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Parameterization of the planetary boundary layer for use in forecasting models*

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सार — स्थिरकल्प दाबघनत्वी ग्रहीय परिसीमा स्तर (पी० बी० एल०) के प्राचलों की गणना के लिए एक एलगोरिथ्म की चर्चा की गई है। अनेक आंकिक प्रयोगों के परिणामों के आधार पर प्रेक्षित मौसम क्षेत्रों एवं ग्रहीय परिसीमा स्तर के आंतरिक प्राचलों में सम्बन्ध स्थापित किया गया है। बहुस्तरीय प्रांभीकरण समीकरण (पी० ई०) निदर्शों में उनके उपयोग के लिए भंवर संवेग अभिवाह और घर्षणी ऊर्ध्वाधर गति के प्राचलनीकरण के लिए एक विधि सुझाई गई है।

ABSTRACT. An algorithm for computaion of the parameters of a quasi-stationary barotropic planetary boundary layer (PBL) is discussed. On the basis of the results of a number of numerical experiments, relations between the observed meteorological fields and the internal parameters of the PBL have been established. A method is proposed to parameterize eddy momentum flux and frictional vertical velocity for their use in multi-level primitive equations (PE) models.

1. Introduction

A multi-level PE model is supposed to be the most general numerical model to study various phenomena of the atmosphere. In order to understand the wide complexities of the tropical atmosphere and simulation of various special features like monsoon circulation, inter tropical convergence zone, Somali jet, tropical cyclones, etc, it is essential to incorporate various dynamical and physical aspects into the models through observational, experimental and theoretical studies.

A few of them are :

(*i*) initial balancing of meteorological fields;

(*ii*) inclusion of the influence of orography;

(*iii*) parameterization of the PBL;

(*iv*) parameterization of cumulus convection;

(*v*) inclusion of moisture and radiation in numerical models.

The planetary bounday layer can have effects on the entire troposphere. The atmospheric boundary layer supplies roughly 50 per cent of the internal energy in the atmosphere and a greater part of the atmospheric kinetic energy is dissipated in this layer. In parallel with these processes, the boundary layer forces vertical velocities and fields of convergence and divergence in the free atmosphere. Kibel (1963) in his investigations found that the introduction of surface friction in numerical weather prediction models considerably influences the forecast results. In the works of Shuman and Hovermale (1968) and Fischer *et al.* (1973) the effect of friction in the PBL was taken into account in the numerical weather prediction models by simple parameterization schemes. Berkovich and Shnaidman (1979) carried out numerical experiments with a hemispheric prognostic model taking into account of the PBL effects by using frictional vertical velocity, for forecasting meteorological fields.

For operational use, a simple and physically sound scheme is desirable as a complex one demands more computational as well as data requirements (Deardorff 1972). The purpose of this study is to propose a simple method of parameterization of the eddy momentum transport from the surface and frictional vertical velocity in the lower layer of a PE model which lays well within the PBL. An algorithm for computation of the internal parameters of a quasi-stationary PBL from the observed meteorological elements is also presented.

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2. Symbols used

The following symbols, which are not defined in the text have been used :

f	—	coriolis parameter
x, y, z	—	horizontal and vertical coordinate system, where the x-axis is directed in the direction of surface wind
u, v	—	velocity components in x and y directions
p	—	pressure
ρ	—	density of air
K	—	eddy viscosity
c_p	—	specific heat at constant pressure
θ	—	potential temperature
P_0	—	turbulent flux of sensible heat at the surface
ϵ	—	rate of kinetic energy disssipation
b	—	turbulent kinetic energy
l	—	mixing length
g	—	gravitational acceleration
T	—	temperature
$\mathbf{V}_g$	—	vector of the geostrophic wind
u_g, v_g	—	the components of the geostrophic wind in the zonal and meriodional directions
τ_x, τ_y	—	the components of the Reynolds stress in x and y directions
v_*	—	friction velocoty
$\vec{\tau}_0 (\tau_{x0}, \tau_{y0})$	—	surface Reynolds stress
H	—	Height of the PBL
κ	—	Von Karman's constant
L	—	internal scale height of the PBL
μ	—	internal parameter for the thermal stratification
Z_0	—	roughness length
v	—	external stratification parameter
γ_a	—	adiabitic lapse rate of the atmosphere
γ_h	—	actual lapse rate at the height h
R_0	—	surface Rossby number
C_g	—	geostrophic drag coefficient
H_{850}	—	geopotential height at 850 mb pressure surface
α	—	cross-isobar angle
F_u, F_v	—	the components of the vertical eddy stress term in x and y directions
u', v', w',	—	deviation of u, v and w components of the velocity from the corresponding mean values of any time and are associated with the turbulent eddies
ω_s	—	vertical velocity at the surface
ω_F	—	frictional vertical velocity at the top of the PBL

