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Study of minimum temperatures employing Markov chains

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सार – ग्रीस के सिट्रस-वर्धन क्षेत्र के लिए मार्कोव श्रृंखला के प्रयोग के साथ स्थिति निर्धारित करने वाले प्रारंभिक मानों के संदर्भ में न्यूनतम तापमानों के सांख्कीय विश्लेषण और पूर्वानुमान का प्रयास किया गया। न्यूनतम तापमानों की अवस्था में सातत्य और अवस्थाओं के मध्य संक्रमण प्रसंभावनायें सामान्य मौसम पैट्नों के साथ संबंधित है।

मार्कोव श्रृंखला का सिद्धांत एक प्रबल सांख्किीय उपकरण के रूप में दर्शाया गया है जिससे कि न्यूनतम तापमानों की काफी प्रागुक्ति (सप्ताह से भी अधिक) की जा सकती है।

ABSTRACT. Statistical analysis and prognosis of minimum temperatures with reference to states-determining threshold values, employing also Markov chains, is attempted for a citrus- growing area of Greece. In-state persistence and between-states transition probabilities of minimum temperatures are associated with common weather patterns.

Markov chains theory is shown to be a vigorous statistical tool allowing considerable (longer than a week) predictability of minimum temperatures.

1. Introduction

Prognosis of minimum temperatures is an important factor in taking tactic or strategic decisions in various economic activities, especially in short and long range agricultural planning.

Synoptic weather forecasting is acceptably reliable up to four or five days, even in the late eighties! Forecasting a certain meteorological parameter on a meso-scale, however, is not an easy task for a period practically longer than two days. Therefore, statistical forecasting of minimum temperatures, based on climatic analysis only, becomes more realistic the longer the forecast period is.

Study of minimum temperatures has been restricted mostly to frost temperatures (*e.g.*, Hocevar and Martsolf 1971; Hatzidaki-Theou 1975; Bootsma 1976; Gerber 1979; Fritton and Martsolf 1981; Liakatas and Demitropoulos 1987). Markov chains, having been applied to investigating meteorological parameters as sunshine duration (Lestienne 1978) precipitation events (Gringorten 1971; Alexandersson 1985) or hourly temperatures (Hansen and Driscoll 1977), could be a powerful tool in studying both below and above-zero minimum temperatures. Their employment will be attempted here, aiming to minimum temperatures statistical analysis and prognosis throughout the year.

2. Using Markov chains in analysis

Assuming X_v the value of a meteorological parameter on day v, like minimum temperature, one of the practical problems in meteorology is forecasting X_v and particularly when it belongs to a subset E_i ($i=1, 2 \ldots, v$), called a state, of a dismemberment of R. Considering frost, for example, it is important to know whether $X_v \in E_1 = R^-$ or $X_v \in E_2 = R^+$, where, $X_v = T_{min} = T_v$.

According to Waldteufel (1980), a good autocorrelation coefficient of X_v and X_{v+k} for $k=1$ should be expected, deminishing rapidly with $k>2$. Therefore, $X_v \in E_i$ and $X_{v+1} \in E_j$ with $i, j \in I_n$ ($I_n=\{1, 2, .., n\}$) are not independent and the Bernoulli product law :

$$P\,(X_v \in E_i, X_{v+1} \in E_j) = P\,(X_v \in E_i)*P\,(X_{v+1} \in E_j) \quad (1)$$

for independent events cannot be applied. Instead, the following is valid :

$$P(X_v \in E_i, X_{v+1} \in E_j) = P(X_v \in E_i)*P\,(X_{v+1} \in E_j / X_v \in E_i) \quad (2)$$

where, $P(X_{v+1} \in E_j / X_v \in E_i)$ are the conditional probabilities of the events $\{X_v \in E_j\}$, given that $X_v \in E_i$.

