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# Network design of stream gauges for water resources assessement of Krishna basin

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सार – प्रवाहमापी स्टेशनों के संजाल के प्रनेक प्रकार हैं जैसे–स्पतन, संतुप्त एवं इच्टतम संजाल । धनेक ग्रवस्यार्घो में, संजाल का ग्रभिकल्पन विशिष्ट उद्येश्य की प्राप्ति को ब्यान में रखकर किया गया या वें स्टेशन चलते रहे शौर उसके बाद में राष्ट्रीय संजाल के बागों को बनाने के काम बाये ।

इस शोध पत्न में भौतिक-सांख्यिक विधि के क्षेत्रीय ग्रमिलक्षण उपयमन के साथ सम्मिलित कर कृष्णाद्रोणी पर प्रयोग में लाया गया है जो कि प्रमुख नदी ट्रोणियों में से एक है। भन्तरमध्यीय स्थितियों में घारा के प्रवाह के धन्तर्वेषण की विश्वसनीयता के निष्कर्ष पर विचार करके कृष्णा दोणी में प्रव.स मापी संजाल बनाया गय. है ग्रौर इसकी दीर्घ ग्रवधि एवं ग्रल्पावधि ग्रावम्यकताओं के लिए पर्याप्त की जांच की गई है। इस शोध पत्न में विचरण एवं सापेक तृटियों की स्थितियों के विभिन्न समुच्चयों के लिए कृष्णा द्रोणी में प्रवाहमापी संजाल विकसित किया गया है ।

ABSTRACT. There are different types of network of stream gauging stations such as minimum, saturation<br>and optimum network. In many cases, the network design was done purely from the point of view of meeting<br>a specific purp

In this paper the physical statistical method combined with the zonal characteristics approach is applied to The stream gauge network in Krishna basin which is one of the principal river basins of India. The stream gauge network in Krishna basin is arrived at by considering the criterion of reliability of interpolation of stream

#### 1. Tatroduction

There are different types of network of stream gauging stations such as minimum, saturation and optimum network etc. In many cases, the network design has been done purely from the point of meeting a specific purpose. If such stations<br>are continued thereafter, they become part of a national network. Primarily, however, there is what is called a basic network or key network to satisfy the following requirements for general water resources assessment :

- (1) To estimate the total water resources in a river basin with a given degree of accuracy.
- (2) To ensure reliable areal interpolation of runoff data at intermediate location.
- (3) In general, to design a network which can estimate runoff at any required point with the same errors as those of measurement of stream flow data.

For this purpose, the zonal characteristics approach is adopted in which the catchment is divided into various zones based on similarity of climate, topography etc. These are then treated as homogeneous areas, and network norms are derived for each of such zones.

In the present paper, an approach to the determination of stream gauge network in Krishna basin has been presented taking into consideration the criterion of reliability of interpolation of stream<br>flow data at intermediate locations and examina tion of its adequacy for long term and short term requirements in respect of water resources assessment.

Studies for determining optimum density of hydrometric network in general and stream gauging network in particular are not extensive in many countries including India. There is con-<br>fusion as to what is network design. Is it determination of the number of observational sites or is it the determination of their locations on the stream? And finally, what is the method that gives a correct answer when a variety of requirements are to be fulfilled? Due to lack of scientific norms, many network densities were being decided on empirical and subjective considerations. In India too, a uniform procedure for determination of basic network does not seem to have evolved. The Indian Standards Institution (ISI)

### **TABLE 1**



Table showing parameters computed for climatic classification

committee on stream gauging methods has recently brought this problem into light by inquiring as to what methods or techniques could be applied to Indian catchments which will enable us to evolve norms keeping in view the WMO guidelines on network design. In a recent study of hydrological network in India, it was stated by Gole (1978) that the WMO norms relate essentially to virgin flows and pre-development flows.

