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A mathematical model on the suppression of hailgrowth

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सार — इस शोधपत्र में गणितीय भाषा में ओलावृद्धि के लुप्तांगता पर एक गतिज निदर्श को तत्वत : प्रस्तुत किया गया है। आर्द्रताग्राही बीजायन और सूक्ष्म भौतिक प्राचलों के मध्य पारस्परिक क्रिया पर विचार किया गया और सतह पर उपलब्ध ओला और वर्षा को कुल राशि पर उनके प्रभावों का निरीक्षण किया गया। वर्तमान निदर्श ओला की वृद्धि के नियंत्रण पर कुछ प्रमुख विशिष्ट लक्षण प्रकट करता है। बीजायन और भ्रूण अर्द्धव्यास के साथ-साथ बीजायन और ओला सघनता के मध्य सहसंबंधों का प्रेक्षण किया गया।

सतह पर ओला की कम उपलब्धता की संभावना के साथ बीजायन के मामले में हिम की सांद्रता को काफी हद तक कम होते पाया गया है। वर्तमान निदर्श की तुलना बनर्जी इत्यादि जैसे लेखकों के उसी ग्रुप द्वारा विकसित निदर्श के साथ की है। इसमें देखा गया है कि आर्द्रताग्राही बीजायन ओला की वृद्धि पर नियंत्रण रखता है।

ABSTRACT. This paper presents essentially a dynamic model on the suppression of hailgrowth in mathematical terms. The interaction between the hygroscopic seeding and microphysical parameters are considered and their effects on the total amount of hail and rainfall available at the surface are investigated. The present model reveals some important characteristic features on the control of the growth of a hailstone. Correlations are observed between seeding and embryo radius as well as between seeding and the density of the hailstone.

The concentration of ice is found to reduce sufficiently in the seeded case with a possibility of less availability of hail at the surface. The present model is compared with the model developed by the same group of authors (Banerji *et al. loc. cit.*). It is found that the hygroscopic seeding controls the growth of hailstone.

1. Introduction

Hailstones are associated with violent thunderstorms which bring about considerable damage to property, aviation, agriculture, etc. The growth of the hailstone which aggravates this destruction is naturally an undesirable phenomenon. Thus, finding effective methods to curb the growth of hailstones has become the forefront of atmospheric research. In recent years much interest is being taken to find methods of controlling the growth of hailstones. The methods available so far are mostly experimental in nature.

The popular ways to suppress the growth of hailstones are by seeding water droplet dry ice, silver iodide in the region of hail formation. The choice of seeding to control the growth are influenced by the conceptual models of hailstones, the hail suppression hypothesis and techniques and, lastly, the equipments available to monitor the seeding operations.

Lacaux *et al.* (1985) analysed data collected from four different seeded storms conducted in Switzerland. They observed a definite correlation between intensity of precipitation and the concentration of seeding chemicals. Farley (1987) used a two-dimensional time dependent cloud model on a simulated multicellular hailstorm case to to study the effects of seeding by means of silver iodide and dry ice. He observed that seeding by dry ice produced a stronger response than by silver iodide in inhibiting the formation of hailstones. In both cases precipitation of ice was found to enhance initially and later slightly reduce with a modest increase in rainfall at the ground.

Cheng's (1987) study was slightly different in the sense that he made a statistical test of different hailstone parameters, *viz.*, number, concentration, kinetic energy.

The previous works have not effectively probed into the role played by intrinsic hailstone parameters in bringing about effective suppression on the growth of hailstones. In view of this the present model has been used to study this essential aspect of the hailstone in a mathematical framework. Suppression of the growth of hailstone is brought about by seeding water droplets on the nucleus. The model, in the parlour of mathematical terms, corroborates not only with the experimental results of Farley (1987), Cheng (1987) and Lacaux *et al.* (1985) but also unravels the dependence of