The decrease of the auto-correlation coefficient for $k>1$ allows the consideration :

$$P\,(X_v \in E_{iv} / X_0 \in E_{io}, X_1 \in E_{i1}, X_2 \in E_{i2} \ldots X_{v-1} \in E_{iv-1}) = P\,(X_v \in E_{iv} / X_{n-1} \in E_{iv-1}) \quad (3)$$

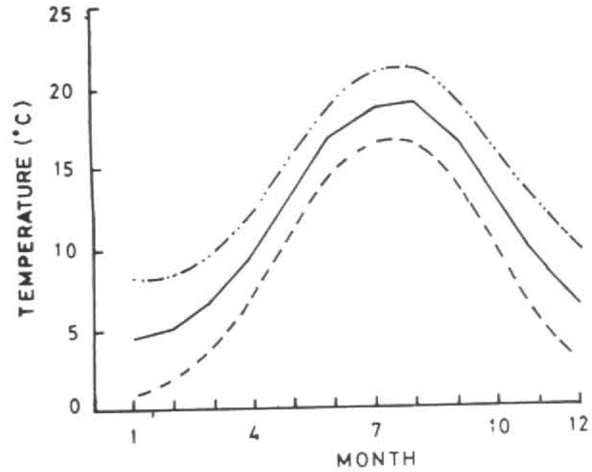

Fig. 1. Yearly variations of mean monthly minimum temp. (—) as well as *plus* (..__..) and *minus* (– —) as standard deviation

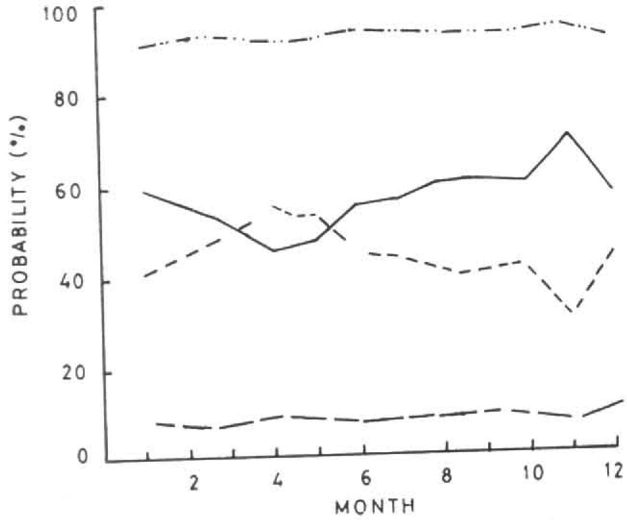

Fig. 2. Yearly variations of persistence (below-limit—·—and above limit ..__..) and transition (from below to above-limit - - - and from above to below limit — —) probabilities with reference to the $T_i—S_i$ limit

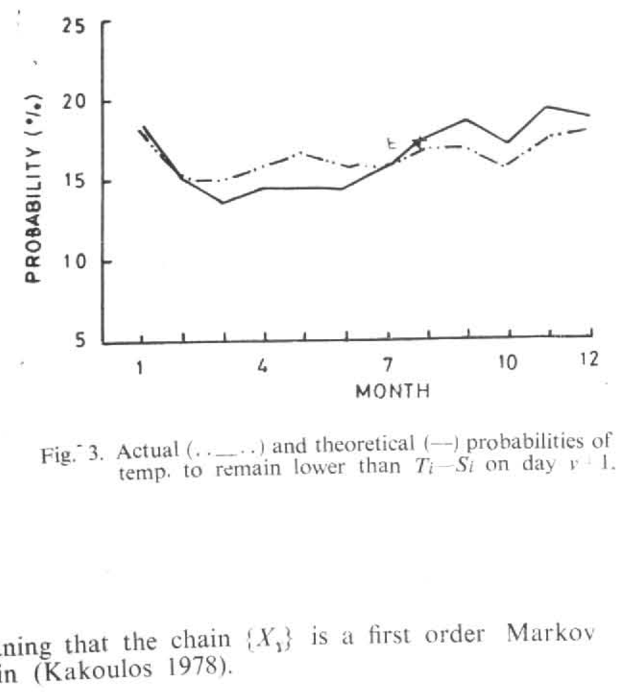

Fig. 3. Actual (..__..) and theoretical (—) probabilities of temp. to remain lower than $T_i—S_i$ on day $\nu+1$.

