intensification*

Cyclone is said to intensify if the speed on RMW increases. Usually intensification with time is associated with contraction of ring of maximum wind, in which case $\left|V\frac{\partial \Psi}{\partial s}\right|$ and $\left|\frac{\partial V}{\partial s}\right|$ increase at all points.

3.7. *Vertical shear* $\frac{\partial \mathbf{V}}{\partial z}$

Vertical shear connotes intensification/weakening with height and also tilt of axis. We will presume that the axis of cyclone is vertical and the streamline configuration is unchanged with respect of height, *i.e.*, $\frac{\partial \Psi}{\partial z} = 0$. Further, cyclones weaken with height and, therefore, $\frac{\partial \log V}{\partial z}$ is negative.

3.8. *Local change* $\frac{\partial \mathbf{V}}{\partial t}$

$\frac{\partial V}{\partial t}$ connotes intensification/weakening with time and also movement of cyclone centre. For simplicity of treatment we will take that the cyclone is not moving and the streamline cofiguration is unchanged with time, *i.e.*, $\frac{\partial \Psi}{\partial t} = 0$. If the cyclone intensifies $\frac{\partial \log V}{\partial t}$ is positive and if it weakens, $\frac{\partial \log V}{\partial t}$ is negative.

3.9. *Pressure field*

We will presume that pressure minimum coincides with spiral centre. In which case, the pressure field is characterised by the cross isobaric inflow in both the hemispheres. In the case of cyclones, rotational feature is predominant compared to inflow. Hence it is realistic to take, θ_{is} as not exceeding 20° for northern hemisphere and not less than 160° in southern hemisphere and that θ_{is} does not change from point to point. These are schematically illustrated in Figs. 6 and 7.

3.10. Neglecting $\frac{\partial \mathbf{V}}{\partial t}$, $\frac{\partial \Psi}{\partial z}$ and $\frac{f_1}{V}$ $\cos(\theta_{is} - \Psi)$,

$$w \simeq \frac{-\sqrt{\left(f + V\frac{\partial \Psi}{\partial s}\right)^2 + \left(K + \frac{\partial V}{\partial s}\right)^2}\ \cos(\theta_{is} + \theta_1)}{\left|\frac{\partial}{\partial z}\log V\right| \cos(\theta_{is} + \theta_2)} = \frac{N}{D}$$

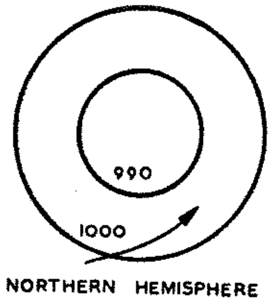

Fig. 6

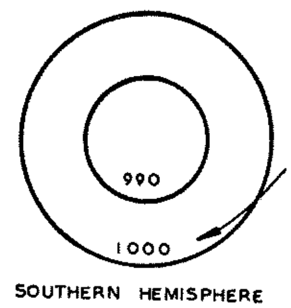

Fig. 7

θ_{is} is not exceeding 20° in northern hemisphere and not less than 160° in southern hemisphere.

The denominator in the case of northern hemisphere is negative at all points and is positive in southern hemisphere. Since $\left(f + V\frac{\partial \Psi}{\partial s}\right)$ is positive and increases and $\left(K + \frac{\partial V}{\partial s}\right)$ is positive and decreases, θ_1 increases in the range $0 < \theta_1 < 90°$ as we proceed from outside RMW towards the ring of maximum wind in northern hemispheric cyclone. Inside the RMW, $\frac{\partial V}{\partial s}$ is negative and θ_1 will continue to increase and will be less than 90°, if $\left(K + \frac{\partial V}{\partial s}\right)$ is positive. If the cyclone is intense, $\left(K + \frac{\partial V}{\partial s}\right)$ can become negative in which case θ_1 has value in excess of 90°. The numerator in the case of intense cyclone will be characterised by a zero isopleth of N inside/outside which N is positive/negative and thus forming the eye. In the case of southern hemisphere, $270° < \theta_1 < 360°$, when it is weak and has values 270° and less than 270° when intense. In this case, N is negative/positive inside/outside the zero N isopleth and note that the denominator D is positive at all points. We conclude that eye of cyclone forms when the cyclone becomes intense as characterised by increase of speed on RMW and contraction of RMW.

