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Return period analysis of extreme rainfall events

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सार — इस शोधपत्र में कृष्णा नदी बेसिन के चुने हुए 15 स्टेशनों पर अधिक वर्षा की घटनाओं की पुनरावृत्ति अवधि के विश्लेषण का अध्ययन किया गया है। अध्ययन में 60 वर्षों (1901–1960) के लिए एक दिन की वर्षा के वार्षिक आंकड़ों का उपयोग किया गया है। दो भिन्न बंटनों अर्थात् गंबेल बंटन और फिशर एवं टिपेट टाइप II बंटन का उपयोग करके पुनरावृत्ति की विविध अवधियों में वर्षा का आकलन किया गया है। दोनों बंटनों के लिए उच्चतम अभिलेखित वर्षा से तत्संबंधित पुनरावृत्ति अवधियों का अन्तर्वेशन किया गया है। परिणामों से पता चलता है कि गंबेल बंटन शीर्षस्थ क्षणों की अति उच्च पुनरावृत्ति अवधियों की ओर इशारा करता है और फिशर एवं टिपेट टाइप II बंटन पुनरावृत्ति अन्तराल के निम्नतर आकलन प्रदान करता है। वर्षा की घटनाएं स्वभावत: यादृच्छिक रूप से घटित होती है, अत: सीमावर्ती क्षेत्रों की पुनरावृत्ति अवधियों का भी समुचित परिशुद्धता के साथ आकलन अपेक्षित है। इस अध्ययन से पता चलता है कि फिशर एवं टिपेट टाइप II बंटन तकनीक का उपयोग करके विभिन्न पुनरावृत्ति अवधियों के लिए वर्षा का आकलन गंबेल तकनीक का उपयोग करके प्राप्त वर्षा के आकलन से बहुत अधिक है। अत: सुझाव है कि अत्यधिक वर्षा की पुनरावृत्ति अवधियों की घटनाओं के मूल्यांकन के लिए गंबेल बंटन की तुलना में फिशर एवं टिपेट टाइप II बंटन को वरीयता दी जाए।

ABSTRACT. The return periods analysis of extreme events of rainfall for selected 15 stations of Krishna river basin have been studied and presented in this paper. The annual one-day rainfall data for 60-years (1901-60) was used for the study. The rainfall estimates were computed for different return periods using two different distributions, namely, Gumbel's distribution and Fisher & Tipett type II distribution. Return periods corresponding to highest recorded rainfall were interpolated for both the distributions. Results show that while Gumbel's distribution seems to indicate very high return periods of extreme events, the Fisher & Tipett type-II disrtibution provides lower estimates of the recurrence interval. As rainfall events are randomly distributed in nature, the return periods of outliers should also be determined with reasonable accuracy. The present study shows that the rainfall estimates for different return periods as obtained by using Fisher & Tipett type II distribution technique are much higher than those obtained by using Gumbel's technique. It is suggested that Fisher & Tipett type II distribution may be preferred for evaluating the return periods of extreme rainfall events to Gumbel distribution.

1. Introduction

In the design of water-way engineering structures the twin factors of safety and economy are both important as in other engineering structures. By safety of the structures, it is implied that the structure should be able to withstand the rainstorms in the catchment or over the site itself in a given period. While safety takes precedence over all other considerations, it is an indication of the technological progress to design a structure in an optimum cost benefit ratio. The return period estimates of the maximum rainfall likely to occur are used by the hydraulic engineers for the design purposes. The optimum design of a structure is made by striking a balance between a calculated risk on estimated return periods of rainfall and the constraints on the availability of funds. The maximum rainfall estimates for specified periods can be made by using suitable extreme value distributions (India Met. Dep. 1972).