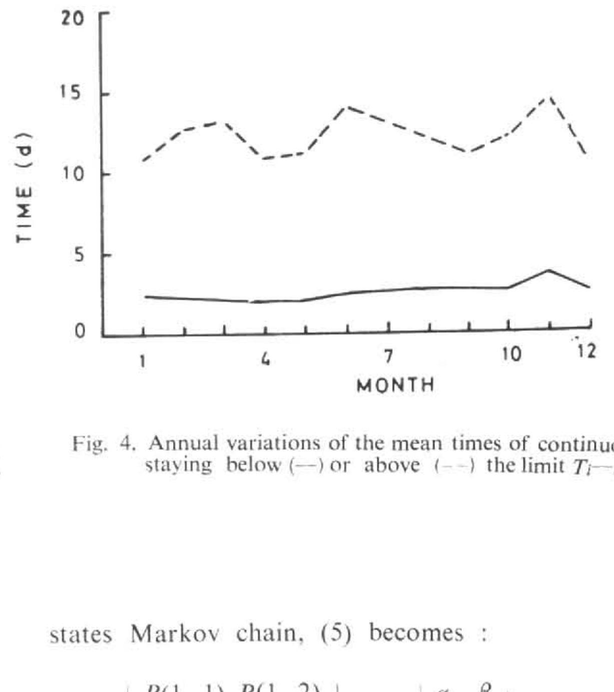

Fig. 4. Annual variations of the mean times of continuously staying below (—) or above (- -) the limit $T_i—S_i$

meaning that the chain $\{X_\nu\}$ is a first order Markov chain (Kakoulos 1978).

Probabilities of the form :

$$P_\nu (i,j) = P(X_{\nu+1} \in E_j / X_\nu \in E_i) \quad (4)$$

$(i,j \in I_2)$ are called transition probabilities from state E_i on day ν to the state j on day $\nu+1$. When $P_\nu (i,j)$ are independent of time (ν) they are stationary and the Markov chain homogeneous. In this case the matrix :

$$P_\nu (i,j) = P (X_{\nu+1} \in E_j / X_\nu \in E_i) = P (X_1 \in E_j / X_0 \in E_i) \quad (5)$$

is called transition matrix. When $n=2$, *e.g.*, a two states Markov chain, (5) becomes :

$$P = \begin{vmatrix} P(1,1) & P(1,2) \\ P(2,1) & P(2,2) \end{vmatrix} = \begin{vmatrix} \alpha & \beta \\ \gamma & \delta \end{vmatrix} \quad (6)$$

For the elements of P the following is obviously valid :

$$\alpha + \beta = \gamma + \delta = 1 \quad (7)$$

Assuming an initial distribution $\pi_0 (i)$, $i \in I2$, where, $\pi_0 (1) = \sigma = P(X_0 \in E_1)$ and $\pi_0 (2) = r = P (X_0 \in E_2)$, the probability $P_\nu(i)$ of the event $\{X_\nu \in E_i\}$ called absolute probability, is given by the relation :

$$P_\nu (i) = P (X_\nu \in E_i) = \sigma * P_\nu (1, i) + r * P_\nu (2, i) \quad (8)$$