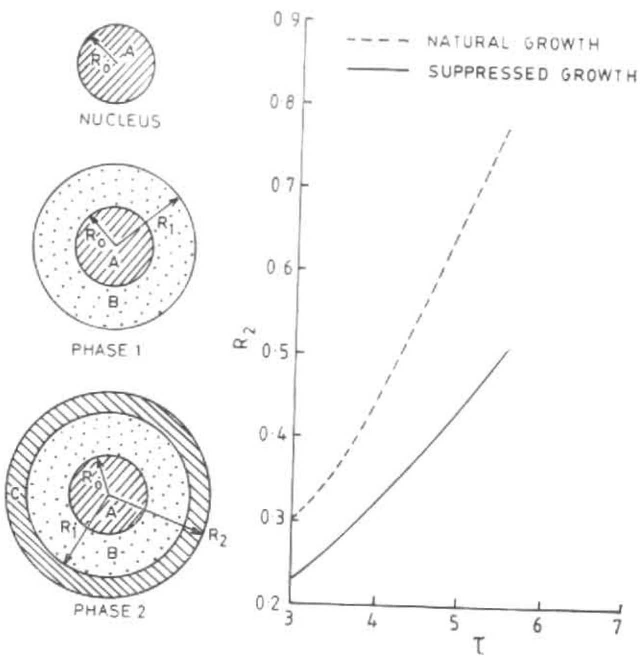

Fig. 1. Schematic change during hailstone growth and hailgrowth against τ for natural and suppressed cases

suppression of growth on hailstone parameters, such as hailstone density, embryo radius and concentration of ice. The important result of the model is that the growth of the hailstone is affected by seeding right from the nucleus stage and detects the parameters responsible to bring about this change. Those aspects of suppression relating to size, density and concentration of ice in this model are different from the other studies and shows that the mathematical model provides a far better understanding of the technique to suppress the growth of hailstone. This claim is supported by graphs based on realistic data. The conclusions obtained in this paper will be of use in the study of seeding methods especially in the Indian context.

2. Mathematical formulation and solution

The hailgrowth model developed by the authors (Banerji *et al.* 1989) involves a two-stage growth procedure of subsequent dry and wet phased. Growth of the hailstone as per the model is envisaged, to begin from a central nucleus, the embryo, which has radius R_0 and concentration C_0. The hailstone grows spherically through its first phase which is a dry growth phase. This indicates that the entire quantity of water that has been accreted freezes. The dry growth continues as long as the concentration of ice particles is more than the critical concentration. Hygroscopic seeding is assumed to be applied at the beginning of this phase of dry growth.

During the subsequent phase of wet growth, the entire quantity of water collected does not freeze immediately. It is soaked into the previously acquired porous rime or is accumulated on the surface, which on freezing, produces hard dense hailstones compatible with samples collected at the surface.

For the collection of ice in the first phase of dry growth, the concentration equation is expressed as :

$$\frac{\partial C_d}{\partial t} = \frac{K}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial C_d}{\partial r}\right], \quad R_0 \leqslant r \leqslant R_1 \qquad (1)$$

where,

C_d = Concentration of ice of the hailstone in first phase of growth,

K = Bulk density of particle,

R_0 = Radius of embryo,

R_1 = Final radius of the first phase.

r = Radial, coordinate.

t = Time.

The concentration equation in the second phase of wet growth is given by :

$$\frac{\partial C_w}{\partial t} = \frac{K}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial C_w}{\partial r}\right] - PC_w \qquad (2)$$

$$R_1 \leqslant r \leqslant R_2$$

where, C_w = Concentration of ice of the hailstone in the second phase of growth,

and P = Rate of collection of ice.

