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A simple approach in verification of numerical models

M. M. ABDEL-WAHAB*

Dept. of Geography, McMaster University, Hamilton, Canada

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सार — सांख्यिकी विधियों एवं ऊर्जा अविनाशिता के नियमों की मदद से प्रायः आंकिक निदर्शों की परिशुद्धता की जांच की जाती है। रुद्धोष्म प्रवाह में विभव भ्रमिलता ($\Omega\theta$) और विभवताप (θ) अविनाशी राशियां हैं। एक आदर्श घटना मे दो क्रमिक समएन्ट्रापी रेखाओं के बीच की राशि स्थिर होती है। प्रस्तुत शोधकार्य में सरल भूविक्षेपी और प्रारंभिक निदर्शों में तुलना के लिए अविनाशिता के इन्हीं नियमों का उपयोग किया गया है।

ABSTRACT. The accuracy of numerical models results are usually compared by the aid of statistical methods and energy conservation laws. In adiabatic flow, the potential vorticity ($\Omega\theta$) and the potential temperature (θ) are conservative quantities. In ideal case, the mass enclosed between two successive isentropic lines must be constant. In the present work these conservation laws are used for the comparison simple geostrophic and primitive models.

1. Introduction

Verification of a numerical model means the entire process of comparison between the predicted and actual charts. In general, there are two types of verification. The first depends upon the study of the statistical characteristics of predicted charts in comparison with the actual charts (correlation coefficient, relative error, etc) ; the second depends on the study of the internal properties of the numerical schemes on the truncation errors, initial data problem, and the choice of boundary conditions.

Abdel-Wahab (1977, 1980) used the analogy of the conservation of vorticity to compare between two barotropic schemes.

In this paper the concept of the conservation of (θ and Ω_θ) is discussed to compare between two baroclinic models. The principle of the use of the conservation laws in numerical models verification was suggested by Charney (1950). In ideal adiabatic flow the mass enclosed between two successive quasi-conservative quantities lines, must remain constant. It must be clear that any conservative quantity may be created or destroyed by the horizontal transport for which the compared scheme must include approximately all the northern hemisphere (assuming that there is no cross equational flow).

2. The compared schemes

The first scheme, if the primitive equation system in the form :

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + v\frac{\partial U}{\partial y} + fv = -g\frac{\partial z}{\partial x}$$

$$\frac{\partial v}{\partial t} + U\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -g\frac{\partial z}{\partial y}$$

$$\frac{\partial U}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \tau_p}{\partial p} = 0$$

$$T = \frac{Pg}{R}\left(\frac{\partial z}{\partial p}\right) \text{ and } \frac{\partial T}{\partial t} + U\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{c^2}{PR} \tag{1}$$

where τ is the vertical velocity and subscripts represent the derivatives.

$$c = \sqrt{\frac{\bar{R}^2 T\ (v_a - v)}{g}}$$

c is the stability parameter

g is the acceleration due to gravity

R is the gas constant.

*Permanent affiliation : Dept. of Meteorology, Cairo University, Cairo, Egypt.

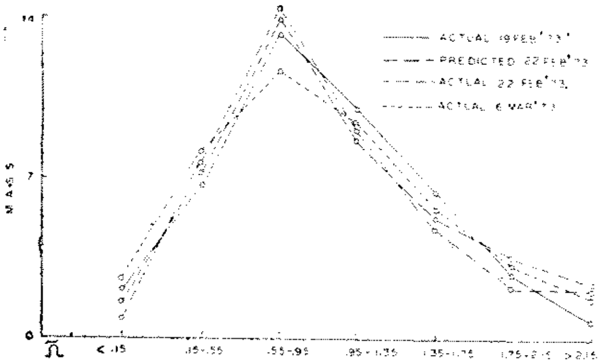

Fig. 1. The distribution of masses with potential temperature intervals for quasi-geostrophic model

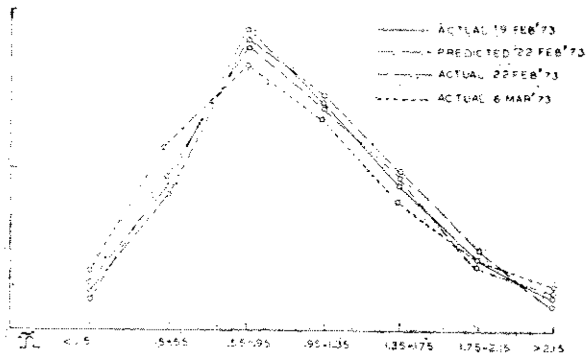

Fig. 2. The distribution of masses with potential temperature intervals for primitive model

