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Objective analysis of wind field over Indian region by optimum interpolation method

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सार – इष्टतम ग्रन्तर्वेशीय विधि का उपयोग करके भारतीय क्षेत्र पर पवन क्षेत्र के वस्तुनिष्ठ विश्लेषण के लिए एक ग्रधि ोजना तैयार की गई है। इसके लिए भारत में 850, 700 एवं 500 मि० बार वाले स्तरों पर पवन क्षेत्र स्वसहसम्बन्ध फलनों एवं संरचना फलनों को ग्रभिकलित किया गया है।

इन सांख्यिकी संरचना फलनों का दिए हुए ग्रिड बिन्दु का संदर्भ देते हुए प्रेक्षणी स्टेशनों के भारीय गुणकों की गणना के लिए उपयोग में लाया गया है ग्रौर 24 जुलाई से 31 जुलाई 1979 तक का 8 दिन का 850 एवं 700 मि० बार स्तरों पर वस्तुनिष्ठ विश्लेषण किया गया है। वस्तुनिष्ठ विश्लेषण का संख्यात्मक मान ज्ञात करने के लिए त्रुटियों के वर्ग के माध्य के वर्ग मूल को वस्तुनिष्ठ विश्लेषण क्षेत्रों के प्रेक्षणी स्टेशनों की पवनों का ग्रन्तर्वेषण करके ग्रभिकलित किया गया है ग्रौर वास्तविक प्रेक्षित पवनों से उनकी तुलना की गई है। वस्तुनिष्ठ विश्लेषण से प्राप्त मानसून प्रणाली के केन्द्रों की व्यक्तिनिष्ट विश्लेषण से भी तुलना की गई है।

ABSTRACT. A scheme for objective analysis of wind field over Indian region was drawn up using optimum interpolation method. For this the autocorrelation functions and the structure functions of the wind field over India have been computed for 850, 700 and 500 mb levels.

These statistical structure functions were used in calculating the weighting factors for the observing stations with respect to the given grid point and objective analyses were made at 850 and 700 mb levels for eight days from 24 July to 31 July 1969. In order to evaluate the objective analyses quantitatively RMS errors were computed by interpolating the winds at the observing stations from the objectively analysed field and comparing with actually observed winds. Also, centres of the monsoon system obtained in objective analyses were compared with those in subjective analyses.

1. Introduction

Meteorological variables observed at randomly situated observing stations have to be interpolated to equally spaced grid points for various prognostic as well as diagnostic studies by numerical methods. Data from satellite, commercial ships, aircrafts and from other non-conventional platforms, observed at non-synoptic hours also have to be utilised in the interpolation of the data. In order to make use in the analysis a large amount of data which are from different sources both conventional and non-conventional types, suitable objective analysis scheme is required. One of the most popular schemes of objective analysis is based on the successive correction method formulated by Cressman (1959). In this method, the anomaly or the deviation of a meteorological variable at a given grid point from the initial guess field is computed as the linear combination of those anomalies at the surrounding observing stations giving weightage to the inverse of distances of the stations from the grid points. Although the weighting functions in this scheme are empirically determined, the scheme provides fairly acceptable results. However, it is logical that the weighting functions should be obtained on the basis of characteristic functions of the statistical structure of the given variable over the given region as stated by Gandin (1963) and Eddy (1967).

Several workers, like Thiebaux (1973, 1974), Schlatter (1975), Peterssen (1973), Rutherford (1973), Kruger (1969), Bergman (1979) and others have since then computed the weighting functions on the basis of statistical structure functions of the given meteorological variable following varying procedures. For tropical regions, Alaka and Elavander (1972A, 1972B) and Ramanathan *et al.* (1973) have made studies respectively over Caribbean region and Indian region. The latter study has been on the computation of structure function and autocorrelation function of wind fields at 500 mb for winter period. The authors (1980) made a preliminary study by computing statistical structure functions for 850, 700, 500 mb levels for the monsoon period over Indian region and drawing up an analysis scheme using the weighting functions based on these computations. This analysis from the optimum interpolation method was compared with two versions of the Cressman's method for a single case for 850 mb level. In the present study, the objective analysis scheme was tested further by making analyses of wind field at 850 and 700 mb levels for eight situations from 24 July to 31 July 1969 and comparing with subjective analyses. RMS errors were also computed by interpolating winds at the observing points from the objectively analysed wind field at the surrounding grid points and comparing with actually observed winds.