TABLE 1

r (km)	$V\frac{\partial \Psi}{\partial s}$ ($\times 10^{-4}$ sec^{-1})	$\frac{\partial V}{\partial s}$ ($\times 10^{-4}$ sec^{-1})	θ_1	θ_2	θ_{is}	$\theta_{is}+\theta_1$	$\theta_{is}+\theta_2$	w (mps)	r (km)	$V\frac{\partial \Psi}{\partial s}$ (10^{-4} sec^{-1})	$\frac{\partial V}{\partial s}$ (10^{-4} sec^{-1})	θ_1	θ_2	θ_{is}	$\theta_{is}+\theta_1$	$\theta_{is}+\theta_2$	w (mps)
				Weak case									Strong case				
20	2.42	−7.76	58.4	180	18.4	76.8	198.4	.35	10	9.70	−3.10	93.4	180	18.4	111.8	198.4	−1.71
40	2.28	−6.39	55.0	180	18.4	73.4	198.4	.43	20	9.13	−2.56	90.4	180	18.4	108.8	198.4	−1.40
60	2.06	−4.41	49.9	180	18.4	68.3	198.4	.55	30	8.26	−1.76	85.1	180	18.4	103.5	198.4	−0.93
80	1.79	−2.15	43.6	180	18.4	62.0	198.4	.69	40	7.18	−0.86	77.8	180	18.4	96.2	198.4	−0.38
100	1.49	0.0	36.9	180	18.4	55.3	198.4	.83	50	6.00	0.0	68.5	180	18.4	86.9	198.4	+0.17
120	1.20	+0.76	30.5	180	18.4	48.9	198.4	.95	60	4.81	+0.71	58.3	180	18.4	76.7	198.4	+0.64
140	0.93	+2.97	25.0	180	18.4	43.4	198.4	1.05	70	3.71	+1.19	48.1	180	18.4	66.5	198.4	+1.01
160	0.69	+3.57	20.4	180	18.4	38.8	198.4	1.11	80	2.75	+1.43	38.5	180	18.4	56.9	198.4	+1.27
180	0.49	+3.65	16.8	180	18.4	35.2	198.4	1.14	90	1.96	+1.46	30.5	180	18.4	48.9	198.4	+1.40
200	0.33	+3.34	14.1	180	18.4	32.5	198.4	1.15	100	1.34	+1.34	24.1	180	18.4	42.5	198.4	+1.44

$(V_i)_{max}$ = 5 mps $(V)_{max}$ = 15.8 mps $(V_i)_{max}$ = 10 mps $(V)_{max}$ = 31.6 mps

$(V_R)_{max}$ = 15 mps σ = 100 km $(V_R)_{max}$ = 30 mps σ = 50 km

3.11. It would be ideal to evaluate vertical motion with actual atmospheric horizontal wind field and pressure field of a cyclone. Since data are not adequate for purposes of calculation, we use a mathematical mode (Lakshminarayanan 1975) $\mathbf{V} = A\,\nabla\phi + B(\nabla\phi \times \mathbf{k})$ where $\phi = \exp\left(-\frac{r^2}{2\sigma^2}\right)$ and σ is the radius of maximum wind. If A and B are taken as positive, we get northern hemispheric cyclone with confluenting counterclockwise curved spiralling streamlines. If A is taken as positive and B as negative, we get confluenting spiralling streamlines with clockwise curvature as in southern hemisphere. A ring of maximum wind of radius σ encloses the spiral centre. $\mathbf{V}_i = A\nabla\phi$ indicates inflow characteristics and $\mathbf{V}_R = B\,(\nabla\phi \times \mathbf{k})$ gives rotational features. In the case of atmospheric cyclones, rotational feature is dominent compared to inflow features. Vertical motion for this cyclone model was evaluated with the following specific features :

(*i*) $\frac{f_1}{V}\cos(\theta_{is} - \Psi)$ is omitted.