The Krishna basin was selected for the study since a committee of engineers had gone in detail into the network requirements for water resources assessment. This committee after examining the<br>rainfall, climatological and flow characteristics,<br>recommended a network of 76 stream gauges in<br>1962 for the whole basin. This works out to a density of one stream gauge for 3400 sq km.

Apart from the methods described in the Quebec Symposium (1965), a collection of case studies on network design is given in WMO Note No. 324  $(1972).$ 

Three types of errors are possible in the estimate of average flow at a point on a stream. They are:

- $(1)$ Interpolation errors
- $(2)$ Sampling errors
- $(3)$ Measurement errors

The present study considers the first two aspects of above and the general principles adopted are as given below:

- (a) Determination of a minimum area which is not subject to further interpolation.
- (b) Consideration of linear gradient in mean annual runoff.
- (c) Correlation of annual flows per unit distance between centres of drainage areas.

Ouantitative limits can be ascribed to the above factors depending upon the accuracy desired. After this is done, an optimum size of the gauging area per station is computed which will lie in between the gradient and correlation functions as suggested by Karasev (1968).

This method while determining the basic required network, stipulates the level of errors and variabilities for which the network is valid. The regional approach, in which the catchment could be divided into zones of similar climatic characteristics peculiar to the basin, is used for delineating those areas. The Koppen's classification limits are applied to the Krishna basin which is the study presented here.

### 2. Zonal characteristics of Krishna basin reference to climate

Of the many elements it is widely accepted that temperature and rainfall can describe the climate



Fig. 1. Climatic classification of Krishna basin  $(Scale 1'' = 20$  Miles)

at a place because temperature denotes heat and rainfall the availability of water which influence life on the earth. Koppen's climatic classification is used because of its simplicity, since parameters rainfall and temperature are easily available for many years.

The Krishna basin has a total area of 2,59,000 sq km and a small portion of this lies in the Western Ghats. The length of the stream is 1093 km and it has a present density of 38 stream gauging stations. The average annual rainfall in this basin is 868 mm and has a range of 576 mm to 1227 mm. Most of the areas have annual rainfall in the neighbourhood of 800-900 mm. The Krishna basin falls in a rather low coefficient of variation of annual rainfall and lies between 25 to 30 %. The monthly rainfall and temperature<br>data for 19 stations in Krishna basin collected The monthly rainfall and temperature from India Meteorological Department is considered in this study for working out the quantitative limits of climat.c classification in the catchment. The stream flow used in this study pertains generally to a period from 1960-1970 and in some cases upto 1973. The annual runoff totals at different gauge sites are used for the computation.

The parameters used for dividing the catchment into similar climatic zones, as per Koppen's classification are given in Table 1. From the<br>limiting values of Koppen's it can be seen that the Krishna basin falls broadly into two climatic zones. They are: (a) Tropical rainy with dry season (AW), (b) Warm temperature rainy (CW).

These delineations are shown in the map of<br>Krishna basin at Fig. 1. There are three variables which can be used to supplement the above classfication, for determining homogeneity :

(i) Uniformity of spatial variation of rainfall in the area.

- (ii) The distribution of coefficient of variation of rainfall.
- (iii) The gradients of stream flow along the river.

As already mentioned the entire Krishna basin has a range of coefficient of variation of annual rainfall between  $25\%$  to  $30\%$  and hence the area is not prone to very large year to year fluctuations of rainfall. In so far as the spatial uniformity of rainfall is concerned, excepting in two small<br>areas, the normal rainfall in the AW zone of the catchment varies from 800 to 900 mm annually. Regarding the third condition the stream gauge network design appears to be quite sensitive to the value of the stream flow gradient chosen. The<br>actual variation of stream flow with distance in Krishna catchment is plotted and is shown in Fig 2. It may be noticed from the figure that stream flow gradient is constant upto a certain distance and beyond this it becomes variable. The detailed study of these fluctuations beyond this point does not form part of the present study and the portion of Krishna basin, which has uniform gradient of flow, i.e., the area from Karad to Maruakonda, is<br>chosen. The study of network design, therefore, is confined to the following homogeneity criteria,  $viz$ , (a) The tropical rainy zone, (b) Spatially uniform variation of rainfall (c) Uniform gradients of stream flow.