2. Method

Gandin (1963) has derived an expression for the weighting functions for the observing stations with respect to the grid points incorporating physical characteristics of the parameter over the given region. This is done by computing autocorrelation functions and structure functions as follows. If we consider f_i, $\hat{f}_i$, $\tilde{f}_i$ and $\hat{f}_i'$ to be the true value, the observed value, the initial guess value and the anomally or the deviation of the observed value from the initial guess value at the station i ,we have the following relations :

$$\hat{f}_i' = \hat{f}_i - \tilde{f}_i$$

$$\hat{f}_i = f_i + \epsilon_i$$

where ϵ_i is the total error in the observed value $\hat{f}_i$ of the parameter. This can be instrumental or observational or that is introduced during coding, decoding and transmission of the data.

The anomalies of the observed values at the observing stations are better correlated with those at the grid points than the actual values. Hence the anomalies at the grid points are computed as the linear combination of the anomalies at the stations (Eqn. 1).

$$f_0' = \sum_{i=1}^{n} \hat{f}_i' \, P_i + I_0 \tag{1}$$

where, P_i is weighting function, n is the total number of the stations around the grid point and I_0 is the error in the interpolation which becomes zero for exact calculation of f_0'. Now, we have

$$\overline{I_0^2} = \overline{(f_0' - \sum_{i=1}^{n} \hat{f}_i' \, P_i)^2} \tag{2}$$

$$\overline{I_0^2} = \overline{[f_0' - \Sigma (f_i' + \epsilon_i) P_i]^2} \tag{3}$$

Now, two important assumptions are made, firstly ϵ_i ,the total error in the valur at the station i, is independent of the anomaly of the true value, f_i' at that place and also independent of the total error ϵ_j at other stations

$$i.e., \quad \left.\begin{aligned} &\overline{f_i' \epsilon_i} = 0 \\ &\overline{\epsilon_i \epsilon_j} = 0 \quad \text{when } i \neq j \\ &\overline{\epsilon_i \epsilon_j} = \sigma^2_{\epsilon i} \text{ when } i = j \end{aligned}\right\} \tag{4}$$

Here $\sigma^2_{\epsilon i}$ denotes the mean square random observational error or simply random error. The second assumption implies that the random error do not effect the values of covariances but increase the true variances by an amount $\sigma^2_{\epsilon i}$ (Gandin 1963, Alaka and Elvander 1972 A).

By invoking the above assumptions (Eqn. 4), the Eqn. (3) can be written as

$$\overline{I^2_0} = \sum_{i=1}^{n} \sum_{j=1}^{n} \overline{f_i' f_j'} \, P_i P_j + \sum_{i=1}^{n} \sigma^2_{\epsilon i} \, P_i^2 - 2 \sum_{i=1}^{n} \overline{f_i' f_0'} \, P_i + \sigma^2_0 \tag{5}$$

In order to have the best interpolation, the mean square interpolation error, $\overline{I_0^2}$ has to be minimum over the given region. This is achieved by the following condition (Eqn. 6) :

$$\frac{\partial \overline{I_0^2}}{\partial P_i} = 0 \tag{6}$$

As a result of this condition (Eqn. 6), we get the following set of equations :

$$\sum_{j=1}^{n} \overline{f_i' f_j'} \, p_j + \sigma^2_{\epsilon i} p_i = \overline{f_i' f_0'} \; (i = 1, 2, 3 \ldots, n) \tag{7}$$

where p_i's are the optimum values of the weighting functions P_i's when $\overline{I_0^2}$ is minimum.

It can be proved easily that $\overline{\hat{f}_i' \hat{f}_j'} = \overline{f_i' f_j'}$ and $\overline{\hat{f}_i' f_0'} = \overline{f_i' f_0'}$ (Alaka and Elvander 1972 B). Hence the set of equations (Eqn. 7) can be written as,

$$\sum_{j=1}^{n} \overline{\hat{f}_i' \hat{f}_j'} \, p_j + \sigma^2_{\epsilon i} \, p_i = \overline{\hat{f}_i' \, f_0'} \; (i = 1, 2, 3 \ldots, n) \tag{7a}$$

Also we get an expression, for the minimum error, E, from Eqns. (5) and (7).

$$E = \overline{I_0^2}_{\,\text{Min}} = \sigma_0^2 - \sum_{i=1}^{n} \overline{\hat{f}_i' \, f_0'} \; p_i \tag{8}$$