The following indices have been used :

n	—	stands for non-dimensional quantities
0	—	taken at Z_0, *i.e.*, at the lower boundary
x	—	the x-component
y	—	the y-component

3. Description of the model

As a system of equations to deal with the PBL models, stationary and horizontally homogeneous turbulent motion in a startified atmosphere is proposed by Laikhtman (1970) and Wippermann (1973). These system of equations are closed by some sort of semi-empirical relations for the profile for viscosity $[K(z)]$ or mixing length $[l(z)]$. Though these types of barotropic PBL models are simple, they are quite capable to represent the main structure of PBL correctly (Wippermann 1973). Further, these types of PBL models are powerful for the purpose of parameterization of boundary layer effects in multilevel prediction and simulation models based on the large-scale variables available at the lowest level (level well within the PBL) of the simulation models. A procedure of closing the system of the PBL equations by a hypothesis for the mixing-length (l) is more advanced and realistic than of a hypothesis for the profile of eddy viscosity $[K(z)]$ (Wippermann 1973). This may be due to the fact that the mixing-length increases monotonically with height starting with a very value l_0 at $Z=Z_0$ upto l_∞ at $Z=H$, where as $K(z)$ has a maximum at a certain height. Therefore, it is much more difficult to specify $K(z)$ profiles than that of $l(z)$.

A closed system of equations for a barotropic PBL based on the above stated assumptions and the mixing length hypothesis are (Kurasaki 1970, Laikhtman 1970 and Wippermann 1973).

$$fv - \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{dK}{dz}\frac{du}{dz} = 0 \qquad (1)$$

$$-fu - \frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{d}{dz}K\frac{dv}{dz} = 0 \qquad (2)$$

$$\rho\, c_p\, K\frac{d\theta}{dz} = -\, P_0 \qquad (3)$$

$$K\left[\left(\frac{du}{dz}\right)^2+\left(\frac{dv}{dz}\right)^2\right] - \alpha_T\frac{g}{T}K\frac{d\theta}{dz} + \alpha_b\frac{d}{dz}K\frac{db}{dz} - \epsilon = 0 \qquad (4)$$

$$K = lb^{1/2} \qquad (5)$$

$$l = -C_1\,\Psi \Big/ \left(\frac{d\Psi}{dz}\right) \qquad (6)$$

where,

$$\epsilon = C_0\, b^{3/2}\, l^{-1} = C_0\, b^2/K \qquad (7)$$

$$\Psi = \frac{\epsilon}{K} = \left(\frac{du}{dz}\right)^2 + \left(\frac{dv}{dz}\right)^2 - \alpha_T\frac{g}{T}\frac{d\theta}{dz} + \frac{\alpha_b}{K}\frac{d}{dz}K\frac{db}{dz} \qquad (8)$$

where α_T, α_b, C_0 and C_1 are non-dimensional constants ($\alpha_T = 1$, $\alpha_b = 0.73$, $C_0 = 0.046$ and $C_1 = 0.4$ as used by Ternopolski and Shnaidman 1974).

The orientation of the coordinate system is fixed in such a way that the x-axis is directed along the surface wind stress $\vec{\tau}_0$.