(*ii*) $\frac{\partial \Psi}{\partial t}$ and $\frac{\partial \Psi}{\partial z}$ are taken as zeros.

(*iii*) $f = .3775 \times 10^{-4}$ sec^{-1} corresponding to Lat. 15°N.

(*iv*) $K = 2.5 \times 10^{-4}$ sec^{-1} (p. 661, *Compendium of Meteorology*)

(*v*) $\frac{\partial \log V}{\partial t} = .08 \times 10^{-4}$ sec^{-1} Speed increases to double the value in 24 hours.

(*vi*) $\frac{\partial \log V}{\partial z} = -2.3 \times 10^{-6}$ cm^{-1} Speed decreases to half the value in 3 km.

(*vii*) $V\frac{\partial \Psi}{\partial s} = \frac{(V_R)_{max}}{\sigma}\, e^{\frac{1}{2}} \exp\left(-\frac{r^2}{2\sigma^2}\right)$

(*viii*) $\frac{\partial V}{\partial s} = \frac{(V_i)_{max}}{\sigma}\, e^{\frac{1}{2}}\left(\frac{r^2}{\sigma^2} - 1\right)\exp\left(-\frac{r^2}{2\sigma^2}\right)$

(*ix*) $\theta_{is} = \tan^{-1}(\frac{1}{3})$ (Isobars are taken as circles with centre coinciding with spiral centre).

Case (i)

Weak cyclone/depression stage :

$(V_i)_{max}$ = 5 mps ; $(V_R)_{max}$ = 15 mps ;

$(V)_{max}$ = 15.8 mps, σ = 100 km

Case (ii)

Intense cyclone :

$(V_i)_{max}$ = 10 mps ;

$(V_R)_{max}$ = 30 mps ;

$(V)_{max}$ = 31.6 mps ;

σ = 50 km.

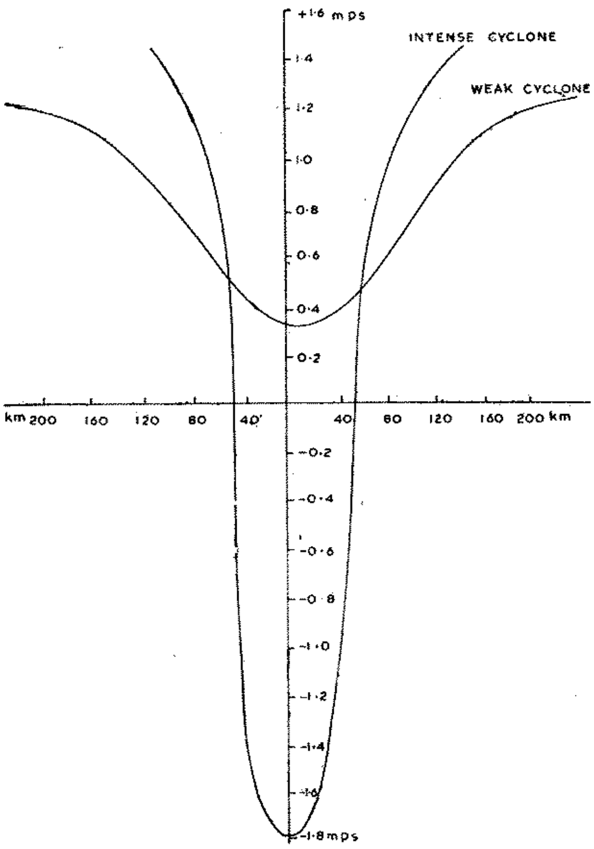

Fig. 8

Table 1 lists $\frac{\partial V}{\partial s}$, $V\frac{\partial \Psi}{\partial s}$, θ_1, θ_2, $(\theta_{is}+\theta_1)$, $(\theta_{is}+\theta_2)$ and w for both the cases.