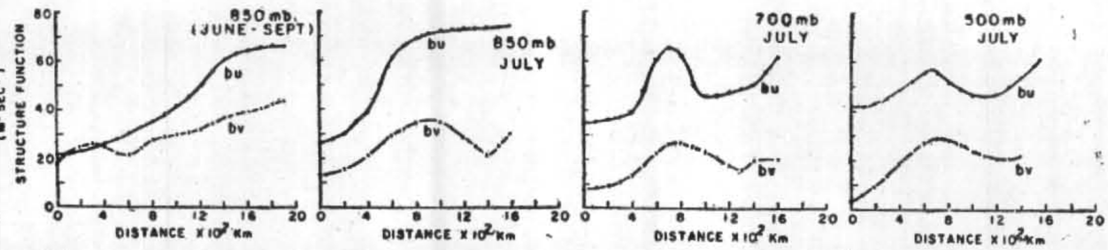

Fig. 1. Structure function of u and v components of wind

Assuming that over the area of synoptic scale, the variances are homogeneous and the covariances are both homogeneous and isotropic, the set of Eqns. (7a) can be normalised by dividing by the corrected mean variance, σ_0^2 which is constant for the given region

$$\sigma_0^2 = \overline{\sigma_i^2} - \sigma^2_{\epsilon_i}$$

The set of Eqns. (7a) and Eqn. (8) respectively become

$$\sum_{j=1}^{n} \mu_{ij} p_j + \lambda^2 p_i = \mu_{0i} \quad (i=1,2,3....n) \quad (9)$$

and

$$E = \sigma_0^2 (1 - \sum_{i=1}^{n} \mu_{0i} P_i) \quad (10)$$

$$\text{where, } \mu_{ij} = \frac{\overline{\hat{f}_i' \hat{f}_j'}}{\sigma^2_0} ; \mu_{0i} = \frac{\overline{\hat{f}_i' f_0'}}{\sigma_0^2} \text{ and } \lambda^2 = \frac{\sigma^2\epsilon_i}{\sigma_0^2} \quad (11)$$

The random error $\sigma^2\epsilon_i$ required in Eqn. (11) in calculating λ^2 is obtained indirectly from the computations of structure function. The structure function, b_f is given by :

$$b_f(\rho) = \overline{(f_i' - f_j')^2} \quad (12)$$

where,

$$\rho = | \mathbf{v}_i - \mathbf{v}_j | \quad (13)$$

$\mathbf{v}_i$ and $\mathbf{v}_j$ are the position vectors of the stations i and j. The estimated structure function is,

$$\hat{b}_f(\rho) = \overline{(\hat{f}_i' - \hat{f}_j')^2} \quad (14)$$

As shown by Gandin (1963), $\hat{b}_f(\rho)$ is given as

$$\hat{b}_f(\rho) = b_f(\rho) + 2\sigma^2\epsilon_i \quad (15)$$

$b_f(\rho)$ becomes zero as ρ becomes zero whereas $\hat{b}_f(\rho)$ need not become zero at the same station, since, if the observations are made by different individuals using different instruments, observed values need not be identical (personal errors in the observations and the instrumental errors are present). So, we have

$$\hat{b}_f(0) = 2\sigma^2\epsilon_i \quad (16)$$

In other words, the curve of the estimated structure function, $\hat{b}_f$ extrapolated to zero distance gives twice the mean random error $2\sigma^2\epsilon_i$.

3. Data and computations

As the interest of this study is on the monsoon period, the statistical structure functions were calculated for 850, 700 and 500 mb levels for the month of July and only for 850 mb level for the monsoon period, *viz*, from June to september. For this daily upper wind data of three July months (1969, 1970 and 1971) from radiosonde stations and pilot balloon stations over Indian region and these wind data from 1 June to 30 September for three years are utilised. Wind field being vector field, has been resolved into u and v components and considering them as two separate parameters, the autocorrelation functions and the structure functions were computed separately.

3.1. *Structure function and autocorrelation function*

The structure function, $\hat{b}_f$ (observed) given by Eqn. (14) were computed at the three levels for u and v components separately and plotted against the distance, ρ (Fig. 1). The mean random errors $\sigma^2\epsilon_i$ at the various levels for both u and v components were obtained from Fig. 1 (Eqn. 16) by extrapolating the structure function curves to zero distance ($\rho = 0$) and these mean random errors were given in Table 1. The covariances $\overline{\hat{u}'_i \hat{u}'_j}$ and $\overline{\hat{v}'_i \hat{v}'_j}$ were computed for every stations with respect to every other stations in the region. They were normalised by dividing by σ^2_0 before plotting them against distance between the two stations i and j. Although the autocorrelation function curve reached zero after certain distance, there was scatter of points. As observed by Alaka and Elvander (1972A), the scatter of points was partly due to an isotropy and non-homogeneity of the true correlations. Hence points within 1° segment were averaged using Eqn. (17) to represent the mid-point of the segment. These auto-correlation function curves are given in Fig. 2. Then, these values of normalised covariances or auto-correlation functions μ are fitted to a curve represented by Eqn. (18).

