

Large scale vertical motion

D. S. UPADHYAY and ONKARI PRASAD

Meteorological Centre, Jaipur

ABSTRACT. The linear second order differential equation in ω (p -velocity), derived from the equation of vorticity, neglecting its solenoidal by tilting terms, has further been reduced to Poisson's equation, by transforming (x, y, p) co-ordinate system into (x, y, P) system where $P = p\sqrt{\sigma/f}$; σ and f being the static stability of the atmosphere and coriolis parameter respectively. This transformed equation has been examined for the computation of diabatic heating temperature advection and vorticity advection components of large scale vertical motion by the methods of three dimensional relaxation.

1. Introduction

The well known quasi-geostrophic ω equation (Haltiner 1971) in (x, y, p) coordinate system may be written as follows :

$$\nabla_h^2 (\sigma\omega) + f^2 \frac{\partial^2 \omega}{\partial p^2} = \nabla_h^2 H - \nabla_h^2 \left(\mathbf{V} \cdot \nabla_h \frac{\partial \phi}{\partial p} \right) - \frac{1}{f} \frac{\partial}{\partial p} \left[J(\phi, \nabla^2 \phi) \right] \quad (1)$$

where,

- σ , is static stability of the atmosphere ;
- f , is coriolis parameter,
- H , is diabatic heating factor defined as

$$H = \frac{1}{c_p T} \frac{dQ}{dt} \text{ in which}$$

Q , is the amount of heat supplied to a unit mass of air and c_p , the specific heat of air at constant pressure;

\mathbf{V} , is the velocity of air,

ϕ is the geopotential of contour heights and the operation

$$\nabla_h^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

First term of R. H. S. of Eq. (1) $\nabla_h^2 H$ represents the effect of differential diabatic heating or cooling. Let us denote it by F_H . The second term $-\nabla^2(\mathbf{V} \cdot \nabla \phi / \partial p)$ contributes to ω due to density or temperature advection (adiabatic heating). Let it be denoted by F_T .

The third term, $-\frac{1}{f} \frac{\partial}{\partial p} [J(\phi, \nabla^2 \phi)]$ say F_τ ,

gives the fraction of ω due to the advection of vorticity. Therefore Eq. (1) may be written as

$$\nabla^2 (\sigma\omega) + f^2 \frac{\partial^2 \omega}{\partial p^2} = F_H + F_T + F_\tau \quad (2)$$

Since this is a linear equation in ω , the effects of terms on R.H.S. can be studied independently taking one at a time and the resultant vertical motion may be obtained by superposition of all these.

2. Transformation of ω equation

By assuming lapse rates γ and γ_d to be constant, σ becomes a function of pressure only. For the present we suppose that stability index is equal to the mean of σ at different standard pressure layers, i.e.

$$\bar{\sigma} = \frac{\int_{\text{Surface}}^{\text{Top}} \frac{R^2 \bar{T}}{g} \left(\frac{\gamma_d - \gamma}{p^2} \right) dp}{\int_{\text{Surface}}^{\text{Top}} dp}$$

Thus $\sigma = \bar{\sigma}$ can be taken as constant.

For the diabatic component only, the ω equation, then can be written as

$$\nabla_h^2 \omega + \frac{f^2}{\sigma} \frac{\partial^2 \omega}{\partial p^2} = \frac{1}{\sigma} F_H \quad (3)$$

Now consider the coordinate system, (x, y, P) , where

$$P = p \sqrt{\sigma} / f \quad (4)$$

we get $\frac{\partial \omega}{\partial p} = \frac{\sqrt{\sigma}}{f} \frac{\partial \omega}{\partial P}$ and $\frac{\partial^2 \omega}{\partial p^2} = \frac{\sigma}{f^2} \frac{\partial^2 \omega}{\partial P^2}$ (5)

Eq. (3) then reduces to

$$\nabla_h^2 \omega + \frac{\partial^2 \omega}{\partial P^2} = \frac{1}{\sigma} \nabla_h^2 H.$$

$$\text{or } \nabla^2 \omega = \frac{1}{\sigma} \nabla_h^2 H \quad (6)$$

$$\text{where } \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial P^2}.$$