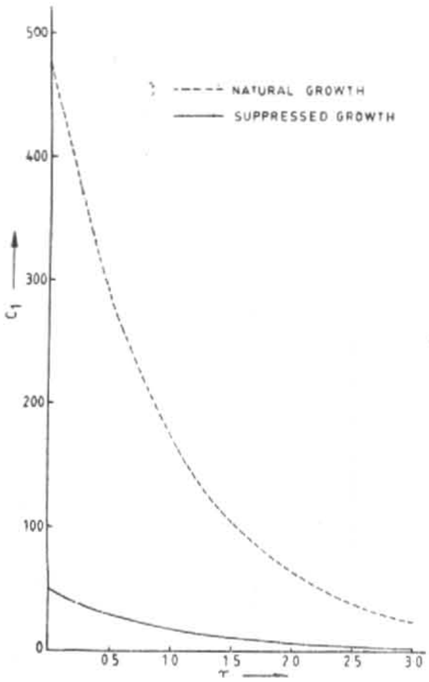

Fig. 2. A comparison of seeded and unseeded variations of concentration with τ

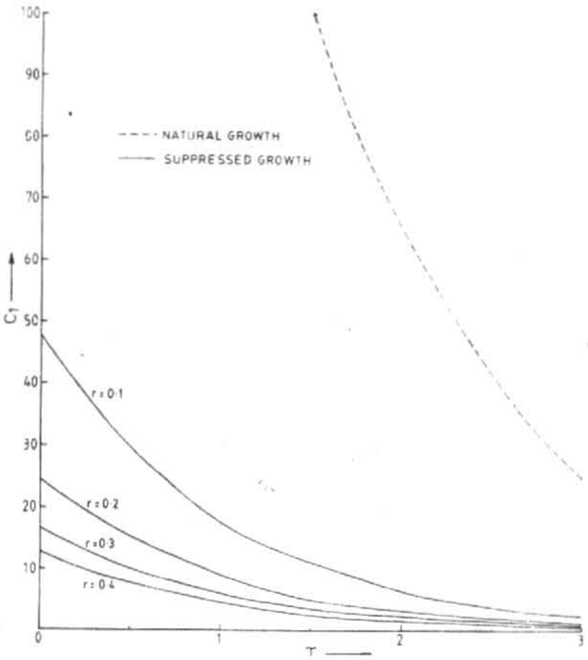

Fig. 3. Variation of concentration (C_1) with τ in dry phase at different radial distances

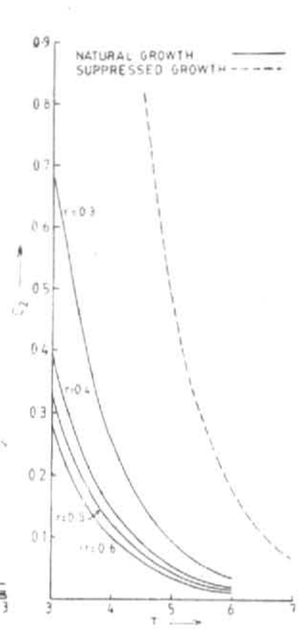

Fig. 4. Variation of concentration (C_2) with τ in wet phase at different radial distances

Seeding of water droplets is initially applied when the hailstone is in the embryo stage.

The initial and boundary conditions are :

$$t=0, r=R_0=R_1=R_2,\ C=C_0=C_0+D_r \quad (3)$$

$$t=0, r=R_0,\quad \frac{dC_d}{dr}=\frac{dC_0}{dr}=D \quad (4)$$

$$r=R_1,\ C_d=C_w \quad (5)$$

$$r=R_2,\ \frac{dC_w}{dr}=0 \quad (6)$$

The rate of growth of hailstone in the first phase of dry region is given by :

$$\frac{4}{3}\pi\frac{d}{dt}(R_1^3)=\frac{4}{3}\pi SR_1^3 \quad (7)$$

where, S=Normal rate of hailstone growth per unit volume per unit time.

In the second phase of wet region the growth rate of hailstone is given by :

$$\frac{d}{dt}\left(\frac{4}{3}\pi R_2^3\right)=\frac{4}{3}\pi SR_1^3+ + \left(S-\lambda\right)\frac{4}{3}\pi R_2^3 \quad (8)$$

where,

λ = Local rate of volume loss per unit volume per unit time.