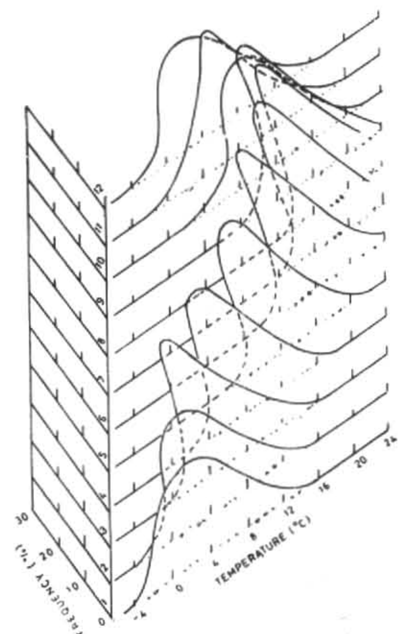

Fig. 5. Minimum temperature frequencies for each month (1 to 12)

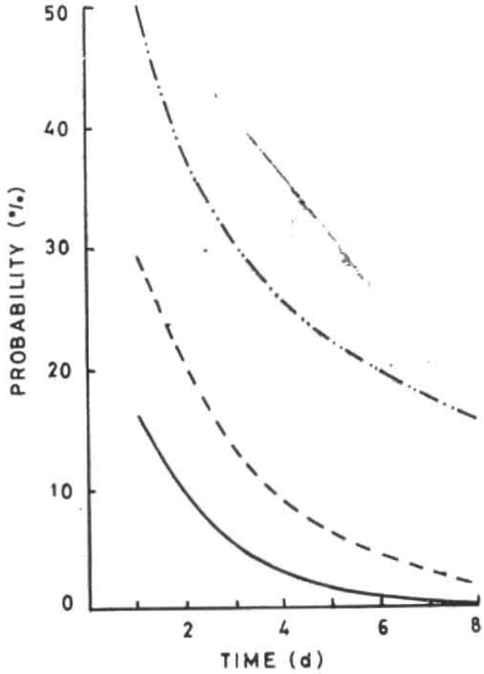

Fig. 6. Average cumulative probabilities of persistence below a threshold (T_i ..—.., T_i—$S_i/2$ — —), T_i-S_i——) up to eight consecutive days

If $k \in N$ and at the initial time $X_0 \in E_1$ the probability that the system remains in E_1 for k consecutive days and then passes to E_2 is :

$$P(A) = \alpha^{k-1} * \beta \tag{9}$$

where, A is the event : $A = \{X_0 \in E_1, X_1 \in E_1, X_2 \in E_1, \ldots\ldots, X_k \in E_2\}$.

The mathematical expectation of staying in the state E_1 is :

$$m_1 = \sum_{k=1}^{m} k*\beta*\alpha^{k-1} = \lim_{m \to \infty} \left(\sum_{k=1}^{m} \beta^* \alpha^{k-1}\right)$$

$$= \beta \{ \lim_{m \to \infty} (1 - \alpha m)/(1-\alpha)\} = 1/\beta \tag{10a}$$

and respectively in the state E_2 :

$$m_2 = 1/\gamma \tag{10b}$$

with appropriate choise of E_1 and E_2 :

$$\lim_{k \to \infty} P^k = B = \begin{vmatrix} \sigma & r \\ \sigma & r \end{vmatrix} = \begin{vmatrix} \gamma/\beta+\gamma & \beta/\beta+\gamma \\ \gamma/\beta+\gamma & \beta/\beta+\gamma \end{vmatrix} \tag{11}$$

meaning that, in a long term ($k \to \infty$) the probability of being in a state is independent of the initial state.

For $\epsilon > 0$ Eqn. (11) implies that there is at least a $n_0 \in N$:

$$\left| P_n - P_{n0} \right| \leqslant \epsilon * \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} \tag{12}$$

for all $n \geqslant n_0$

For more details reference could be made to the works of Hoel, Port and Stone (1972), Isaacson and Madsen (1975) and Vasiliou (1983).