Then the momentum equations (1 and 2) become:

$$v - \mathbf{V}_g \sin \alpha = -\frac{1}{f}\frac{d\eta}{dz} \quad (9)$$

$$u - \mathbf{V}_g \cos \alpha = \frac{1}{f}\frac{d\sigma}{dz} \quad (10)$$

where,

$$\eta = K\frac{du}{dz} = \frac{\tau_x}{\rho} \quad (11)$$

$$\sigma = K\frac{dv}{dz} = \frac{\tau_y}{\rho} \quad (12)$$

The system of Eqns. (1)-(8) governing the PBL are non-dimensionalised. Such non-dimensionalization can be achieved by using the internal parameters of the PBL like the internal scale height of PBL ($L = \kappa v_*/f$) and the friction velocity ($v_* = \tau_0/\rho$) (Laikhtman 1970 and Wippermann 1973). The non-dimensionalized PBL model is independent of the geographical region under consideration. Variables, used in the PBL model (Eqns. 1-8), are non-dimensionalized in the following manner :

$$\begin{aligned} Z_n &= Z/L \\ \eta_n &= \eta/v_*^2 \\ \sigma_n &= \sigma/v_*^2 \\ K_n &= K/\kappa v_* L \\ \epsilon_n &= \kappa L \epsilon / v_*^3 \\ b_n &= C^{1/2} b / v_*^2 \\ l_n &= l/\kappa C^{1/4} L \\ \mathbf{V}_{gn} &= \kappa \mathbf{V}_g / v_* \\ u_n &= \kappa u / v_* \\ v_n &= \kappa v / v_* \end{aligned} \quad (13)$$

The lower index n stands for non-dimensionalized variables. By differentiating the momentum Eqns. (9)-(10) with respect to z and by using above non-dimensional variables (13) in place of the corresponding dimensional variables, the system of the equations for the barotropic PBL can be written in the non-dimensional form as :

$$\frac{d^2\eta_n}{dz_n^2} + \frac{\sigma_n}{K_n} = 0 \quad (14)$$

$$\frac{d^2\sigma_n}{dz_n^2} - \frac{\eta_n}{K_n} = 0 \quad (15)$$

$$\frac{\eta_n^2 + \sigma_n^2}{K_n} - \mu - \frac{b_n^2}{K_n} + \beta \frac{d}{dz_n} K_n \frac{db_n}{dz_n} = 0 \quad (16)$$

$$K_n = l_n b_n^{1/2} \quad (17)$$

$$l_n = -\left(\frac{d}{dz_n} \log_e \frac{b_n}{K_n}\right)^{-1} \quad (18)$$

$$\text{where, } \mu = \mu_0 \frac{K_n}{Z_n}\left(1 + \frac{v Z_n^{m+1}}{\mu_0 H_n^m}\right) \quad (19)$$

$$m = 1$$

$$\beta = 0.54$$

For solution of the above system of equations for the PBL, the following boundary conditions in non-dimensional form are required :

$$Z_n = Z_{no} \qquad \eta_n = 1,\ \sigma_n = 0,\ l_n = 0,\ b_n = 1$$

$$Z_n \to \infty \qquad \eta_n \to 0,\ \sigma_n \to 0,\ b_n \to 0 \quad (20)$$

4. Numerical procedure

The system of Eqns. (14)-(18) for the barotropic PBL model along with the boundary conditions (20) are solved by iteration method. As initial approximation for the iteration scheme over the eddy viscosity $K_n(Z_n)$, the vertical profile of $K_n(Z_n)$ is given to be linear function of the non-dimensional height Z_n. Finite difference approximation of the differential equations are carried out over an irregular vertical grid, which provides fine resolution close to the surface.

For computation of the turbulent characteristics and the vertical profiles $K_n(Z_n)$, $b_n(Z_n)$, $l_n(Z_n)$, $\eta_n(Z_n)$ and $\sigma_n(Z_n)$ with the help of the non-dimensional PBL model [Eqns. (14)-(18) and (20)], the following non-dimensional parameters are to be prescribed :