Fig. 8 gives the profile of vertical motion along west-east oriented line passing through the cyclone centre. The vertical motion in the weak stage is upwards everywhere with a minimum value at the centre of cyclone. $(\theta_{is}+\theta_1)$ increases as we proceed from outside the RMW to the interior of the cyclone but the value is less than 90°. In the intense stage, the vertical motion has still a minimum at the cyclone centre but the minimum is characterised by negative value. The eye is formed now surrounding which vertical upward motion still takes place as in the weak stage. $(\theta_{is}+\theta_1)$ increases as we proceed from outside RMW into the interior where its value is now greater than 90°. The isopleth of $(\theta_{is}+\theta_1) = 90°$ is the zero isopleth of w forming the cyclone eye boundary inside/outside which vertical motion is downward/upward.

3.12. *Expansion/contraction of cyclone eye*

The cyclone eye is shown to occur when the cyclone was intense. We now investigate the expansion/contraction of cyclone eye. R_e the radius of cyclone eye is evaluated on the following assumptions :

(*i*) $\frac{\partial \mathbf{V}}{\partial t}$ is small and hence omitted.

(*ii*) Isobars are circles and minimum pressure and spiral centre are the same.

(*iii*) Cyclone wind field is given by

$\mathbf{V} = A\,\nabla\phi + B\,(\nabla\phi \times \mathbf{k})$ in which case,

$$\tan\theta_{is} = \frac{A}{B} = \frac{(V_i)_{\max}}{(V_R)_{\max}} = \alpha$$

From Eqn. (3), a necessary condition for eye to occur is

$$w = 0 \Rightarrow \cos(\theta_{is}+\theta_1) = 0 \Rightarrow \tan\theta_{is}\tan\theta_1 = 1$$

$$\tan\theta_{is} = \frac{(V_i)_{\max}}{(V_R)_{\max}} = \alpha$$

$$\tan\theta_1 = \frac{f + V\frac{\partial\Psi}{\partial s}}{K + \frac{\partial V}{\partial s}} = \frac{1}{\alpha}$$

$$V\frac{\partial\Psi}{\partial s} = \frac{(V_R)_{\max}}{\sigma}\, e^{\frac{1}{2}} \exp\left(-\frac{r^2}{2\sigma^2}\right)$$

$$\frac{\partial V}{\partial s} = \frac{(V_i)_{\max}}{\sigma}\, e^{\frac{1}{2}} \left(\frac{r^2}{\sigma^2} - 1\right)\exp\left(-\frac{r^2}{2\sigma^2}\right)$$

$$(V_R)^2_{\max} + (V_i)^2_{\max} = (V)^2_{\max}$$

$$(V_R)_{\max} = \frac{V_{\max}}{\sqrt{1+\alpha^2}}\ ;$$

$$(V_i)_{\max} = \frac{\alpha}{\sqrt{1+\alpha^2}}(V)_{\max}$$

Representing r at $w = 0$ as R_e, radius of eye we get

$$\alpha V\frac{\partial\Psi}{\partial s} - \frac{\partial V}{\partial s} = K - f\alpha$$

$$\Rightarrow \left(1 - \frac{R_e^2}{2\sigma^2}\right)\exp\left(-\frac{R_e^2}{2\sigma^2}\right)$$

$$= (K - f\alpha)\,\frac{\sqrt{1+\alpha^2}}{\alpha}\,\frac{1}{2e^{1/2}}\left(\frac{V_{\max}}{\sigma}\right)^{-1}$$

We note the following :

(*i*) $\left(1 - \frac{R_e^2}{2\sigma^2}\right) \exp\left(-\frac{R_e^2}{2\sigma^2}\right)$ has maximum of one when $\frac{R_e}{\sigma}$ is zero, minimum of $-\,e^{-2}$ when $\frac{R_e}{\sigma} = 2$ and tends to zero when $\frac{R_e}{\sigma}$ tends to infinity or is $\sqrt{2}$.