Rao and Krishnan (1958) applied Gumbel's and Jenkison's methods for the determination of maximum probable rainstorms. They applied the above methods of the data of Damodar river catchment for the period 1891 to 1950. They computed rainfall estimates for various return periods for the rainstorms of 5 to 7 days. The comparison of the results by two methods showed marked difference between the computed and observed frequencies. They suggested that all rainfall amounts rather than extreme values should be considered while evaluating probabilities. Harihara Ayyar and Tripathi (1971) studied the heaviest rainfall over India recorded in relation to return period. For the purpose, 50 stations all over the country and 50 years one day annual maximum rainfall data were utilised and applying Gumbel's distribution to the series the return period values for the extreme events were computed by them. The return periods of the extreme events presented by above authors vary from 21 to 2883 years; they did not discuss the implication of this wide range, and left it to the users to decide appropriate methodology for selecting the design value. This gives rise to an under-estimation of the rainfall in a given return period for design criterion endangering the

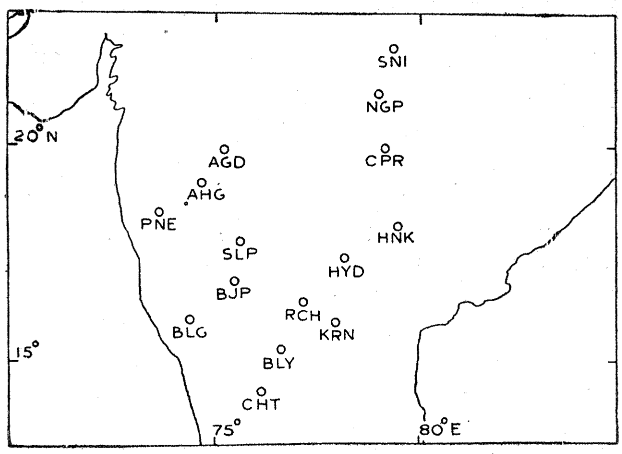

Fig. 1. Locations of stations used in the study

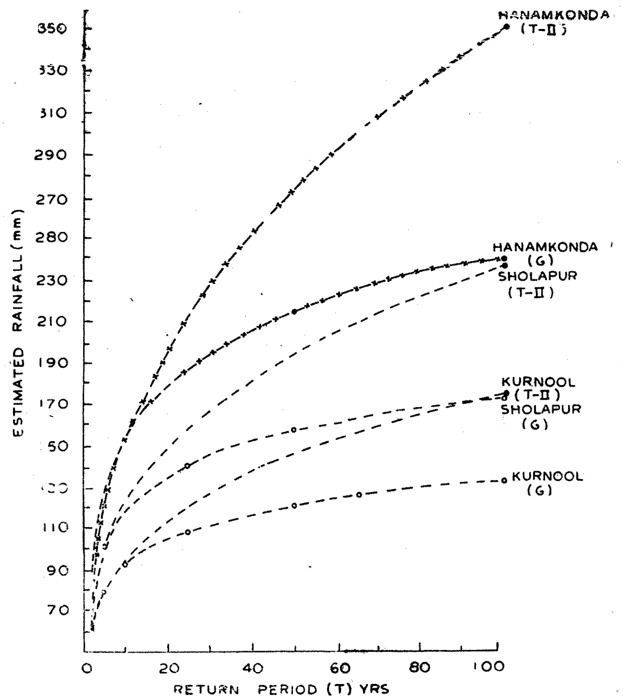

Fig. 2. Graph showing estimated rainfall in respect of 3 selected stations for different return periods

safety of structures. The reason for low value is that Gumbel's extreme value distribution does not estimate outliers, within reasonable degree of accuracy.

Dhar and Kulkarni (1970) estimated one-day rainfall estimates for different return periods in Uttar Pradesh. They selected 226 stations in plains of Uttar Pradesh and computed the 2-year and 100-year return periods rainfall estimates. They presented generalised chart of 2-year one-day rainfall. The ratios of 100 to 2-year rainfall estimates were worked out for the districts in UP. However, the problem of outliers was not discussed.