(*i*) internal stratification parameter of the PBL at Z_0, μ_0,

(*ii*) thermal stratification at upper level of the PBL v and

(*iii*) the Rossby number R_0

where,

$$\mu_0 = \kappa^2 g P_0 / f \rho_0 c_p T v_*^2 \quad (21)$$

$$v = \kappa^4 (\gamma_a - \gamma_h) g / f^2 T \quad (22)$$

$$R_0 = \mathbf{V}_{g0} / f z_0 \quad (23)$$

It is obvious from the above relations (21)-(23) that v and R_0 are the external parameters whereas μ_0 is the internal one of the PBL. As P_0 is not known to us, so μ_0 cannot be determined by relation (21). A method to determine μ_0 was proposed by Ternopolski and Shnaidman (1972). According to that method μ_0 is computed by the relation :

$$\mu_0 = \left[M + \frac{v H_n C_g}{2\kappa^2}\right] \frac{\kappa^2}{C_g \log_e (\kappa^2 H_n R_0 C_g)} \quad (24)$$

where,

$$M = \frac{g}{f T \mathbf{V}_g}\left[(\theta_{850} - \theta_{1000}) - (\gamma_a - \gamma_h)\Big/_{850}^{700} H_{850}\right] \quad (25)$$

$$C_g = \frac{\kappa\, v_*}{\mathbf{V}_g} \quad (26)$$

The non-dimensional height of the PBL (H_n) is a function of v and μ_0 and geostrophic drag coefficient C_g is a function of v, μ_0 and log R_0, therefore μ_0 can be expressed by the functional relation as :

$$\mu_0 = \mu_0 (M,\ v,\ \log R_0) \quad (27)$$

Thus μ_0 can be obtained by the numerical solution of the system of equations of the PBL or from the functional relation (27). A graphical representation of the relationship (27) based on a number of numerical experiments is illustrated in Fig. 1. This nomogram can be used to determine μ_0 from the given values of v, log R_0 and M, which are the external non-dimensional parameters of the model.

The replacement of μ_0 by M, v, log R_0 and the nomogram (Fig. 1) paves the way for the solution of the system of equations of he PBL model by use of the observed meteorological parameters in the lower troposphere (*i.e.*, geopotential height, wind velocity and temperature), roughness length and latitude of the point under consideration.

The following steps are carried out for obtaining the characteristic parameters of the PBL model:

(a) from usually available meteorological informations v, R_0 and M are calculated by the relations (21, 22, 25),

(b) with use of R_0, v and M, μ_0 is obtained from the nomogram as discussed above (though μ_0 is an internal parameter of the PBL, as it is obtained from external parameters of the PBL by a nomogram, it is referred in the text as an external one).

(c) R_0, v and μ_0 are used to obtain H_n, C_g and α directly from the PBL model or from the semi-empirical relations obtained by numerical experiments (discussed in paragraph-6).

(d) friction velocity v_* and x and y components of the surface Reynolds stress (τ_{x0} and τ_{y0}) are computed by the relations :

$$v_x = \kappa\, C_g\, /V_g/ \quad (28)$$

$$\tau_{x0} = \rho_0\, \kappa^2\, C_g^2\, /V_g/\, (u_g \cos\alpha - v_g \sin\alpha) \quad (29)$$

$$\tau_{y0} = \rho_0\, \kappa^2\, C_g^2\, /V_g/\, (u_g \sin\alpha + v_g \cos\alpha) \quad (30)$$

(e) Vertical profiles of the parameters $K_n(Z_n)$, $b_n(Z_n)$, $\eta_n(z_n)$ and $\sigma_n(Z_n)$ are computed directly by numerical solution of the PBL model.

5. Method of parameterization

In this paper two ways of parameterization of the integral effect of the PBL in the atmospheric models are discussed. They are :

(*i*) incorporation of frictional forces (eddy diffusion terms) in the momentum equations and

(*ii*) inclusion of frictional vertical velocity, due to boundary layer convergence, in the numerical models by the simple device of changing the lower boundary condition for vertical velocity.

In the PE models for a turbulent atmosphere the horizontal and vertical variation of the eddy diffusivity terms are appeared in the momentum equations. However, the vertical gradients of the eddy momentum are much larger than the horizontal gradients. Further, in the boundary layer the vertical eddy stress terms are quite large and comparable to the other forcing terms of the momentum equations. This necessiates the incorporation of the vertical eddy stress in the equations of motion for the lower level of the atmospheric models, which lies well within the boundary layer.