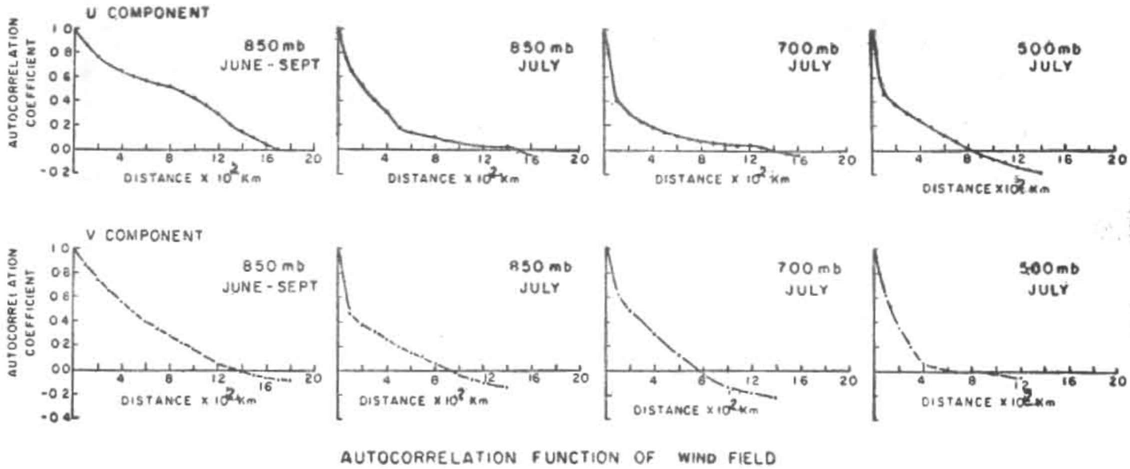

Fig. 2. Autocorrelation function of *u* and *v* components of wind

TABLE 1

Estimates of random errors

	Period	Level	Extrapolated value of structure function at distance $\rho=0$	$\sigma^2_{\epsilon i}$ (m^2s^{-2})	Random error $\sigma_{\epsilon i}$ ($m\ s^{-1}$)
u-Component	June-Sept	850	21.5	10.75	3.28
	July	850	26.0	13.0	3.61
	July	700	35.0	17.5	4.18
	July	500	41.5	20.75	4.55
v-Component	June-Sept	850	17.5	8.75	2.96
	July	850	12.5	6.25	2.50
	July	700	8.0	4.0	2.00
	July	500	4.0	2.0	1.41

$$\mu(d) = \sum_{i=1}^{N_T} \mu(\rho_i) \frac{N_i \left[\frac{1}{2} + \frac{1}{2} \cos \frac{\pi(\rho_i - d)}{100} \right]}{\sum\limits_{j=1}^{N_T} N_j \left[\frac{1}{2} + \frac{1}{2} \cos \frac{\pi(\rho_j - d)}{100} \right]} \tag{17}$$

N_T is the total number of covariance values contributing to μ.

N_i is the number of points within 1° segment.

$$\mu(\rho) = (A e^{-B \rho^C} + 1 - A) \cos D\rho \tag{18}$$

where A, B, C and D are constants. The $\mu(\rho)$ curve becomes 1 at $\rho=0$ and it decreases exponentially and reaches zero after certain distance. The constants A, B, C and D were evaluated separately for u and v components at 850, 700 and 500 mb levels. These constants for various cases are given in Table 2.

3.2. *Weighting functions and objective analysis*

As we have assumed earlier that the autocorrelation function to be homogeneous over the region, $\mu(\rho)$ for any given distance, ρ could be computed from Eqn. (18), knowing the constants A, B, C and D for u and v components separately. Thus μ_{ij} for the pair of stations i and j and μ_{oi} for the station i and the grid point, 'o' were obtained, $\sigma^2_{\epsilon i}$ was obtained from structure function curves b_u and b_v for u and v components (Fig.1) and subsequently, λ^2. Thus the set of Equations (Eqn 9) was formed and then solved to get the weighting functions, p_i's of the stations ($i=1,2..n$) with respect to each of the grid points in the region. As there is no restriction on the number of station considered around the grid points, there has to be a constraint on the sum of the weighting functions and hence the sum of the weighting functions of the stations with respect to a grid point is not allowed to exceed one

$$i.e., \quad \sum_{i=1}^{n} p_i \leqslant 1 \tag{19}$$

$$\text{if} \quad \sum_{i=1}^{n} p_i > 1$$

$$\text{then} \quad p_i = \frac{p_i}{\sum\limits_{i=1}^{n} p_i} \tag{20}$$

Knowing the weighting functions, the anomalies at the grid points were computed as the linear combination of anomalies at the surrounding stations giving appropriate weights to them (Eqn.1). The climatological values at the grid points were the initial guess field. The u and v components were separately analysed daily from 24 July 1969 to 31 July 1969 at 850 and 700 mb levels. During

TABLE 2

Values of the constants of the empirical curves fitted for auto-correlation functions