In the present study, an attempt has, therefore, been made to estimate the rainfall values for different return periods, by two extreme value distributions, namely Gumbel's distribution and Fisher & Tipett Type-II distribution. The annual rainfall series were tested for the outliers. If the extreme value exceeded 3 times the median value of the data set, that value was treated as off-shoot or an outlier (UK, Flood Report 1975).

2. Data used

The input variate for the study has been taken as the one day annual maximum rainfall. Fifteen stations in Krishna river basin (Fig. 1) were considered and 60 years (1901-60) rainfall data have been utilised.

3. Return period analysis

The probability density functions used for the estimation of return periods corresponding to the extreme rainfall recordded over a station, are given below :

(a) *Using Gumbel distribution*

Let $x_1, x_2, \ldots\ldots\ldots\ldots x_n$ be the extreme value annual series of one day rainfall over the station. Gumbel's distribution (1958) is given by :

$$F(x) = \exp\left[-e^{-\alpha(x-u)}\right] \quad (1)$$

where the parameters α and u may be estimated using the method of moments as suggested by WMO (1981) leading to relation. :

$$\alpha = 1/0.78\,s \quad (2)$$

and

$$u = \bar{x} - 0.58/\alpha \quad (3)$$

Here $\bar{x}$ and s are the mean and standard deviation of the extreme rainfall series.

The rainfall estimate x_T for a given return period T may be computed by :

$$x_T = u + y_T/\alpha \quad (4)$$

where y_T is the reduced variate given by

$$y_T = -\log_e \log_e [\,T/(T-1)\,] \quad (5)$$

(*b*) *Fisher and Tipett type II distribution*

The type II distribution is also known as log Gumbel distribution (Flood Report 1975). The density function of x is given by

$$F(x) = \exp\left[-e^{-a(z-u)}\right] \quad (6)$$

and where $z = \log x$. The parameters α and u are estimated by method of moments. These are :

$$\alpha = 1/0.78\,s \quad (7)$$

and

$$u = \bar{z} - 0.58/\alpha \quad (8)$$

where $\bar{z}$ & s are the mean & standard deviation of the log of rainfall series.

The rainfall estimate x_T for any given return period T may be computed by :

$$x_T = \text{antilog}\,(u + y_T/\alpha) \quad (9)$$

where y_T is given by Eqn. (5). The rainfall estimates as computed from Eqn. (9) for various return periods are given in Table 1. The return periods of highest recorded rainfall as estimated by this technique are shown in Table 2.

4. Standard errors of quantiles estimates

From Eqn. (4) we find that x_T is a function of mean $\bar{x}$ and standard deviation s :

$$x_T = x_T(\bar{x}, s)$$

It is asymptotically normally distributed with variance (Flood study Rep. 1975) given by :

$$\text{var}\,x_T = \left(\frac{\partial x_T}{\partial \bar{x}}\right)^2 \text{var}\,\bar{x} + \left(\frac{\partial x_T}{\partial s^2}\right)^2 \text{var}\,s^2 + 2\left(\frac{\partial x_T}{\partial \bar{x}}\right)\left(\frac{\partial x_T}{\partial s^2}\right)\text{cov}\,(\bar{x}, s^2)$$

where the partial derivatives are evaluated at the expected values of $\bar{x}$ and s^2 and are given by :

$$\frac{\partial x_T}{\partial \bar{x}} = 1, \quad \frac{\partial x_T}{\partial s^2} = \frac{(-0.45+0.78\,y_T)\,0.78\alpha}{2}$$

The variance and covariance terms are given by :

$$\text{var}\,\bar{x} = \frac{1}{0.78^2\alpha^2 N}$$

$$\text{var}\,s^2 = \frac{4.40}{(0.78\,\alpha)^4 N}$$

$$\text{cov}\,(\bar{x}, s^2) = \frac{1.14}{(0.78\,\alpha)^3 N}$$

The moment ratios $\mu_3/\mu_2^{3/2}$ & μ_4/μ_2^2 are the coefficients of skewness & kurtosis which are 1.14