The vertical eddy stress terms (F_u and F_v) in the horizontal components of equations of motion are :

$$F_u = -\frac{1}{\rho}\frac{\partial\, \overline{\rho\, u'\, w'}}{\partial z} = \frac{1}{\rho}\frac{\partial \tau_x}{\partial z} \quad (31)$$

$$F_v = -\frac{1}{\rho}\frac{\partial\, \overline{\rho\, v'\, w'}}{\partial z} = \frac{1}{\rho}\frac{\partial \tau_y}{\partial z} \quad (32)$$

Assuming that there exists a PBL characterised by a vanishing (or at least very small) stress at the top of the layer and surface eddy momentum fluxes denoted by τ_{x0} and τ_{y0}, from Eqns. (31) and (32) we obtain :

$$F_u = -\tau_{x0}\, /\rho H \quad (33)$$

$$F_v = -\tau_{y0}\, /\rho H \quad (34)$$

In the non-dimensional form the height of the PBL is expressed as :

$$H_n = H/L \quad (35)$$

Thus from (33, 34) with use of (35) the relations are :

$$F_u = -\tau_{x0} f / \rho\kappa v_*\, H_n \quad (36)$$

$$F_v = -\tau_{y0} f / \rho\kappa v_*\, H_n \quad (37)$$

Using the relations (28)-(30) for v_*, τ_{x0} and τ_{y0} and assuming ρ to be constant in the PBL from Eqns. (36), (37) we can obtain the relations for F_u and F_v in their final form as :

$$F_u = -\frac{C_g f}{H_n}(u_g \cos\alpha - v_g \sin\alpha) \quad (38)$$

$$F_v = -\frac{C_g f}{H_n}(u_g \sin\alpha + v_g \cos\alpha) \quad (39)$$

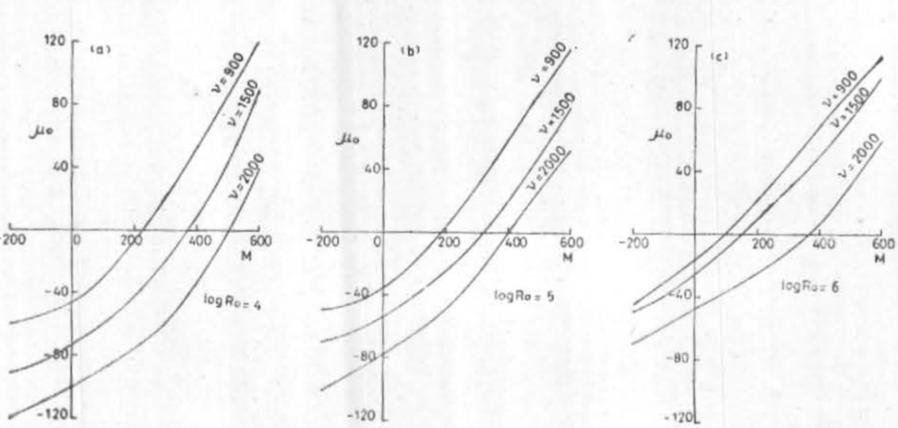

Fig. 1. Internal parameter for thermal stratification at Z_0 as a function of the external parameters υ and M for (a) $\log R_0=4$, (b) $\log R_0=5$ and (c) $\log R_0=6$

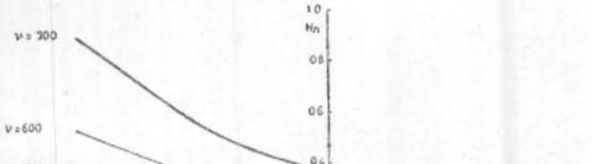

Fig. 2. Non-dimensional height of the PBL as a function of thermal stratifications υ and μ_0

The second approach for the parameterization of the integral effect of the PBL in the atmospheric models is the incorporation of the vertical velocity caused by the frictional convergence in the PBL which is known as Ekman pumping. This goal can be achieved by a simple device of putting the frictional vertical velocity (ω_F) as the lower boundary condition for vertical velocity at the lowest level of the atmospheric model, which lies well within the PBL.

From the equations of motion for a turbulent atmosphere and continuity equation in the pressure coordinate system, we can write :

$$\frac{\partial \omega}{\partial p} = \frac{g}{f} \frac{\partial}{\partial p} (\mathbf{k}.\ \nabla \times \vec{\tau}) \tag{40}$$

Integrating the Eqn.(40) from the surface to the top of the PBL, where the eddy stresses assumed to be zero, we will have :

$$\omega_F = \omega_s - \frac{g}{f} \left(\frac{\partial \tau_{y0}}{\partial x} - \frac{\partial \tau_{x0}}{\partial y} \right) \tag{41}$$

The relations (29) and (30) for τ_{x0} and τ_{y0} can be used to evaluate ω_F at the lowest layer of the atmospheric models with known value of ω_s.