No.	Period/ Month	Level (mb)	Constants for *u*-component				Constants for *v*-component				Distance where ρ becomes zero	
			A	B	C	D	A	B	C	D	*u*-comp.	*v*-comp.
1	June-Sept	850	0.789	0.688	0.402	0.920	0.770	1.145	0.860	1.140	14.8°	12.6°
2	July	850	1.250	1.180	0.560	1.140	0.825	2.240	0.350	1.680	13.0°	8.5°
3	July	700	0.975	2.400	0.410	1.080	1.060	1.100	0.420	2.030	12.8°	7.0°
4	July	500	1.028	1.170	0.210	1.890	1.080	3.410	0.920	2.370	7.6°	6.0°
5	July*	850	0.895	2.120	1.060	0.441	1.070	2.050	1.500	0.000	—	—
6	July*	500	1.020	1.710	1.360	0.190	1.140	2.130	1.830	0.340	—	—

*(After Alaka and Elvander 1972 A)

TABLE 3

Centres of monsoon depression/low in subjective and objective analysis and the vector difference of centres in both cases

No.	Date	Analysis at 850 mb			Analysis at 700 mb		
		Subjective	Objective	Vector diff. (km)	Subjective	Objective	Vector diff. (km)
1	24 July 1969	19° 30′N 86° 0′E	20° 30′N 85° 30′E	124	20° 30′N 86° 0′E	21° 0′N 85° 0′E	124
2	25 July 1969	20° 0′N 84° 30′E	21° 0′N 84° 30′E	110	20° 30′N 85° 0′E	21° 0′N 85° 0′E	55
3	26 July 1969	26° 0′N 80° 0′E	25° 0′N 82° 0′E	248	—	—	
	*	20° 30′N 87° 30′E	—	—	20° 30′N 87° 30′E	22° 0′N 87° 0′E	175
4	27 July 1969	21° 30′N 88° 0′E	23° 0′N 86° 30′E	235	20° 0′N 88° 0′E	20° 30′N 87° 30′E	78
5	28 July 1969	20° 30′N 89° 0′E	21° 0′N 88° 0′E	124	19° 30′N 89° 0′E	19° 30′N 87° 30′E	166
6	29 July 1969	20° 0′N 89° 0′E	21° 0′N 89° 0′E	110	20° 0′N 88° 0′E	19° 0′N 88° 0′E	110
7	30 July 1969	21° 30′N 87° 0′E	21° 30′N 87° 30′E	55	21° 30′N 86° 30′E	22° 0′N 87° 0′E	78
8	31 July 1969	22° 30′N 85° 30′E	22° 30′N 85° 0′E	55	22° 0′N 85° 0′E	22° 30′N 85° 30′E	78

* There were two systems in subjective analysis at 850 mb.

TABLE 4

RMS errors obtained by comparing observed winds with objectively analysed field

Date	RMS Error in mps					
	850			700		
	u-comp	*v*-comp	wind	*u*-comp	*v*-comp	wind
24 July 1969	2.3	2.6	3.5	4.5	3.1	5.5
25 July 1969	2.5	2.6	3.6	5.0	2.7	5.7
26 July 1969	2.3	2.9	3.7	5.2	2.5	5.8
27 July 1969	2.5	2.3	3.4	3.1	2.5	4.1
28 July 1969	2.0	2.5	3.2	4.6	3.7	5.9
29 July 1969	2.1	3.8	4.3	5.2	3.7	6.4
30 July 1969	2.4	3.5	4.2	5.1	3.8	6.4
31 July 1969	2.8	2.6	3.9	5.9	3.9	7.1

this period, on the first two days there was a weak system and on 26th another low was formed at north Bay, then intensified into deep depression and moved inland. Thus we have different situations for making the objective analysis and examinig how the monsoon system has been depicted on different days. The objective analyses and the corresponding subjective analyses for three typical cases at 850 mb and for two typical cases at 700 mb are given in Fig. 3 (a-e).

For examining the objective analyses quantitatively, rms errors were computed by interpolating the values at the observing stations from the objectively analysed field at the grid points, and comparing them with the observed values. The rms errors for all the days at the two levels are given in Table 4.