3. Scope and data

3.1. Analysis of minimum temperatures with Markov chains using 23 years (1961-1983) data from a northwestern Greek meteorological station (Arta) is attempted, aiming to determine :

(*a*) The actual probabilities of persistence of T_v in the same state for up to eight consecutive days, having defined the states :

$$\gamma_1 : E_{i1} = (-\infty, T_i - S_i) \text{ and } E_{i2} = R - E_{i1}$$

$$\gamma_2 : E_{i1} = (-\infty, T_i - S_i/2) \text{ and } E_{i2} = R - E_{i1}$$

bars indicating average values and S being the standard deviation, with $i \in I_{12}$ the month index.

(*b*) For each of the above states the values of σ_i and r_i and the transition matrices P_1 (k, λ).

(*c*) The n_0 for which Eqn. (12) is true for $\epsilon = 10^{-5}$.

3.2. Similarly, by defining 16 states with : $E_1 = (-\infty, -6)$, $E_{16} = (22, +\infty)$ and $E_i = (-10+2^*i, -8+2^*i)$, $i = 2, 3, \ldots, 15$, and using the same data to estimate:

(*a*) The actual probabilities of persistence of T_v in in the same state for up to five days and the transition tables for each month.

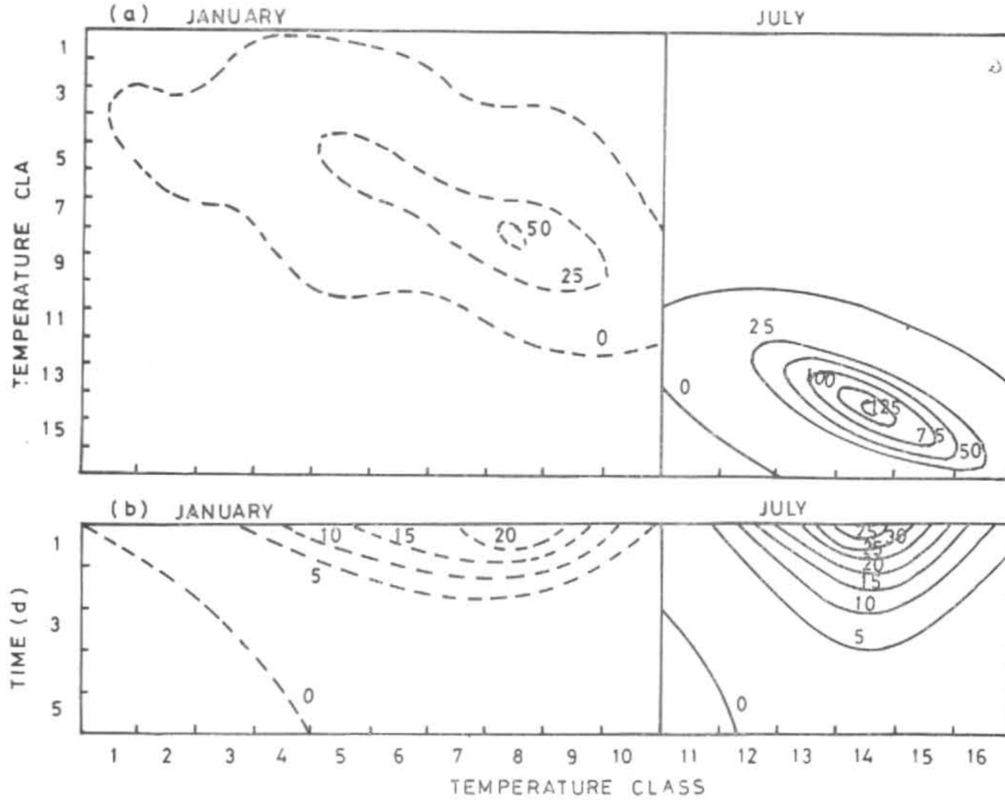

Fig. 7. (a) Frequency of occurrence of a temperature state on day $\nu+1$, starting from a certain temperature on day ν for January and July, and (b) Persistence time of the most temperature state of January and July

(*b*) The days of first and last appearance of upper limit of each state and for the corresponding time scan (in a two-states space) $E_1=(-\infty$, upper limit), $E_2=$ (upper limit, $+\infty$) the absolute and the transition probabilities.