Thus, these two approaches of the parameterization of certain effects of the PBL in numerical models involve the determination of H_n, C_g and α which are the internal parameters of a PBL model and can be obtained by the numerical solution of the PBL model, proposed above, with the given values of the external parameters.

6. Numerical experiments

Relationships between the internal (C_g, α and H_n) and external (υ, R_0 and μ_0) parameters of the PBL can be established by the solution of the system of equations governing the turbulent flow in the lower troposphere. As a result of the iterative solution of the system (14)-(18) for the PBL (Babileba 1970 and Ternopolski and Shnaidman 1972), we obtain the vertical profiles of the non-dimensional parameters $\eta_n(Z_n)$, $\sigma_n(Z_n)$, $K_n(Z_n)$ and b_n (Z_n). The non-dimensional height of the PBL (H_n) is obtained from the definition for the upper boundary condition (*i.e.*, the Raynolds stress should vanish) as

$$\text{at } \sqrt{\eta_n^2 (Z_n) + \sigma_n^2 (Z_n)} \leqslant E \quad H_n = Z_n \tag{42}$$

where E is a very small quantity ($E=0.01$).

In order to obtain expressions for C_g and α in terms of $\eta_n (Z_n)$ and σ_n (Z_n) the equations (9) and (10), are non-dimensionalized by the use of the relations (13) and the lower boundary conditions of the PBL (at $Z_n \to Z_{n0}$ $u_n = v_n = 0$) are applied. Further, the relation between V_{gn} and C_g as $V_{gn} = \kappa\ V_g/v_* = 1/C_g$ is used to obtain the equations :

$$\frac{d\eta_n}{dZ_n} = \frac{1}{C_g} \sin \alpha$$
$$\frac{d\sigma_n}{dZ_n} = -\frac{1}{C_g} \cos \alpha \tag{43}$$

From these Eqn. (43) the expressions for C_g and α can easily be obtained as :

$$C_g = 1 \bigg/ \sqrt{\left(\frac{d\eta_n}{dZ_n}\right)^2 + \left(\frac{d\sigma_n}{dZ_n}\right)^2} \tag{44}$$

$$\alpha = \text{arc tan} \left[- \left(\frac{d\eta_n}{dZ_n} \right) \bigg/ \left(\frac{d\sigma_n}{dZ_n} \right) \right] \tag{45}$$

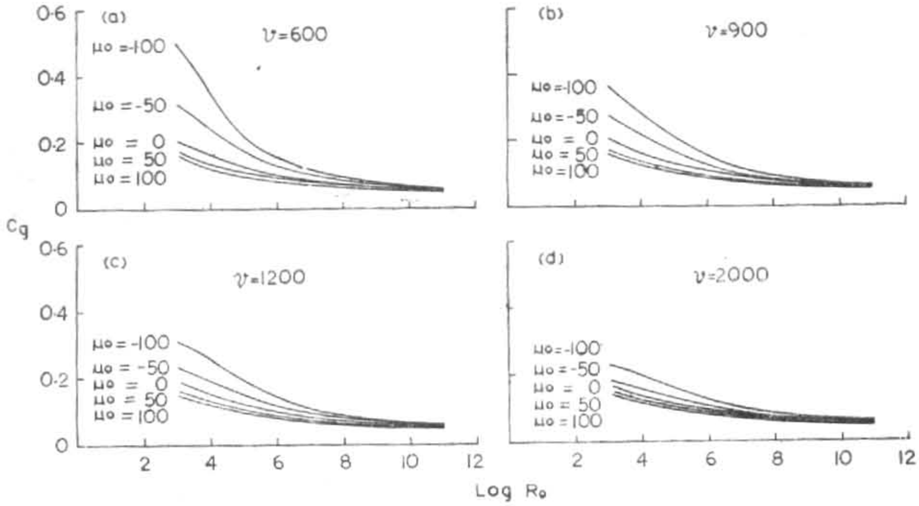

Fig. 3. Geostropic drag coefficient as a function of μ_0 and log R_0 for (a) $v = 600$, (b) $v = 900$, (c) $v = 1200$ and $v = 2000$.