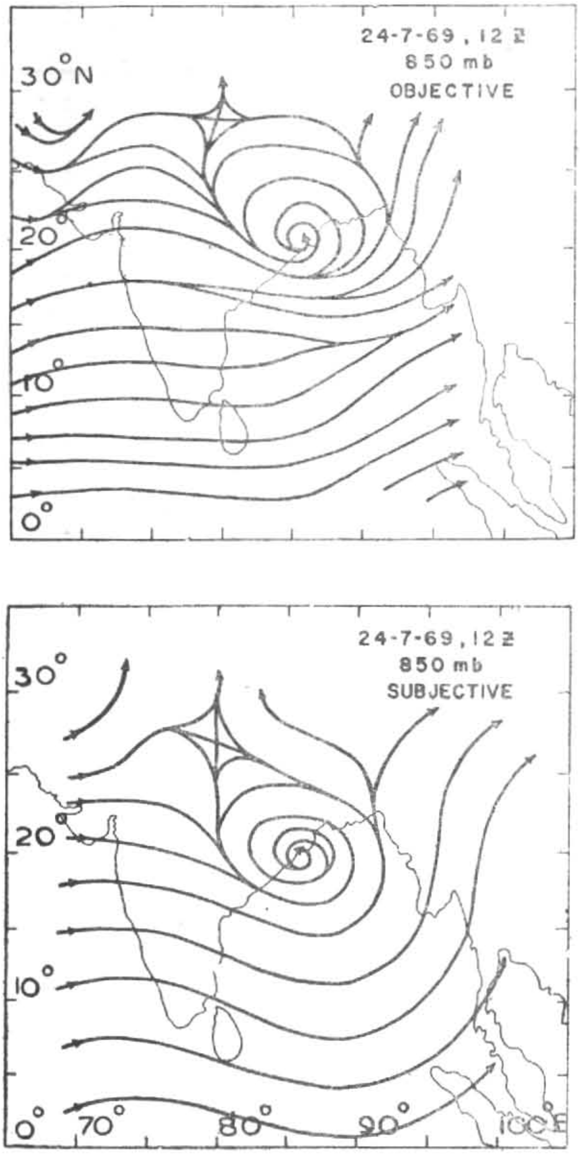

Fig. 3(a). Objective and subjective analyses for 24 July 1969 at 850 mb level

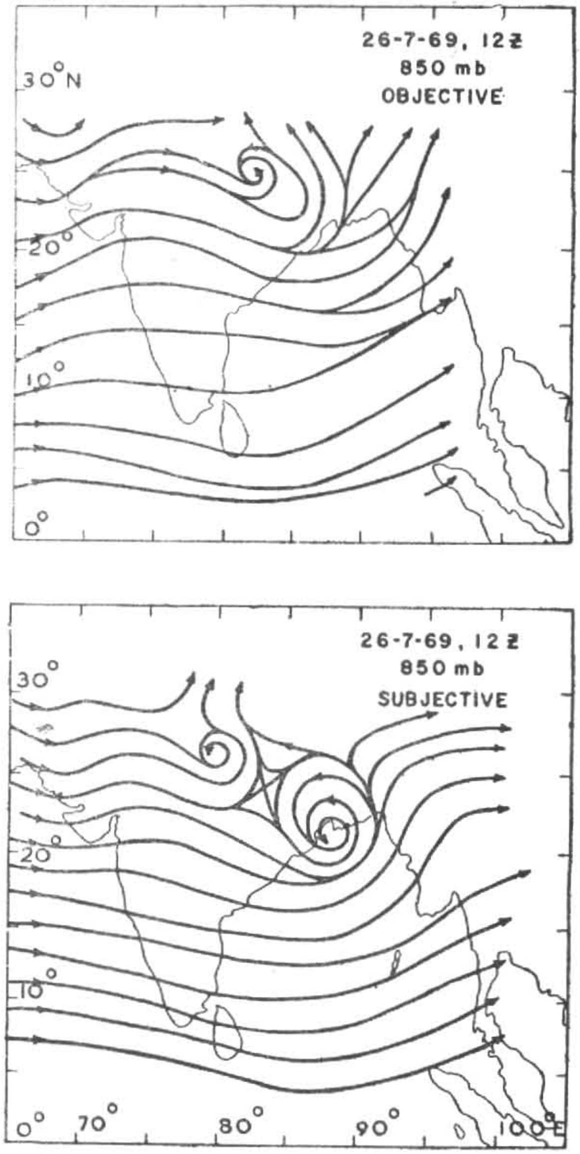

Fig. 3(b). Objective and subjective analyses for 26 July 1969 at 850 mb level

4. Discussion of results

4.1. *Statistical structure functions*

The autocorrelation curves show that for u and v components, they decrease with distance and become zero respectively at 13.0° and 8.5° at 850 mb level, at 12.8° and 7·0° at 700 mb levels and at 7.6° and 6° at 500 mb level. In other words, this means that waves of wave-length of about 2500 km affect 850 and 700 mb levels whereas much smaller waves affect 500 mb level. Ramanathan *et al.* (1973) have computed autocorrelation functions and found that in winter period at 500 mb, the auto-correlation function μ becomes zero at greater distance. Also they have obtained the correlation coefficient patterns to be elliptic whereas the authors (1980) have obtained the correlatoin coefficient patterns to be almost circular at this level (500 mb). Hence it may be inferred that whereas in winter at 500 mb longer waves affect the region and the atmosphere is not isotropic, in July shorter wave affect the region and the atmosphere is nearly isotropic. It is relevant to mention here that in the monsoon season 500 mb level is a transition level since the lower level southwesterlies/westerlies change to easterlies around this level.