4. Persistence and transition probabilities

4.1. *Reference to statistical limits of temperature*

From a three-dimensional graph analysis of minimum temperatures frequencies (Fig. 5), it is obvious that they are almost normally destributed within each month and so are the monthly maximum frequencies within the year. Temperature scattering is greater in winter months, whereas the more peaked distribution curves in the summer may possibly mean easier statistical forecasting of minimum temperatures in the warm season. Thus, the probability of the most frequent temperature in a month is highest in June, reaching 35% and lowest in January, not exceeding 20%. The range of the annual variation of average monthly minimum temperatures is approximately 14°C above 4° C in January, whereas S_i varies from about 3.5° C in January to 2° C in July (Fig. 1). However, T_ν may be as low as −6° C in January (Fig. 5).

To study extreme temperature cases (with probabilities lower than 16%) the persistence and transition probabilities for the states $E_{1i}=(-\infty, T_i-S_i)$ and $E_{2i}=R-E_{1i}$ are plotted for each month in Fig. 2. The probability that a minimum temperature remains higher also the next day is approximately 90% throughout the year. The complementary probability (10%) indicates the chance of the temperature to fall below the limit. The probability that the temperature remains below or jumps above the limit varies up to 20% around the 50% level, possibly in agreement with the weather changeability, determined by the passage frequency of synoptic systems of various patterns (Laliotis 1977 and Flocas 1984), as well as the proximity of the sea. Thus, for a temperature lower than T_i-S_i, its probability of becoming higher is more than 50% only in spring, whereas the probability of staying in the same state E_1^{11}, is as high as 70% in November when passage of cloud systems over west Greece also shows relevant persistence (Laliotis 1977).

The actual along with the theoretical (estimated with Eqn. 3) probabilities of temperature to remain lower than T_i-S_i on day $\nu+1$ are plotted in Fig. 3. Their yearly variations are in phase and quite those to each other, though the theoretical values are slightly underestimated in spring and overestimated in autumn.

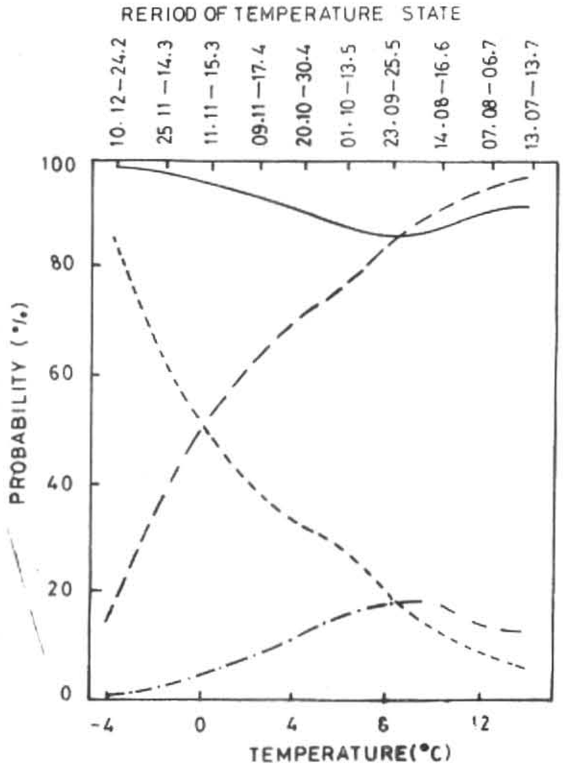

Fig. 8. Variation of the transition matrix elements α(—), β (.__.), γ (- - -) and δ (—-—) along ten minimum temperature states

The annual variations of the mean times of continuously staying below or above the limit $T_i - S_i$ estimated by Eqns. 10(a) and 10(b), are shown in Fig. 4. Below-limit persistence is more constant (about two days) whereas persistence above the limit varies between 10 days in April and 14 days in November, in agreement with corresponding probability variations in Fig. 2.