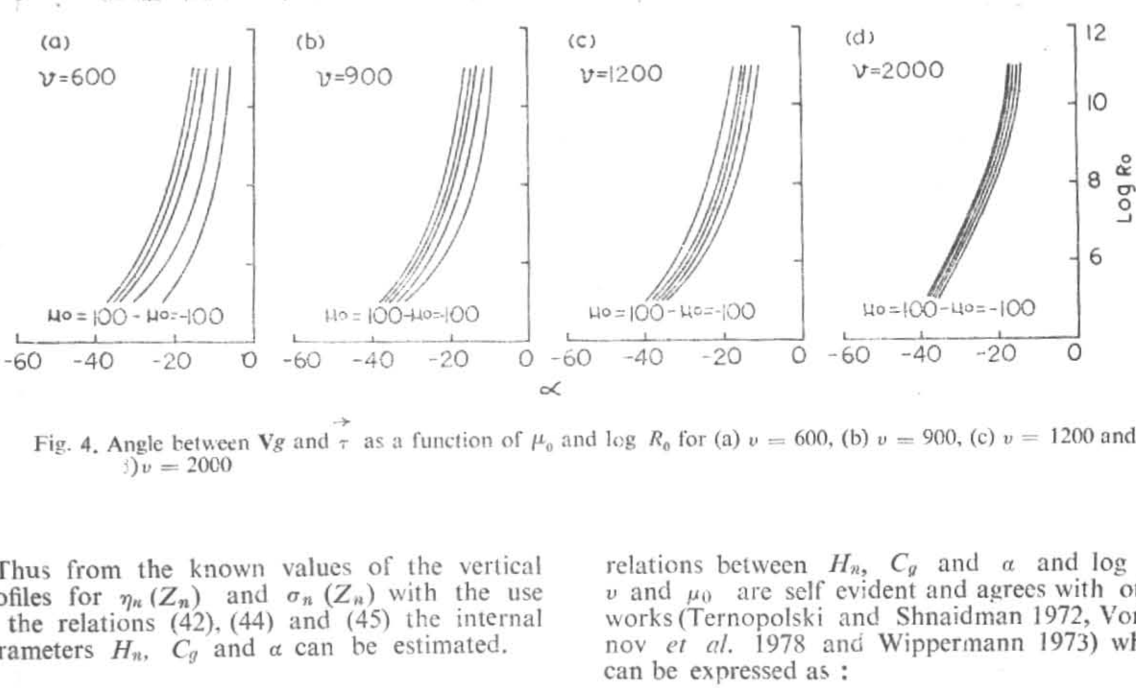

Fig. 4. Angle between $\mathbf{V}g$ and $\vec{\tau}$ as a function of μ_0 and log R_0 for (a) $v = 600$, (b) $v = 900$, (c) $v = 1200$ and (d) $v = 2000$

Thus from the known values of the vertical profiles for $\eta_n(Z_n)$ and $\sigma_n(Z_n)$ with the use of the relations (42), (44) and (45) the internal parameters H_n, C_g and α can be estimated.

A number of numerical experiments with the PBL model Eqns. (14)-(18) are carried out with various values for log R_0, v and μ_0 as input and the corresponding values for H_n, C_g and α are computed from $\eta_n(Z_n)$ and $\sigma_n(Z_n)$ profiles. Examples of the relationships between the external and internal parameters of the PBL are illustrated in Figs. 2-4. From these figures, the functional relations between H_n, C_g and α and log R_0, v and μ_0 are self evident and agrees with other works (Ternopolski and Shnaidman 1972, Vordanov *et al.* 1978 and Wippermann 1973) which can be expressed as :

$$
\begin{aligned}
H_n &= H_n(v, \mu_0) \\
C_g &= C_g(\log R_0, v, \mu_0) \\
\alpha &= \alpha\ (\log R_0, v, \mu_0)
\end{aligned}
\tag{46}
$$