TABLE 1

Return period estmates of rainfall (mm)

S. No.	Station	x-25yr		x-50 yr		x-100 yr		x-150 yr		x-200 yr		x-500 yr	
		G	T-II	G	T-II	G	T-II	G	T-II	G	T-II	G	T-II
1	Nagpur	203	221	228	270	252	330	266	370	276	402	308	521
2	Chanda	194	214	217	260	238	315	252	353	261	382	290	492
3	Seoni	230	255	260	319	289	399	306	454	318	498	357	667
4	Aurangabad	146	168	164	208	181	258	191	292	199	319	221	423
5	Hyderabad	132	143	148	173	163	209	172	234	178	283	197	325
6	Belgaum	182	194	205	240	228	296	241	335	251	365	281	482
7	Poona	119	130	132	156	145	187	152	207	158	234	175	283
8	Kurnool	109	121	121	146	133	176	140	196	145	211	161	270
9	Bellary	132	150	147	185	168	227	171	256	178	278	198	364
10	Bijapur	124	147	139	185	154	233	162	266	168	292	188	394
11	Hanamkonda	189	212	215	273	241	352	256	407	466	452	300	530
12	Chitradurg	128	152	143	189	159	236	167	268	173	293	193	391
13	Ahmednagar	153	185	171	231	190	288	200	328	204	359	231	480
14	Raichur	126	149	141	185	155	230	164	260	170	284	189	377
15	Sholapur	141	159	157	195	173	238	183	267	189	290	211	377

G — Gumbel's estimate   T - II — Fisher & Tipett type II   x — Rainfall estimates

TABLE 2

Return period values of the extreme events

S. No.	Station	Highest rainfall (mm)	Date	Gumbel's method parameters			Type-II parameters		
				Mean $\bar{x}$ (mm)	St. Dev. s (mm)	R.P. value (yr)	$\bar{z}$ (mm)	s (mm)	R.P. value (yr)
1	Nagpur	315.0	12-6-1911	112.03	44.36	652	2.022	0.158	88
2	Chandrapur	249.4	14-9-1959	112.46	63.71	141	2.018	0.153	44
3	Seoni	281.9	2-8-1912	120.61	53.68	88	2.044	0.178	35
4	Aurangabad	245.1	2-9-1891	85.45	45.73	1648	1.877	0.170	87
5	Hyderabad	190.5	1-8-1954	74.16	28.32	380	1.843	0.152	74
6	Belgaum	279.4	10-7-1975	95.35	42.23	485	1.945	0.168	85
7	Poona	149.1	14-9-1892	69.89	23.80	130	1.821	0.144	43
8	Kurnool	200.1	9-9-1888	63.45	22.24	4830	1.777	0.149	164
9	Bellary	162.3	21-5-1940	74.38	28.06	100	1.841	0.164	34
10	Bijapur	176.0	30-7-1964	67.81	27.30	318	1.795	0.182	44
11	Hanamkonda	304.8	29-9-1908	91.84	47.44	585	1.916	0.201	70
12	Chitradurg	181.6	21-5-1955	71.77	37.63	322	1.823	0.175	45
13	Ahmednagar	190.0	18-9-1970	85.07	32.98	102	1.903	0.166	28
14	Raichur	170.5	7-10-1975	71.85	26.62	210	1.826	0.171	40
15	Sholapur	191.0	12-8-1940	80.19	29.66	208	1.876	0.160	47

TABLE 3

Standard error (mm) of the rainfall estimates for different return periods (yr)