With these, it is assumed that the area of study satisfies the condition of homogeneity both meterologically and physiographically. However, there is still considerable uncertainty as to what constitutes homogeneous regions and as yet it has not been possible to parameterise the various difinitions on homogeneity.

#### 3. Application of physical-statistical method of stream gauge network design to Krishna basin

The areal distribution of run-off Q, may be taken to be made up of two components, one is the normal runoff and the other is the deviation from normal.

It can be expressed as :

$$
Q_{\epsilon}, j = Q_{\epsilon} + Z_{\epsilon}, j \tag{1}
$$

where  $\overline{Q_e}$  is a long term average runoff of N years at  $e^{t\lambda}$  observational point, and Z<sub>ci</sub> is observational point, and  $Z_{\epsilon, j}$  is the deviation from the normal runoff in  $j<sup>t</sup>$  year and at e<sup>th</sup> observational point. The principle of hydrological regionalisation is applied to bring the stream flow field to homogeneous conditions. For each such hydrological region, values of long term average runoff  $Q_{e}$ , at observational points, co-efficient of variation of runoff,  $C_v$ 's are re-<br>quired to be computed. The dispersion of annual stream flow is given by :

$$
\sigma_{e}^{3} = \frac{1}{N} \sum_{j=1}^{N} (Q_{e,j} - \overline{Q}_{e})^{3}
$$

$$
= (C_{e} \overline{Q}_{e})^{3} \approx \text{const.}
$$
 (2)

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where  $C_v$  is the coefficient of variation of runoff. Let us consider the first term of expression (1), which gives normal runoff. The variation of normal runoff over the whole area is characterised by the stream flow gradient :

$$
\text{Grad } \left( \, \overline{\varrho} \, \right) = \overline{\varrho}_{\mathbf{t}}' = \frac{d}{de} \left( \, \overline{\varrho}_{\mathbf{t}} \right) \tag{3}
$$

The relative gradient can be written as

$$
\text{Grad}_{\circ} \left( \overline{\varrho} \right) = \frac{\overline{\varrho}_{\text{e}}'}{\overline{m}} \tag{4}
$$

Where,  $\overline{m} = 0.5(\overline{Q}_e + \overline{Q}_{e+b})$ 

and S is the distance between the watershed centres.

Now consider the second term of expression (1), the correlation function can be expressed  $2R$ 

$$
r_q(S) = \frac{\sum\limits_{j=1}^{N} Z_{e,j} \cdot Z_{e+s,j}}{N \sigma_{e} \cdot \sigma_{e+s}}
$$
(5)

where  $\sigma_e$  and  $\sigma_{e+s}$  are standard deviations at two stations separated by a distance S.

$$
= \frac{C_q(S)}{\sigma_{\epsilon} \cdot \sigma_{\epsilon+1}}
$$

where,  $C_q(S)$  is the co-variance term

$$
r_q(S) = \frac{C_q(S)}{(C_v \overline{Q}_e)^2}
$$
 in view of assumption

in Eqn. (2), *i. e.*,  $\sigma_{\epsilon} = \sigma_{\epsilon + \delta}$ 

 $r_q$  (S) can also be expressed as

$$
r_q(S) = 1 - a.S \tag{6}
$$

where,  $a = 1/S_0$ ;  $S_0$  is the radius of corre-<br>lation at which the correlation function has the zero value. From values of  $r_q(S)$  and S, the value of  $S<sub>o</sub>$  is determined.

For the reliability of stream flow data, two conditions are given corresponding to two terms of expression  $(1)$ 

- (i) The criterion with regard to spatial variation of normal stream flows.
- (ii) The criterion on interpolation of stream flow data at intermediate points for an individual year.

The above two conditions represent the two terms in the RHS of Eqn. (1), thus satisfying it.