For convenience in paractical application of the PBL parameterization in the numerical models of the atmosphere, we can approximate the relationships between the internal and external

TABLE 1

Values of coefficients in the polynomials (47) for calculating H_n, C_g and α

a_i (i=1, 5)	b_j (j=1 ,9)	c_j (j=1, 9)
0.7080	0.04688	—94.435
$-0.7448.10^{-3}$	$-0.3180.10^{-1}$	$-0.8875.10^{-2}$
$-0.5699.10^{-2}$	$-0.8707.10^{-3}$	$-0.8757.10^{-2}$
$0.3707.10^{-5}$	$-0.9792.10^{-2}$	14.2642
$0.2632.10^{-6}$	$-0.1568.10^{-7}$	$0.2457.10^{-5}$
$-0.2308.10^{-5}$	$0.1088.10^{-6}$	$0.8289.10^{-4}$
	$0.1534.10^{-5}$	$0.7450.10^{-3}$
	$-0.4275.10^{-5}$	$0.2174.10^{-4}$
	$0.9956.10^{-4}$	$-0.1919.10^{-2}$
	$0.2930.10^{-2}$	—0.6751

parameters of the PBL with non-linear second-order multiple regression equations. Thus, the empirical relations for the system (46) may be written as :

$$H_n = a_0+a_1v+a_2\,\mu_0+a_3\,v\mu_0+a_4\,v^2+a_5\,\mu_0^2$$

$$C_g = b_0+b_1\,v+b_2\,\mu_0+b_3\,\log R_0+b_4\,v^2+b_5\,v\mu_0+ +b_6\,v\log R_0+b_7\,\mu_0^2+b_8\,\mu_0\log R_0+b_9\,(\log R_0)^2$$

$$\alpha = C_0+C_1v+C_2\,\mu_0+C_3\log R_0+C_4v^2+C_5v\,\mu_0+ +C_3v\log R_0+C_7\,\mu_0^2+C_8\,\mu_0\log R_0+C_9(\log R_0)^2$$

In order to find out the coefficients a_i, b_j and C_j (i=0, 5 ; j=0, 9) of the second order polynomials (47), method of least square was applied. The values of the coefficients are presented in Table 1.

For given values of log R_0, v and μ_0 the corresponding values of H_n, C_g and α can be obtained from the ploynomial relations (47) with known values of the regression coefficients (Table 1).

7. Conclusions

The proposed algorithm for evaluation of the surface friction the eddy flux of momentum and frictional vertical velocity as the integrated effect of the PBL in the dynamic models of the atmosphere has the following summerized scheme. Suppose that the numerical model of the atmosphere gives us the components of the velocity vector, the geopotential height, the temperature, the lapse rate and the horizontal temperature gradient at the upper boundary of the PBL and a specified temperature, at the surface is assumed. First, using the thermal wind relation $\mathbf{V}_{g0}$ can be estimated and then the non-dimensional external parameters, R_0, v and M can be calculated by the relations (22), (23) and (25). Using the nomogram (Fig 1.), μ_0 can be determined from known values of R_0, v and M. Then the non-dimensional internal parameters of the PBL H_n, C_g and α can be obtained from the estimated values of log R_0, v and μ_0 by use of polynomial relations (47). The unkown values of the surface friction and frictional vertical velocity can be determined from known values of H_n, C_g and α by the relations (38), (39) and (41). Further, the heat flux at the surface P_0 can also be estimated from the known value of μ_0 by the relation (21) (Vordanov *et al.* 1978).

Thus the eddy fluxes of momentum and heat and the frictional vertical velocity can be obtained at each grid point of a numerical model of the atmosphere from the known dependent variables of the model by simple relations as discussed above, without solution of the PBL model and as such minimizes the computational time considerably to make it practicable to include the role of the PBL in the dynamics of the atmosphere.

The mathematical model of the PBL, discussed in detail, can also be used to find out the structure and characteristics of the boundary layer with different synoptic conditions. Further, the results of the model can be utilised directly in various studies, such as, air pollution and aeronautical meteorology for finding out vertical profiles of components of the vector velocity, turbulent diffusion coefficient $K(z)$, stability, etc.

An extension of this study to include the effect of the baroclinicity of the tropical atmosphere will be presented in a separate paper.

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