On solving Eqns. (1) and (2) analytically under conditions (3) - (6) and writing the terms in dimensionless form, the ice concentration in first and second phases, C_d and C_w are expressed respectively as :

$$\sigma_d=\exp(-\bar{\alpha}\tau)\,.\,[\sigma_1(\sigma_2\sigma_3)+\sigma_4]/\sigma_5 \quad (9)$$

where,

$$\sigma_1=\sigma_0\sqrt{\alpha\sigma/K}+D\xi_0\,,$$

$$\sigma_2=\xi_0\sqrt{\sigma}\cos\sqrt{\sigma}\,\xi_0-2\sin\sqrt{\sigma}\,\xi_0\,,$$

$$\sigma_3=\xi_0\sqrt{\sigma}\cos\sqrt{\sigma}(\xi-\xi_0)+\sin\sqrt{\sigma}(\xi-\xi_0),$$

$$\sigma_4=D\xi_0\sin\sqrt{\sigma}\,\xi$$

and

$$\sigma_5=\xi\sigma\sqrt{\alpha\sigma/K}(\xi_0\sqrt{\sigma}\cos\sqrt{\sigma}\,\xi_0-\sin\sqrt{\sigma}\,\xi_0)$$

$$c_w=\sigma_d\,\xi_1[\sigma_6+\sigma_7]/\xi[\sigma_8+\sigma_9] \quad (10)$$

where,

$$\sigma_6=\sqrt{\alpha-P}\cos[\sqrt{\sigma(\alpha-P)/\alpha}\,(\xi-\xi_2)]$$

$$\sigma_7=\sqrt{\alpha/\sigma}\,/\,\xi_2\sin[\sqrt{\sigma(\alpha-P)/\alpha}\,(\xi-\xi_2)]$$

$$\sigma_8=\sqrt{\alpha-P}\cos[\sqrt{\sigma(\alpha-P)/\alpha}\,(\xi_1-\xi_2)]$$

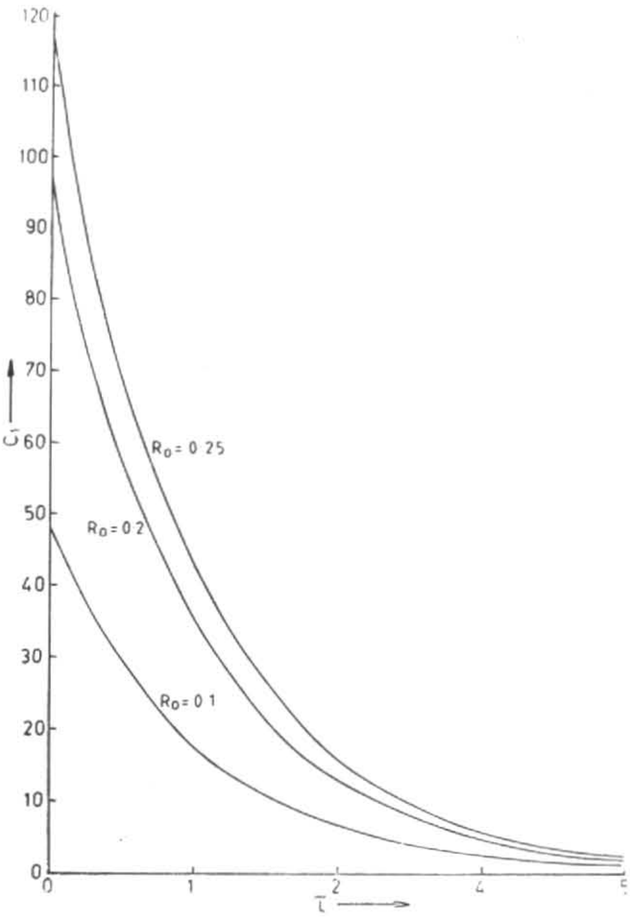

Fig. 5. Variation of concentration against τ for different embryo radii (R_0)