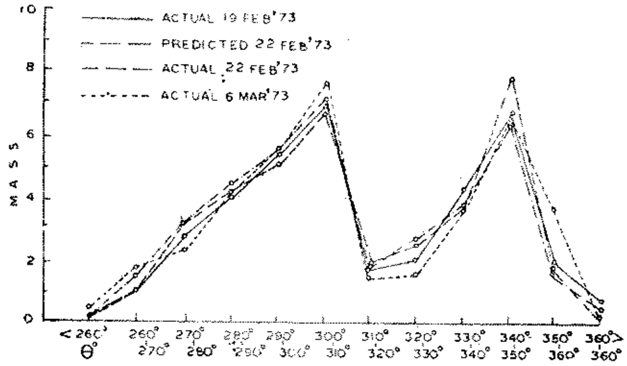

Fig. 3. The distribution of masses with potential vorticity intervals for quasi-geostrophic model

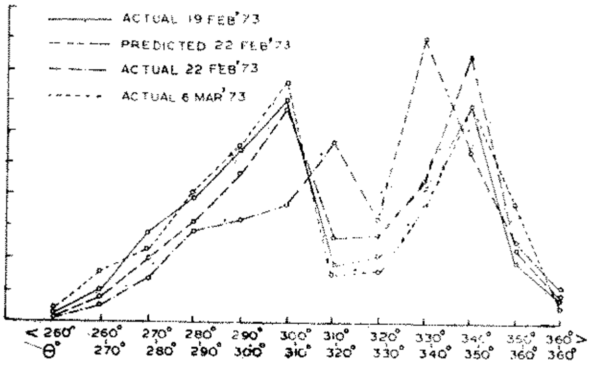

Fig. 4. The distribution of masses with potential vorticity intervals for primitive model

TABLE 1

Criteria day	$\triangle OP_1$	$\triangle OP_2$	$\triangle OP_3$	$\triangle_3 P_3$	$\triangle O_3$
3.II	2.71 6.150	4.662 13.74	7.976 16.180	16.21 19.41	14.27
16.II	3.146 10.97	6.08 18.17	8.14 24.91	15.87 29.74	13.51
14.III	2.54 5.18	6.6466 10.64	8.7727 14.72	16.13 18.19	13.06
19.IV	2.09 5.841	5.7987 9.9611	7.6868 12.869	17.72 20.06	10.97
20.IV	2.64 7.17	7.379 13.92	10.467 17.85	26.17 19.66	16.66
22.IV	2.15 12.6	7.679 16.173	10.437 19.907	14.69 32.159	17.75
25.IV	3.516 9.126	7.561 14.55	9.65 19.61	21.15 21.19	12.71
Mean	2.67 8.16	6.46 13.88	9.95 17.06	17.56 23.29	13.99

The upper and lower numbers belonging to the primitive and the geostrophic models respectively.

This set of equations are solved using splitting method Marchuk (1969) ; under the two following boundary conditions :

(a) The vertical boundaries :

$$\text{at } P = P_0 \qquad \tau = 0;$$
$$P = 0 \qquad \tau = 0;$$

(b) The horizontal boundaries :

$$\frac{\partial z}{\partial t} = 0; \quad \frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = 0$$

The second scheme is the geostrophic model based upon the equation :

$$\left(\triangle + f^2 \frac{\partial}{\partial \zeta} \frac{\zeta^2}{c^2} \frac{\partial}{\partial \zeta} \right) \frac{\partial z}{\partial t} = F(x, y, \zeta, t) \qquad (2)$$

$$F = f^2 \frac{R}{g} \frac{\partial}{\partial \zeta} \frac{\zeta}{c^2} A_T - A_\Omega$$

where , A_Ω is the geostrophic vorticity advection

A_T is the temperature advection.

ζ is the relative height parameter P/P_0

$\triangle$ is the Laplacian $\frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial y^2}$.

This equation is solved by Liebman relaxation methcd under the following boundary conditions :

$$\text{at } \zeta = 1 \qquad P = P_0 \qquad \tau = \frac{gP}{RT_1} \frac{\partial z}{\partial t}$$

using this condition in last couple equations in set (1) it is easy to express the upper boundary condition in the form :

TABLE 2

Statistical verification of the models (R.M.S.)