### 3.1. Gradient criterion

The first criterion corresponds to the determination of the confidence interval for the esti-<br>mation of normal stream flow at the mid-point of the distance between the centres of the two regions.

The error for the estimation of normal stream flow at the midpoint is given by :

 $-22$ 

$$
\sigma_2^2 = \frac{1}{N} \sum_{j=1}^N \left[ \frac{Q_{e+3,j} + Q_{i,j}}{2} - \frac{\overline{Q}_{e+1} + \overline{Q}_e}{2} \right]^2
$$

$$
= \frac{1}{4} \left[ \sigma_e^2 + \sigma_{e+1}^2 + 2 \sigma_e \sigma_{e+1} r_s \right]
$$

$$
= \frac{\sigma_e^2}{2} \left[ 1 + r_e \right] \tag{7}
$$

in view of assumption in Eqn. (2). Confidence interval at  $P=0.95$  assuming normal distribution is

$$
= 1.96 \frac{\sigma_{\epsilon}}{\sqrt{2}} \frac{\sqrt{1+r_{\epsilon}}}{\sqrt{N}}
$$

$$
= \frac{1.96}{\sqrt{2}} \frac{C_{\theta}}{\sqrt{N}} \sqrt{1+r_{\theta}} \overline{Q}_{\epsilon}
$$

$$
= \frac{1.96}{\sqrt{2}} \sigma_0 \overline{Q}
$$
(8)

where  $\sigma_0$  is the relative error and N is the number of years of record.

The condition of sufficiency of stream flow gradient between two observational points is given by:

$$
S_{gr} \geqslant \frac{2 \cdot 82 \sigma_{o m}}{grad \ (\overline{Q})} \tag{9}
$$

This criterion expresses the minimum distance S to determine the stream flow variation caused by zonal characteristics.

#### 3.2. Correlation criterion

The correlation criterion for network density depends upon the maximum error of interpolation of stream flow values for individual years for a<br>basin situated at middle of S. The interpolation error is assumed to be the same as the error of stream flow estimation and as given by

$$
\sigma_{s}^{2} = \frac{1}{N} \sum_{j=1}^{N} \left[ (Q_{\epsilon,j} - Q_{\epsilon+s,j}) - (\bar{Q}_{\epsilon} - \bar{Q}_{\epsilon+s}) \right]^{2}
$$

$$
= \frac{1}{N} \sum_{j=1}^{N} \left[ (Q_{\epsilon,j} - \bar{Q}_{\epsilon}) - (Q_{\epsilon+s,j} - \bar{Q}_{\epsilon+s}) \right]^{2}
$$

$$
= \frac{1}{N} \left[ \sum_{j=1}^{N} Z_{\epsilon,j}^{2} + \sum_{j=1}^{N} Z_{\epsilon+s,j}^{2} - 2 \sum_{j=1}^{N} Z_{\epsilon,j} Z_{\epsilon+s,j} \right]
$$

$$
= \sigma_{\epsilon}^{2} + \sigma_{\epsilon+s}^{2} - 2 r_{\epsilon} \sigma_{\epsilon} \sigma_{\epsilon+s}
$$

$$
= 2 C_{v}^{2} \bar{Q}_{\epsilon}^{2} [1 - r_{s}]
$$
 (10)

The empirical structure function is given by:

$$
C_{a}^{a} = 2C_{b}^{a} (1 - r_{a}) + 2 C_{b}^{a}
$$
  
= 2C\_{b}^{a} (a, S) + 2 C\_{b}^{a}  

$$
C_{s12}^{a} = \left(\frac{C_{a}}{2}\right)^{2} = \frac{2 C_{a}^{a} (a, S) + 2 C_{b}^{a}}{4}
$$
  

$$
r C_{s12} = \frac{1}{\sqrt{2}} \left(C_{b}^{a} a \cdot S + C_{b}^{a}\right)^{\frac{1}{2}}
$$
 (11)

Since  $C_0$  and  $\sigma_0$  are inter-related, and taking S as  $S<sub>c</sub>$  at zero correlation,

Then,  $C^2$ <sub>v</sub>  $a S_c + \sigma_0^2 = 0$ This satisfies the condition

 $\Omega$ 

$$
S_c \leqslant \frac{\sigma_o^3}{a C_v^3} \tag{12}
$$

where the distance between two watershed centres is  $S_c$ .