The mean random errors, $\sigma^2_{\epsilon_i}$ obtained from the structure function curves, vary from 1.5 to 5 mps. The structure functions as well as the auto-correlation functions for this period July are also different from those obtained by Ramanathan *et al.* (1973) for winter period over Indian region and by Alaka and Elvander (1972A) for winter

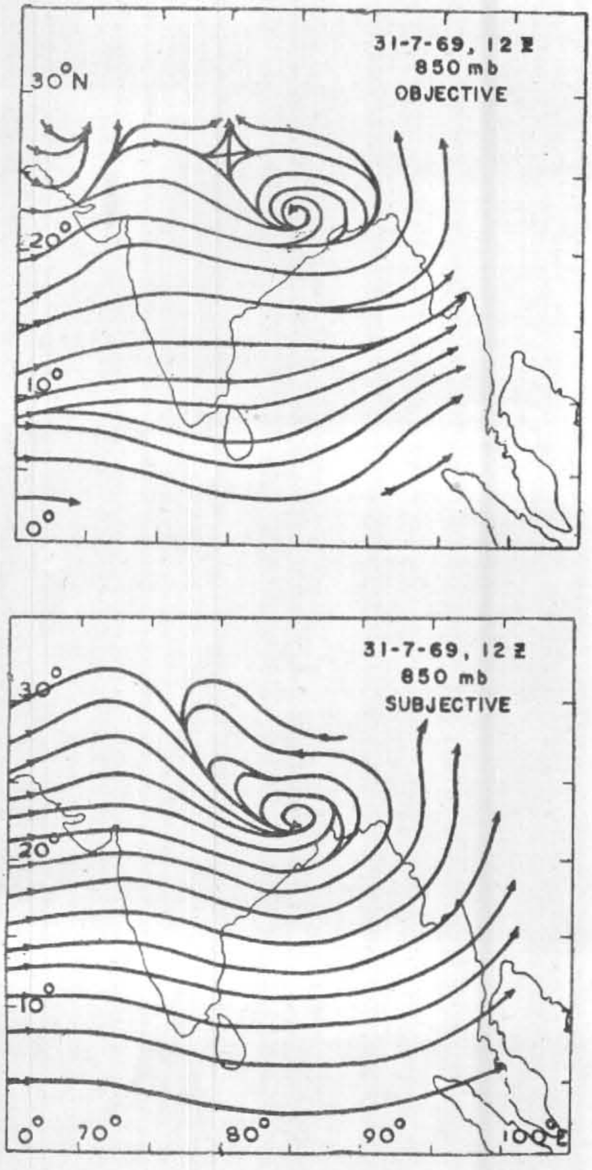

Fig. 3(c). Objective and subjective analyses for 31 July 1969 at 850 mb level

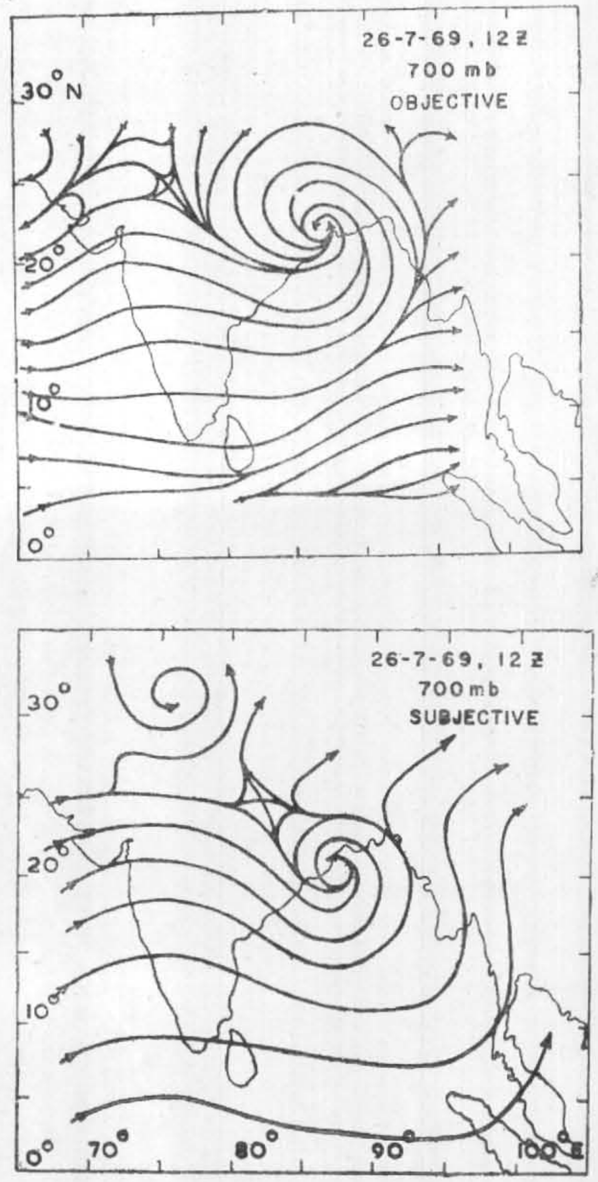

Fig. 3(d). Objective and subjective analyses for 26 July 1969 at 700 mb level

and summer periods over north American tropics. This suggests that these functions are different for different regions and different periods and hence have to be determined separately for each region and for every season for using them in the computation of weighting functions.

4.2. *Analyses*

The analyses of wind field at 850 mb and 700 mb levels obtained by this optimum interpolation scheme for the days from 24 July to 31 July 1969 compare fairly well with the corresponding subjective analyses. To illustrate this, a few cases of the analyses at 850 mb and 700 mb levels are given in Fig. 3(a-e). Although, in most of the cases both analyses agreed well, on 26 July, the subjective analysis at 850 mb level shows two systems whereas in the objective analysis only one