S. No.	Station	S.E. G	25 yrs T-II	S.E. G	50 yrs T-II	S.E. G	100 yrs T-II	S.E. G	150 yrs T-II	S.E. G	200 yrs T-II	S.E. G	500 yrs T-II
1	Nagpur	16.1	29.4	19.2	43.0	22.4	61.2	24.3	74.4	25.9	85.3	29.7	129.3
2	Chanda	14.6	27.6	17.4	40.1	20.3	56.7	22.0	68.8	23.4	78.6	26.9	118.2
3	Seoni	19.4	38.1	23.2	57.1	27.1	83.3	29.3	102.8	31.3	118.9	35.9	186.4
4	Aurangabad	11.6	24.0	13.9	35.7	16.2	51.6	17.6	63.4	18.8	73.1	21.5	113.1
5	Hyderabad	10.4	18.2	12.4	26.5	14.4	37.5	15.6	45.4	16.7	51.8	19.2	77.8
6	Belgaum	15.5	27.4	18.6	40.6	21.7	58.4	23.5	71.7	25.0	82.5	28.8	127.2
7	Poona	8.6	15.7	10.3	22.5	12.0	31.5	13.0	37.9	13.9	43.1	16.0	63.8
8	Kurnool	8.0	15.1	9.5	21.6	11.1	30.7	12.1	37.1	12.9	42.2	14.8	63.1
9	Bellary	10.8	20.7	13.0	30.5	15.1	43.7	15.3	53.4	16.3	61.3	20.1	93.9
10	Bijapur	10.1	22.6	12.1	33.9	14.0	49.8	15.3	61.7	16.3	71.6	18.7	113.0
11	Hanamkonda	17.2	35.9	20.6	55.4	24.0	83.2	26.1	104.6	27.8	122.5	31.9	199.6
12	Chitradurg	10.1	22.3	12.1	33.3	14.0	48.5	15.3	59.7	16.3	69.0	18.7	107.7
13	Ahmednagar	11.9	24.4	14.2	36.1	16.6	51.9	18.0	63.6	19.2	73.1	22.0	112.4
14	Raichur	9.6	21.4	11.5	31.8	13.5	46.1	14.7	56.6	15.6	65.3	17.9	101.1
15	Sholapur	10.8	21.3	12.9	31.2	15.1	44.5	16.4	54.2	17.5	62.1	20.0	94.4

TABLE 4

Confidence bands of return periods of extreme events

S. No.	Station	Highest rainfall on record up to 1979 (mm)	Median (mm)	3× median (mm)	Return period: Gumbel upper limit (yrs)	Return period: Type II upper limit (yrs)
1	Nagpur*	315.0	101.5	304.5	2004	153
2	Chanda	249.4	100.6	301.8	307	74
3	Seoni	281.9	107.3	323.4	166	55
4	Aurangabad*	245.1	70.5	211.5	4122	150
5	Hyderabad	190.5	87.7	263.1	820	127
6	Belgaum*	279.4	82.6	247.8	1339	147
7	Poona	149.1	64.9	194.7	274	72
8	Kurnool*	200.1	62.3	187.0	>5000	339
9	Bellary	162.3	71.1	213.4	200	52
10	Bijapur	176.0	65.8	197.4	729	71
11	Hanamkonda*	304.8	85.7	257.1	1752	117
12	Chitradurga	181.6	67.4	202.2	690	75
13	Ahmednagar	190.0	77.3	231.9	195	42
14	Raichur	170.5	67.9	203.8	458	63
15	Sholapur	191.0	77.2	231.7	436	72

*For outliers

and 4.40. Using above values, var x_T can be computed and the standard error of estimate is given by :

$$\text{S.E.}\ (x_T) = \frac{1}{\alpha}\left(1.17+0.196\,y_T+1.099\,y_T^2\right)^{1/2}\Big/N \qquad (10)$$

where y_T is reduced variate given by Eqn. (5).

If x_T is estimated from type II distribution, then the standard error estimate can be computed by :

$$\text{S.E.}\ (z_T) = \frac{1}{\alpha}\left(1.17+0.196\,y_T+1.099\,y_T^2\right)^{1/2}\Big/N$$

where z_T is the rainfall estimate.