Considering  $F_{gr}$  and  $F_c$  as limiting areas of the gradient function and correlation function, then we get

$$
F_{\varrho r} \geqslant 8 \left[ \frac{\sigma_o \overline{Q}}{\text{grad}(\overline{Q})} \right]^2 \tag{13}
$$

$$
F_e \leqslant \frac{\sigma_o^4}{a^2 C_v^4} \tag{14}
$$

assuming the following relationships:

and

$$
L \approx 2 F^{0.5}
$$
  

$$
S \approx F^{0.5}
$$
 (15)

where  $L$  is the length of the river,  $F$  is the drainage area and S is the distance between the centres of basin.

The optimum area for determining the gauge density, therefore, will lie between  $F_{q_r}$  and  $F_c$ .

### 4. Determination of optimum number of gauging stations in Krishna basin

(Between Karad and Maruakonda, covering an area of 205038 sq km)

S is considered as the distance between Karad and Maruakonda = 552.47 km.

The correlation coefficient between annual flows at Karad and Maruakonda =  $0.7359$ .

 $r=1-a.S$ , where  $a=1/S_3$ .

 $S_0$  is the radius of correlation for which correlation function is zero.

$$
1 - \frac{S}{S_o} = 0.7359
$$

 $\alpha$ r

S

$$
h = \frac{552.47}{1 - 0.7359} = 2091.9
$$
 km

The mean annual flow at Karad  $\overline{Q}_k$  is equal to 170.45 thousand million cubic feet (TMC) or 4826.6 million cubic meters (MCM)

The mean flow at Maruakonda  $\bar{Q}_v$  is 1335.31 TMC or 37811.71 MCM.

$$
\overline{m} = 0.5 \left( \overline{Q_k} + \overline{Q_v} \right)
$$
  
= 752.88 T.M.C. or 21319.15 MCM  
Grad. ( $\overline{Q}$ ) =  $\frac{(1335.31 - 170.45)}{552.47}$  = 2.1084 TMC/km  
or 59.7048 MCM/km

relative gradient, Grad<sub>o</sub>(
$$
\overline{Q}
$$
) =  $\frac{\text{grad}(\overline{Q})}{\overline{Q}}$  =  $\frac{59.7059}{21319.15}$ 

 $= 0.0028005 MCM/km$ 

The error of estimation  $\sigma_q$  of normal stream<br>flow at Karad and Vijaywada comes to 14% and 15% respectively.

## For  $C_{\nu}$  at Karad = 0.30

For a relative error of 5%,  $\sigma_0 = 0.05$ 

then, 
$$
\frac{\sigma_0}{C_s} = \frac{0.05}{0.30} = 0.1667
$$
  
 $\frac{2.82 \times \sigma_0 \times \overline{m}}{\text{grad } \overline{Q}} = \frac{2.82 \times 0.05}{0.0028005} = 50.35 \text{ km}$ 

$$
S_{\text{gr}} \geqslant 50.35 \text{ km}
$$

$$
S_{\bullet}
$$
 is given by

 $S_4 \leq 2091.90 \times (0.1667)^3 = 58.13$  km

$$
F_{\rm gr} \ge 8 \left[ \frac{\sigma_0}{\text{grad}_o \,\bar{Q}} \right]^2 = 8 \left[ \frac{0.05}{0.0028005} \right]^2
$$

$$
\geqslant 2534.93
$$
 sq km

For a  $C_v$  of 30% and a relative error of 5%.  $F<sub>e</sub>$  is calculated as

$$
F_c = \left(\frac{\sigma_0}{C_v}\right)^4 \times (2091.90)^2 = (0.1667)^4 \times (2091.90)^3
$$
  

$$
\leq 3379.28 \text{ sq km}
$$

Similar computations are carried out varying the coefficient of variability and the relative error, and these results are tabulated in Table 2.