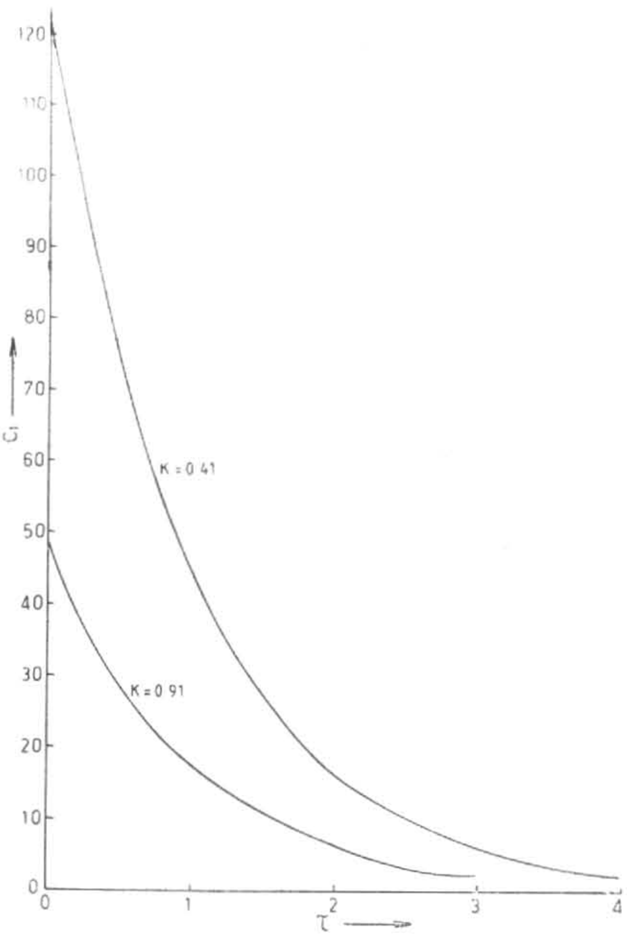

Fig. 6. Variation of concentration C with τ for different densities (K)

and

$$\sigma_9 = \sqrt{\alpha / \sigma} \; / \; \xi_2 \sin \left[\sqrt{\sigma (\alpha - P) / \alpha} \, (\xi_1 - \xi_2) \right]$$

σ_d, σ_w, σ, ξ, ξ_1, ξ_2, τ, a are dimensionless forms of C_d, C_w, C, r, R_1, R_2, t and α respectively.

Solving Eqns. (7) and (8) using conditions (3) - (6) the dry as well as the wet growth of hailstone in dimensionless form are expressed respectively as :

$$\xi_1 = \xi_0 \exp (\tau / 3) \tag{11}$$

and

$$\xi_2 = (\lambda / S) \, \xi_0^3 \exp (\tau) + \xi_0^3 (1 - S/\lambda) \times \exp [(1 - \lambda / S) \, \tau] \tag{12}$$

3. Results and discussions

The ice concentration of hailstone in the dry phase, C_d, and in the wet phase, C_w as well as the growth pattern of the hailsone in both the phases after hygroscopic seeding are given in Eqns. (9), (10), (11) and (12) respectively. The model lends support to the possibility of seeding on the embryo of the hailstone in the preliminary stage of growth (Fig. 1). The concentration of the embryo is assumed to be constant due to its extremely small dimensions, *viz.*, $C_0 = 5E - 4m^{-3}$. Three sizes of embryos have been considered, *viz.*, of radii 0.10 cm, 0.20 cm and 0.25 cm. The value of the embryo density is taken to be 0.91 gm/cm^3 and 0.41 gm /cm^3 according to Xu (1983) and the critical concentration C is fixed at 1. The value of the constant $\gamma = \lambda/S$ is arbitrarily chosen at 0.5. Figs. 2 & 3 reveal that cloud seeding at embryonic stage of hailstone considerably reduces the initial concentration of the seeded stone. Table 1 clearly elucidates a comparison of variation of ice concentrations in both seeded and unseeded cases of hailstone growth. The successful application of seeding at the initial stage of growth is revealed by the marked decrease in concentration from 500 to 47.71. The ice concentration in both phases are considerably reduced at different radial distances from the embryo (Figs. 3 & 4). In other words, the water content of the hailstone can be assumed to increase following the seeding operation. From above it can be suggested that precipitation particles can be induced to develop earlier in the life of the storm cloud with precipitation on the ground possibly occurring sooner than in unseeded cases. This is in conformity with experimental results obtained by Farley (1987).