% error in geopotential height

Pressure level (mb)	Forecast extent (hours)		
	24	48	72
850	6.4 10.9	12.8 19.2	28.2 34.6
500	5.1 9.7	11.1 17.2	20.7 28.7
300	5.6 8.6	10.6 13.9	23.2 31.4

The upper and lower numbers refer to primitive and geostrophic model respectively.

$$\frac{\partial}{\partial \zeta} \frac{\partial z}{\partial t} + \alpha \frac{\partial z}{\partial t} = -\frac{R}{g} A_T$$

where, $\alpha = c^2/RT_1 = R \frac{(v_a - v)}{g} = .1$

and the lower boundary condition :

$$\text{when } \zeta \to 0 \quad (P \to 0) \quad \zeta \frac{\partial^2 z}{\partial z \partial t} \to 0$$

3. Method of verification

The potential temperature (θ) and potential vorticity ($\Omega\theta$) were calculated from the actual and predicted geopotential heights z, at all the grid points. The potential temperature can be evaluated directly from the hydrostatic equation. The potential vorticity is calculated due to Obukhov (1964) by the formula :

$$\Omega_\theta = \frac{P^*_{(\theta)} (v_a - v)}{P(v_a - v^*_{(\theta)})} \Omega_a \qquad (3)$$

where, v is the vertical gradient of temperature (lapse rate)

v_a is the adiabatic lapse rate,

Ω_a is the absolute vorticity,

$P^*_{(\theta)}$ is the mean pressure at isentropic surface θ,

$v^*_{(\theta)}$ is the mean vertical gradient of temperature at isentropic surface θ.

The mass enclosed between the equi-interval of θ and Ω_θ was computed, using the location of the different points on the charts.

The vertical velocity in the integrated domain for different isobaric levels, was evaluated as shown by Abdel-Wahab (1981) by the aid of the equation :

$$\tau_p = -\frac{1}{x} P \int_{P_0}^{P} D^* \exp(\bar{x} P)\, dP$$

where, $\bar{x} = -\frac{\gamma g}{R}\frac{1}{\bar{T}}$

$\bar{T}$ is mean layer temperature at any grid point,

D is horizontal divergence,

g is acceleration due to gravity,

R is gas constant of the air,

$\gamma = c_p/c_v$ is the ratio between specific heat at constant pressure to the specific heat at constant volume.

4. Results and discussion

The computations were carried out for 18 cases in different synoptic situations through February 1973. The models were applied over an area covering approximately all the northern hemisphere. The vertical structure of the models extended from $P=1000$ mb up to 100 mb isobaric level.

The mass distribution through θ and Ω_θ are shown in the actual atmosphere and for both models during the period of forecasting. The Fig. 1 shows the actual distribution of masses against the potential temperature intervals for the geostrophic model. Fig. 2 also shows the same distribution for the primitive models.

In Figs. 3 & 4 the distribution of masses by potential vorticity intervals for the geostrophic and non-geostrophic models respectively are shown. For tubes of (θ, Ω_θ) dimensions, the deviations of masses between the initial and the first, the second and the third day forecasting ($\triangle OP_1$, $\triangle OP_2$, $\triangle OP_3$) respectively are evaluated.

where O, 3 denote the initial and final actual fields,

P_1, P_2, P_3, denote the prognostic first, second and third day,

$\triangle_3 P_3$ is the mass deviation of 72 hours predicted charts about its corresponding actual ones.

$\triangle O_3$ is the mass deviation between the initial chart and its actual ones after 72 hours.

The distribution of potential temperature and potential vorticity for both models and for actual weather show that there are characterised behaviour for those invariants (Figs.1-4).

Table 1 shows that the mass deviation in the case of primitive model is less than in geostrophic. By comparing $\triangle O_3$ and $\triangle OP_3$ it can be seen that the change in mass enclosed between (θ or Ω_θ) lines in primitive scheme is less than in the actual weather which represents the deviation of the actual weather from the adiabatic predicted weather.

5. Conclusion

(1) There are constant distribution laws for the masses with the potential temperature and potential vorticity intervals as in the actual weather and in the used numerical models. The maximum values existing at intervals (300-310)° K, (340-350)° K (Figs. 2 and 4) of potential temperature and at (.55-.95) (Figs. 1 and 2) of the potential vorticity.

(2) The mass distribution with the potential vorticity intervals gives a good agreement in the two models with the actual atmosphere.

(3) Verification by the proposed method indicates that primitive model is more accurate than geostrophic model (Table 1).

This method can be used in comparison between different time difference schemes.

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