(*ii*) $(K - f\alpha)\,\frac{\sqrt{1+\alpha^2}}{\alpha}\,\frac{1}{2e^{1/2}}\left(\frac{V_{\max}}{\sigma}\right)^{-1}$

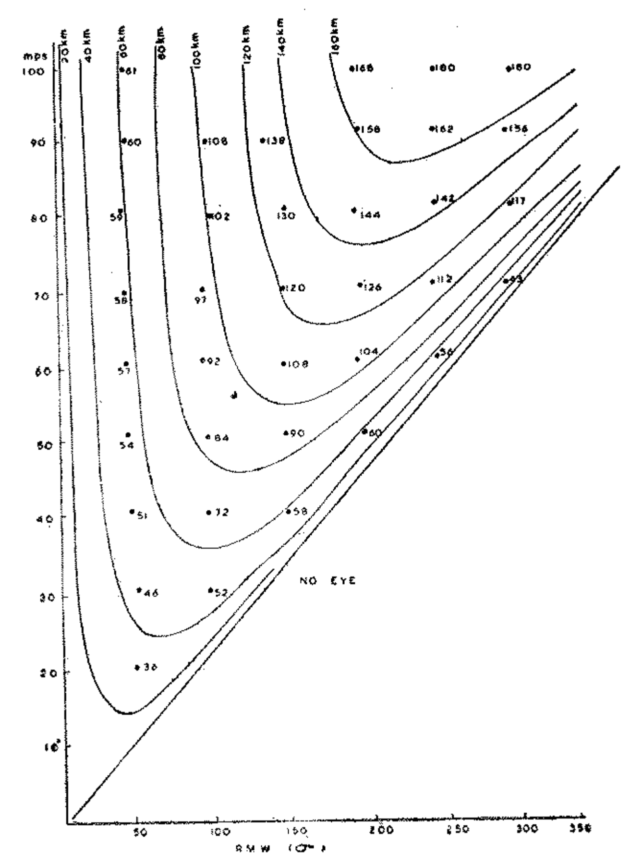

Fig. 9

is positive for cyclones and cyclone eye occurs if it is less than one. Cyclone eye will not occur if it is equal to one or greater than one.

Taking $K = 2.5\times10^{-4}$ sec^{-1}, $f = .3775 \times 10^{-4}$ sec^{-1} and $\alpha = 1/3$ the radius of cyclone eye has been evaluated for various values of σ and V_{max}. Fig. 9 gives the plot of cyclone eye radius on the (σ, V_{max}) plane where isopleths of R_e are also drawn. From Fig. 9 we note following for a given set of values of K, f and α :

(*i*) Taking that radius of ring of maximum wind is unchanged with time, the eye of cyclone does not exist when maximum wind is below a specific value. The eye is formed and expands rapidly when V_{max} increases and exceeds the specific value. As V_{max} increases further, the expansion of the eye continues but the rate becomes sluggish and the eye radius reaches an asymptotic value of $\sqrt{2}\sigma$.

(*ii*) Taking that V_{max} is unchanged with time, the eye of cyclone does not exist when the radius of maximum wind is greater than a specific value. As RMW contracts and is less than the specific value the cyclone eye is formed and expands rapidly. With further contraction of RMW, the cyclone eye expands and reaches a maximum value whereafter the cyclone eye contracts with further contraction of RMW.