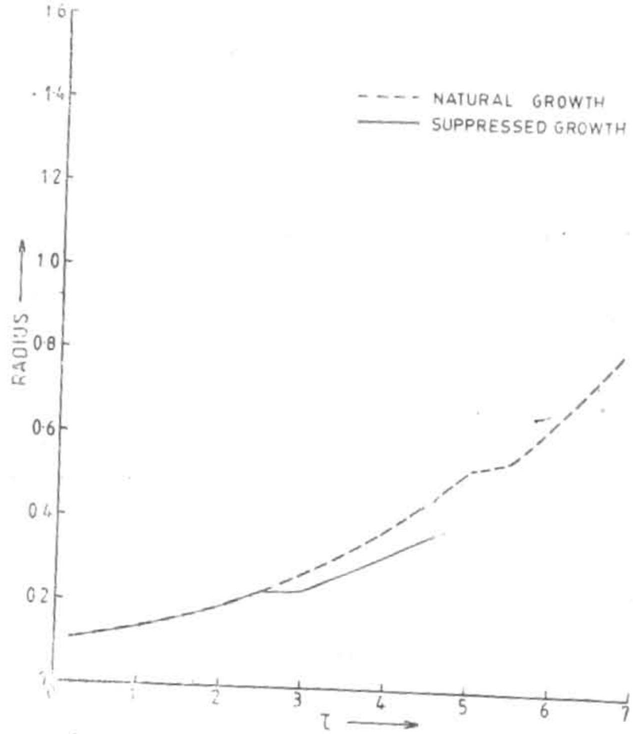

Fig. 7. A comparison between natural and suppressed hailstone growth

TABLE 1

Variation of ice concentration in dry phase with τ at different radial distances in the cases of seeded and unseeded hailstone growth for constant values of $\bar{a}=1$, $\xi_0=0.1$, $\sigma_0=500$, $K=0.91$, $D=+1$

τ	ξ_0	σ_d Unseeded	σ_d Seeded
0	0.1	500	47.71
	0.2	245.36	24.27
	0.3	172.47	16.46
	0.4	131.54	12.55
1	0.1	183.94	17.55
	0.2	93.29	8.93
	0.3	48.39	6.06
	0.4	33.33	4.62
2	0.1	67.67	6.46
	0.2	34.42	3.29
	0.3	25.20	2.23
	0.4	17.80	1.70
3	0.1	24.89	2.38
	0.2	12.66	1.21
	0.3	6.55	0.82
	0.4	4.51	0.65

TABLE 2

Variation of concentration with τ for different densities (K) for constant values of $\xi_0=0.1$, $D=+1$

τ	K	σ_d	τ	K	σ_d
0	0.91	47.71	2	0.91	6.46
	0.41	121.97		0.41	16.51
1	0.91	17.55	3	0.91	2.38
	0.41	44.87		0.41	6.07

In order to study the correlations between suppression and embryo radius and suppression and density, the model is tested for different values of radii and density. Fig. 5 reveals an exponential variation of embryo radius with seeding. It is found that embryos of smaller radius are more responsive to effects of suppression than larger embryos. Density is inversely proportional to the concentration of ice as shown in Table 2. Density of the growing hailstone bears a direct proportionality with seeding. Fig. 7 exhibits that with denser hailstones the degree of suppression is far higher than in less dense hailstone. The growth pattern in Fig. 7 reveals a break at the boundary of dry and wet phase. It has been observed that the growth of the hailstone in the case of hygroscopic seeding is much less than in the unseeded case. Though initial growth of the dry phase in both seeded and unseeded cases are the same, the wet growth stage begins much earlier in the case of suppressed growth. This conforms to the experimental results obtained by Dennis & Musil (1973) and others.

Thus, it can be concluded that hygroscopic seeding by water droplets onto the embryonic stage of hailstone is an important device to suppress the hailgrowth. The constraint of the model is that it has assumed the hailstone to grow in spherical shapes only which is not true in real situations.

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