system has been depicted (Fig. 3b). The objective analyses in other cases Fig. 3 (a, c, d), agree well with subjective analyses. The positions of the centres of the systems in both analyses and their vector differences are given in Table 3. This shows the vector differences are in general about 100 km (about 1°) with maximum value 250 km (2. 5°) and minimum value 55 km (0.5°). From this, it may be inferred that the analyses of both the methods agree fairly well. The objectively analysed wind field was compared with actually observed winds by computing rms errors, which vary from 3mps to 7mps (Table 4).

5. Concluding remarks

For optimum interpolation method, it is necessary that the statistical structure functions and weighting functions are separately calculated for each level, each region and for each season.

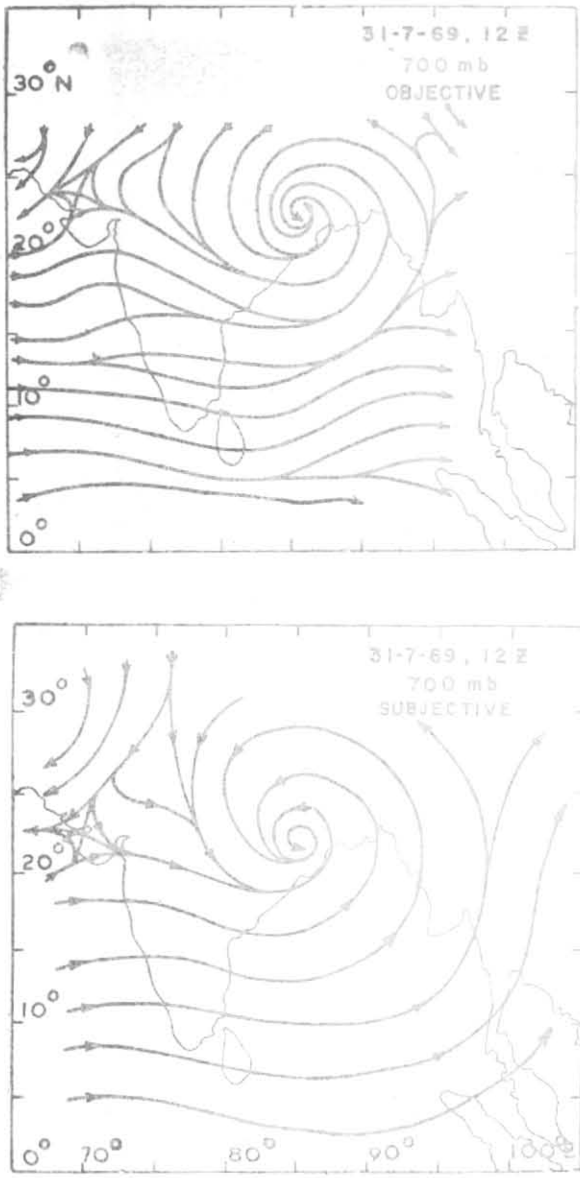

Fig. 3(e). Objective and subjective analyses for 31 July 1969 at 700 mb level

The analyses obtained at 850 and 700 mb levels show that they compare well with the subjectively made analyses. In this study, data of three July months or 90 days data were used. In order to see that the values of the autocorrelation functions etc obtained are stable irrespective of further increase of data, the computations have to be made with 5/6 years data and examined.

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