3. Three dimensional relaxation scheme

The equation $\sigma \nabla^2 \omega = \nabla_h^2 H$ can be solved for ω by 3-dimensional relaxation method. A suitable 3-dimensional grid consisting of say, 10 points ($i=0, 2 \dots 9$) along latitude (x-axis), 6 grid-points ($J=0, 1, 2, 3, 4, 5$) along longitude (y-axis) and 5 points ($k=0, 1, 2, 3, 4$) corresponding to the pressure levels 900, 700, 500, 300 and 100 mb along vertical P-axis may be constructed.

Suitable grid length of 2° latitudes (approximately) (h) may be taken along x-and y-axes.

Thus $h = 2 \times 10^5$ m.

Along P-direction the grid length (l) becomes

$$l = (\sqrt{\sigma/f}) \times 200 = 55800 \text{ units,}$$

where $\sigma = 4^\circ\text{C}/100 \text{ mb}$ and $f = 7.3 \times 10^{-5} \text{ sec}^{-1}$.

Eq. (6) can be written in the form of difference equation as

$$\begin{aligned} &\omega(i+1, j, k) + \omega(i-1, j, k) + \omega(i, j-1, k) \\ &+ \omega(i, j+1, k) + r^2 [\omega(i, j, k+1) + \omega(i, j, k-1)] \\ &- 2(2+r^2) \omega(i, j, k) - \frac{1}{\sigma} [H(i+1, j, k) + \\ &H(i-1, j, k) + H(i, j+1, k) + H(i, j-1, k) - \\ &4H(i, j, k)] = 0 \end{aligned}$$

$$\text{where } r^2 = \frac{h^2}{l^2} = 12.8$$

$$\text{or } r = 3.6$$

4. Computation of H-field

The atmosphere gets heated by the infrared terrestrial radiation coming from the surface of the earth. The quantity of heat received at any point of the atmosphere depends on the position of the point, which exponentially decreases along longitude and vertical. If we take the reference of (x, y, P) co-ordinate axes, the quantity of heat H at any point in the space can be written as,

$$H = f(L, x, y, P),$$

where L is the wave length of earth's radiation.

The expression given below as

$$H = A \sin(2\pi x/L) \exp(-\mu y) \cdot \exp[-\lambda(P_0/P)]$$

may be justified for giving satisfactory values of H at the grid points of the atmospheric parallelepiped considered under the present scheme. Also experimental evaluations have shown that the following values of the constants A, L, μ and λ can be acceptable.

$$A = 6.0^\circ \text{ C/day}$$

$$L = 15.0 \text{ microns} = 15.0 \times 10^{-6} \text{ m. and}$$

$$\mu = \lambda = 3$$

5. Boundary conditions

ω may be assumed to be zero at top of the atmosphere and also at each grid points on the side walls of the parallelepiped. It is not justified to make the ω vanished at the bottom of the atmosphere, which is 900-mb level in the present case. At this level vertical motion depends mainly on the divergence in the frictional layer (100 to 1000 m above ground) and can be computed with the help of Ekman spiral.

As suggested by Das (1969) vertical velocity at the top of frictional layer may be given by—

$$\omega_0 = - \frac{g\rho_0}{f^2} \sqrt{\frac{Kf}{2}} \sin 2\delta \cdot \nabla^2 \phi$$

where K is the coefficient of eddy viscosity and δ represents the angle between actual and geostrophic winds.

For $g = 980 \text{ cm sec}^{-2}$, $\rho_0 = 10^{-3} \text{ gm. cm}^{-3}$
 $f = 10^{-4} \text{ sec}^{-1}$, $K = 10^4 \text{ cm}^2 \text{ sec}^{-1}$ and $\delta = 15^\circ$,
the vertical velocity at $p_0=900\text{-mb}$ level becomes

$$\omega = - 35 \times 10^6 \nabla^2 \phi$$

which may be taken as the initial values at the bottom grid points of the parallelepiped.

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