Averaged over all months (due to small yearly variations) actual cumulative probabilities of persistence below a certain threshold up to eight consecutive days appears in Fig. 6. As expected [because of Eqn. (9), suggesting an exponential pattern of probability variation, and the rather good agreement of probability variation tracks in Fig. 3] probabilities decrease exponentially with increasing time and lowering of the limit from T_i to $T_i - S_i$ starting with 50% for the higher and 16% for the lower limit. This variation pattern would allow study of persistence with the Poisson distribution (Prezerakos 1979). The theoretical values were almost indistinguishable from the actual values and this is why they do not appear in the graph. Eqn. (12) is valid with an accuracy $\epsilon = 10^{-5}$ for $\nu_0 = 12$ days for all months except April and May, where $\nu_0 = 8$.

Consequently, the rather good fit of theoretical to actual probabilities curves allows employment of a vigorous statistical theory — the Markov model — to estimate probabilities of extremely low minimum temperatures.

4.2. *Reference to critical temperature thresholds*

Since there are certain temperature values critical for plant growth and development (Alessi and Power 1971; Watts 1972; Landsberg 1975), animal life and production or human activities, it is advisable to use these as temperature thresholds in studying persistence and transition probabilities. Therefore, the minimum temperatures range was subdivided into sixteen states, as in Sec. 3.2.

The frequency of occurrence of a temperature state on day $\nu+1$, starting from a certain temperature on day ν, is given for January and July in Fig. 7(a). Centralization of maximum frequencies along the main diagonal in both months shows that the most probable value of $T_{\nu+1}$ is T_ν. The density of the frequency isopleths suggests, once more, the broader scattering of temperature in January, in comparison with July, whereas the central isopleths determine the most common states; 6° to 8° C in January and 18° to 20° C in July. The tendency of a temperature state to repeat itself the next day is not valid for temperatures below —4° C, meaning that extremely low minimum temperatures do not persist, therefore, considered transient phenomena. Such low temperatures are usually due to the quick southward-passage of cold fronts associated with polar air masses. Persistence time of the most common temperature state may be four days in July, about double that in January (Fig. 7b). As temperature departs from its most common value the probability, that it changes the next days, increases.

In Fig. 8, the variations of the transition matrix elements α, β, γ and δ for the first ten minimum temperature states are presented. Also given are the time periods determined by the dates of the first and last appearance of the upper limit of each state. Study of α, β, γ and δ was restricted to within these periods to insure homogeneity of Markov chains. Fig. 8 allows generalization of a previous observation; when the state upper limit is relatively low, lower than this minimum temperatures do not easily persist, whereas higher temperatures tend to repeat themselves. The probability of persistence below the state limit also the next day increases the higher this limit becomes (δ). In agreement with this, the frost period is defined between 11 November and 15 March, whereas temperatures lower than 14° C may be observed almost throughout the year (19 July-7 July). Transition probabilities of above-limit temperatures (β) also increase with limit becoming maximum when the limit is equal with the average for the corresponding period temperature. γ— and α— curves have a mirror-effect pattern, as being complementary to δ and β respectively. It can also be observed that frost has 50% persistence and transition probabilities whereas at both sides of the 8° C limit (approximately the yearly mean minimum temperature) persistence probabilities are equally high (about 85%) and transition probabilities equally low (about 15%).

5. Conclusions

Markov chains theory is a vigorous statistical tool that can be used for analysis and forecasting of minimum temperatures below or above a threshold value. Thresholds may be determined either by statistical parameters (statistical thresholds) or by the response patterns of plant, animal and human physiological processes to temperature (critical thresholds).

From the previous considerations it is obvious that when the forecasted (with the use of synoptic models) minimum temperature has a rather low (lower than about 30%) probability to occur, there is a great demand for supplementary assistance from Markov matrices or estimated parameters (*e.g.*, mean persistence time and mean return time).

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