### 5. Sensitivity of gradient function

In order to examine the sensitivity of the gradient function, three cases of gradients were considered as noted below

Case 1 : The gradient based upon reach between, Karad and Dhannur.

Case 2 : Gradient based upon the reach between Karad and Maruakonda.

Case 3 : Gradient based on the reach between Karad and Deosagar.

### TABLE 2

Limiting values of  $F_g$  and  $F_c$  for reach between Karad and Maruakonda in Krishna basia catchment area,  $F = 2,05,038$  sq km



These gradients are also shown in Fig. 2. In each of the above cases the gradients were computed and also the gradient functions for various values of relative errors ranging from<br>04 to .10. The limiting area for each of these gradient functions are shown in the table b.low:



It will be seen from the above table that as the gradient increases from case 1 to case 3, the network of gauges required will be more. In other words if there is an error in the computation of gradients. then it will not considerably affect the number of stream gauges required for a given area.

6. Comparison of network arrived in one stage and two stages

In order to see as to what will be the effect in the network density if a reach is taken as a whole or is treated in parts, the following cases have been taken for study (see Fig 3).



Fig. 2. Krishna basin-mean stream flow v/s distance (Read Maruakonda instead Morvakonda)



Fig. 3. Krishna basin - gradient sensitivity selected for of gradient analysis function

parison of stream net work gauge design

Case 4 : Stream gauge stations required between Karad and Deosagar.

Case 5 : Stream gauge stations required in the reach between Deosagar and Maruakonda.

Case 6 : Stream gauge stations required in the reach between Karad and Maruakonda.

For all the above three cases again the gradients were computed and the results of networks required are shown in the table in each case.



From the above table, it will be seen that for a relative error of .05, while the case 6 gives 81 stream gauges, a total number of 77 gauges are required when the whole reach is divided into two parts. Since the gradient between Deosagar and Maruakonda is smaller than the gradient between Karad and Deosagar, the number of gauges required in the former case was quite small, consistent with the area between the reach.

#### 7. Conclusions

In the study presented here, the physicalstatistical method has been applied for the determination of basic network of stream gauging stations for general water resources assessment in Krishna basin.

On the basis of the above analysis, the optimum distance between observational points<br>was computed and shown in Table 2. For a<br>relative error of  $5\%$ , the number of gauges required in Krishna basin between Karad and Maruakonda comes to 81 (see Table 2).

Table 2 also gives the limiting areas worked out for Krishna basin in the case of correlation function as well as the gradient function. It is noticed from the results presented in Table 2 that the network is more sensitive to changes in correlation function. This is so because  $F_c$ (correlation function) varies as the fourth power of the ratio  $\sigma_o/C_v$  other factors remaining constant. An examination of the sensitivity of gra-<br>dient function has been made in some detail. The gradient function changes rather gradually for different gradients of stream flow and hence can be a very reliable indicator. Therefore, even if stations are established with net-work which is not very ideal, it would give an idea of the gradient function and since the gradient function is not so sensitive the net-work can be expanded for keeping the interpolation errors low. In conclusion, it may be stated that for a relative error of  $5\%$ and a coefficient of variation of 30% the network requirement in Krishna basin for stream gauges is one gauge for 2,534 sq km.

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### Explanation of symbols used

- $\overline{Q}$ ,  $\overline{Q}_e$  Average of long term runoff and<br>runoff at individual stations respectively.
- Criteria of minimum distance based  $S_{gr}$ on gradient function.
- Relative error of the mean.  $\sigma_{\rm o}$
- Inverse of correlation radius at  $\mathcal{C}$ which the correlation function has zero value.
- $F_{gr}$ ,  $F_c$  Gradient and correlation functions respectively.

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