5. Results and discussion

5.1. *Gumbel distribution*

The 60 years (1901-60) data of 15 stations of Krishna river basin were analysed by Gumbel and Fisher & Tipett type II distributions. Rainfall estimates for return periods of 25, 50, 100, 150, 200 and 500-year were computed and listed in Table 1 (lower return periods not shown). Corresponding return periods of extreme events on record were interpolated by using both methods, which may be seen in Table 2. According to Gumbel's estimate, return periods of extreme event on record varies from 88 years to 4830 years. Such a large variation in return periods of extreme events on record suggest need for trying an alternative methodology.

As a matter of fact, the Gumbel's distribution does not give correct rainfall estimates for various return periods for the rainfall series containing outliers. The outliers were worked out as a value exceeding 3 times the median of the data series. These outliers are listed in Table 4. It may be mentioned that 5 out of 15 stations fell in the category of outliers. The Gumbel's estimate of return periods of these outliers varies from 8 to 80 time of the length of the series. This shows that these outliers should be treated with an alternative method. The authors suggest the application of Fisher Tipett type II distribution for the computation of rainfall estimates for various return periods for those stations having outliers in the series in order to avoid under estimation of design rainfall which may otherwise have serious consequences.

5.2. *Fisher & Tipett Type II distribution*

Application of Fisher & Tipett type II distribution for estimation of return periods of extreme events was made (Table 2). The return period estimates for outliers varies from 70 to 164 years.

It may be added that the above distributions can be expressed by the equation :

$$x = a\,(1 - e^{-ky})$$

where $k = 0$, represents Gumbel's distribution and $k < 0$ stands for type II distribution. Gumbel's distribution when plotted is a straight line fit and type II is a curve with no upper bound. Perhaps that is why the return period estimates are very high in case of type II distribution beyond return periods of 50 years.

5.3. *Standard error of estimates*

The rainfall events are random in nature and at 67% level of confidence are expected to be within the limits of variate $\pm$ standard error. Therefore, standard errors were computed for extremes under the probability density functions given by Gumbel and type II for all the return periods. These are shown in Table 3. By using both methods, return periods were also computed for the rainfall events (extreme on record) for lower and higher limits of extreme rainfall, *i.e.*, variate $\pm$ S.E. These are shown in Table 4. The lower limit of the variates are already covered under the extreme events itself and its return period carries no significance for statistical analysis of extremes. The return period of upper limit have further gone up in both methods. The type II values vary from 118 years to 329 years for outliers and also for significant extreme.

The fitting of Gumbel and type II distributions have been shown in Fig. 2. Three stations were chosen taking into account highest event, lowest amongst outliers and a medium event from series. Graphical representation is shown upto a return period of 100-yr only. From the graphs it can be seen that for lower values of return periods, say less than 25 years, the estimate by both methods do not differ significantly. This suggests that wherever return period values of 25 years or less are required by design engineers, Gumbel's method can be applied safely. Further, the graphs showed a large and signficant variation for the higher return periods of 50-year or 100-year which are most needed for design purposes. For such return periods due cognisance should be made of the events on record and if the highest or 2nd highest event of a series corresponds to a very high return period (more than 4 to 5 times the length of the series) Gumbel's technique should not be applied. Else, the series may be treated by type II ditribution for high return period estimates of 50-year or 100-year.

6. Conclusions

The following conclusions can be made from the study:

(1) For return periods of more than 25 years Gumbel's technique does not estimate some highest rainfall events within acceptable degree of accuracy, for certain conditions of rainfall occurrence.

(2) If Gumbel's method shows a return period of an extreme events on record more than 4 to 5 times of the length of series, type II distribution may be applied to estimate design rainfall for required return periods (50 years or more).

(3) Test for outliers should be made before application of any method for return period analysis.

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