In the case of tropical cyclones, intensification is characterised by increasing V_{max} and contraction of RMW and decay by decreasing V_{max} and expansion of RMW in which case, it is realistic to conclude that cyclone intensification will be associated with birth of cyclone eye and a very rapid rate of eye expansion initially whereafter eye expansion either is sluggish or there may be sluggish contraction and cyclone weakening will be associated with either sluggish expansion/sluggish contraction followed by rapid contraction with ultimate vanishing of the eye. However, Dennis J. Shea and William M. Gray (1973) *say* "Colon (1963) has noted that weakening storms are invariably accompanied by widening of the eye. The present data agrees well with this conclusion". This conclusion is, however, not shared by all. For instance, Raghavan and Rajagopalan (1980) state that association of the expansion of eye with weakening may be operationally risky. There is need to study the cyclone eye expansion and contraction in greater detail with individual cyclone data.

4. Discussion

4.1. The two important aspects meriting critical comments are (*i*) wind field of cyclone and (*ii*) the equation of motion for horizontal wind where frictional term is represented as — $K\mathbf{V}$. We discuss in particular whether the inner area of cyclone is characterised by confluenting and spiralling-in, streamlines with inflow, speed convergence and horizontal convergence or by difluenting and spiralling-out streamlines with outflow, speed divergence and horizontal divergence features. Next we discuss the different types of representation of frictional term. It will be advantageous even though it is not essential, to refer to papers of (*i*) William M. Gray and Dennis J. Shea (1973) in connection with the wind field in a cyclone eye and (*ii*) Francis E Fendell's Tropical Cyclone (1974).

4.2. Equation of continuity is very often used to estimate vertical motion in the atmosphere :

$$w\Big|_1^2 = -\int_1^2 \left\{\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \frac{1}{\rho}\frac{d\rho}{dt}\right\} dz$$

It is customary to ignore $\frac{1}{\rho}\frac{d\rho}{dt}$ and take w as zero at the earth's surface in which case vertical upward motion will occur in an atmospheric layer with convergence and downward motion will occur in layer with divergence. In the context of cyclone eye, convergence prevails and the vertical motion is downward. If so, $\frac{1}{\rho}\frac{d\rho}{dt}$ cannot be ignored but must be present as a positive quantity such that $\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \frac{1}{\rho}\frac{d\rho}{dt}$

is positive. If we persist that $\frac{1}{\rho}\frac{d\rho}{dt}$ is small and neglect $\frac{1}{\rho}\frac{d\rho}{dt}$ even in the cyclone eye, then there must be divergence. Gray and Shea obtained divergence and radial outflow in cyclone eye. They split upper wind observations obtained by aircraft into radial V_r and tangential component V_θ. Positive/negative V_r connote radial outflow/inflow. Gray and Shea obtained mean radial outflow. They calculated divergence using $\frac{\partial V_r}{\partial r}+\frac{V_r}{r}$ and the average value of divergence relative to RMW was found to be positive from which vertical downward motion was obtained. Radial outflow coupled with divergence if real will necessitate difluenting spiralling-out pattern of streamlines whereas we have taken confluenting spiralling inwards pattern of streamlines. In this connection, it may be pointed out that the radial and tangential wind components were obtained on a polar coordinate system where origin was made to coincide with the cyclone centre. For illustrative purposes, let us take a polar coordinate system where origin coincides exactly with the spiral centre of confluenting spiralling-in streamline as chosen in this paper. We obtain V_r as negative at all points and interpret this as radial inflow. If the centre does not coincide with the spiral centre and differs by a small quantity, we obtain negative as well as positive V_r. From this, we cannot conclude that the streamline has "*outflow*". Gray and Shea are aware of this source of error. Divergence is invariant with respect to coordinate system. Hence even when the origin of polar coordinates and spiral centre do not coincide and we have a region of positive V_r interpreted as radial outflow, divergence using the correct expression $\frac{\partial V_r}{\partial r}+\frac{V_r}{r}+\frac{1}{r}\frac{\partial V_\theta}{\partial \theta}$ will be negative. If $\frac{1}{r}\frac{\partial V_\theta}{\partial \theta}$ were omitted, one can possibly get positive value of divergence for positive V_r. Gray and Shea are aware of the error due to non-coincidence of the origin of polar coordinates and spiral centre in the context of evaluation of divergence but believe that errors "should largely be eliminated in the averaging process". The resolution of the difficulty posed by Gray and Shea requires experimental observation of horizontal wind field in the neighbourhood of cyclone centre to answer the query " Are the streamlines confluenting and convergent or difluenting in the cyclone centre".

4.3. *Frictional term*

The external force other than pressure gradient has been taken as friction and expressed in the form as formulated by Guldberg and Mohn (p. 661, *Compendium of Meteorology*). Theoretical formulation of horizontal component of frictional term is $\frac{\mu}{3}\left(\mathbf{i}\frac{\partial}{\partial x}+\mathbf{j}\frac{\partial}{\partial y}\right)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial z}\right)+\mu\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\left(u\,\mathbf{i}+v\,\mathbf{j}\right)$. In the case of Ekman spiral divergence term and $\mu\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(u\mathbf{i}+v\mathbf{j}\right)$ are omitted and frictional term has the form $K\frac{\partial^2\mathbf{V}}{\partial z^2}$ (Godske *et al.* 1957). Gray and Shea (1973) take it as $C_D V^2$ along the streamline.

We note that there is no unique representation of friction. Let $\mathbf{F}_r=(F_r)_s\frac{\mathbf{V}}{V}+(F_r)_n\frac{(\mathbf{k}\times\mathbf{V})}{V}$ represent the horizontal component of friction. $(F_r)_s$ is along and $(F_r)_n$ normal to streamline. So long as frictional term does not contain w and/or its derivatives, the expression for w as in theorem needs a small recasting of numerator as :

$$N=-\sqrt{\left\{\frac{(F_r)_s}{V}+\frac{\partial V}{\partial s}+\frac{\partial\log V}{\partial t}\right\}^2+\left\{\frac{(F_r)_n}{V}+f+V\frac{\partial\Psi}{\partial s}+\frac{\partial\Psi}{\partial t}\right\}^2}\quad\cos\left(\theta_{is}+\theta_1\right)$$

where,

$$\tan\theta_1=\frac{\frac{(F_r)_n}{V}+f+V\frac{\partial\Psi}{\partial s}+\frac{\partial\Psi}{\partial t}}{\frac{(F_r)_s}{V}+\frac{\partial V}{\partial s}+\frac{\partial\log V}{\partial t}}$$

If, however, frictional term contains w and/or its derivatives, the expression given for w in this paper cannot be used since the right hand side of equation requires information of w to evaluate w on the left hand side. We have taken $K=2.5\times10^{-4}\ \text{sec}^{-1}$ from page 661, *Compendium of Meteorology* with a view to illustrate the role of friction. At higher levels, K will be having much lower values. In the context of accurate evaluation of vertical motion using dynamical equation of motion, frictional term needs specification and coefficients exactly determined.

4.3.1. Formation of the eye and its area in cyclone is related to maximum speed, radius of the RMW and the cross isobaric flow. As the cyclone intensifies characterised by higher values of $V_{\max}$ and / or contraction of the ring of maximum wind, speed convergence exceeds the frictional coefficient. Since there is cross-isobaric inflow, the eye will be formed even before occurrence of this stage. The area of the cyclone eye can be expected to increase with further increasing speed of $V_{\max}$ and/or further contraction of RMW. Likewise area of the eye can be expected to decrease and ultimately vanish with weakening of the cyclone.

5. Conclusion

The inner area of cyclone has minimum vertical motion which is positive when the cyclone is less intense but becomes negative when the cyclone is very intense resulting in eye formation.

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