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# The continental thermal energy storage in a zonally averaged global thermodynamic model

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सार — ग्रादम (1962, 63, 64 क) के जवलायु निदर्श में जब तापगति के पहले नियम को महाद्वीपों के उपयोग में लाया गया था तब जर्ध्वाधर प्रक्षोभ के आवागमन से विकिरण ऊर्जा की अधिकता और तापीय ऊर्जी संवेद्य एवं वाष्पन की (संचलन और मंडारण के प्रभावों से तापीय ऊर्जा के परिवर्तन को नगण्य मानकर) गुप्त उष्मा के हास के मध्य संतुलन को ध्यान में रखकर समीकरण को हल किया गया है।

इस गोधपत में यह दर्शाने का प्रयास किया गया है कि ऊर्जा के महाद्वीपीय भण्डार नगण्य नहीं है। इस जांचपड़ताल से महाद्वीपीय सतह तापमान के परिणामों में सुधार हुआ है।

यह महाद्वीपीय तापमान भूमितल को निकट वायू के तापमान, मेधाण्ठन्तता, धवलता, सौर ग्रौर पार्थिव विकिरण, लघुतंरग में अवकोषण गुणांक, ऊर्जा के भण्डार एवं संवेद्य एवं वाष्पन की गुप्त उष्मा के उध्वधिर प्रक्षोभ के आवागमन के कारण तापीय ऊर्जा में ह्रास या वृद्धि का फलन हे ।

ABSTRACT. In the climatic model of Adem (1962, 63, 64 a), when the first law of thermodynamics is applied to the continents, the equation is simplified considering a balance between the excess of radiant energy and the loss of thermal energy — sensible and latent heat of evaporation — by vertical turbulent transport, neglecting changes of thermal energy by conduction and storage effects.

In this paper we have attempted to show that the continental storage of energy is not negligible. This finding improves the results of continental surface temperature.

This continental temperature is a function of the temperature of the air near the ground, the cloudiness, the albedo, the solar and terrestrial radiation, the absortion coefficients in short wave, the storage of energy and the loss or gain of thermal energy by turbulent vertical transport of sensible and latent heat of evaporation.

## 1. Introduction

The thermodynamic model is used by the U.S. Weather Bureau to forecast monthly and seasonal climatological variables of the northern hemisphere, (Adem & Jacob 1968, Adem 1979), being also used in research of climatic changes (Shaw and Donn 1971, Adem 1973). With the purpose of verifying the use of the model on a global scale, Buendia et al. (1975, 1978) integrated it on zonal average basis, obtaining satisfactory results on the temperature fields and the excess of radiation from the ocean surface and the middle troposphere (500 mb), with better results for the tropical zones than that obtained by Adem (1963), who used in his hemispheric model a null zonal wind at the equator as a boundary condition, giving in this way contamined solutions in lower latitudes, although this is an adequate hypothesis.

For the continental surface temperatures the calculated values were quite different from the ob-

served ones, because they were neglected in agreement with Adem(1964 b), the continental energy storage and the fluctuations of thermal energy by conduction.

The model uses the principle of conservation of energy (Adem 1963-73) in a tropospheric vertical columm of 11 km height on one cm<sup>a</sup> of surface, in the oceans and continents is used per unit of area vertical columns 100 m depth, (Adem 1968) and 12 m in agreement with Bryson-Dittberner (1976) respectively.

With the object to join the thermodynamic and dynamic effect we use also the equations of continuity and movement, where this last one is simplified with a geostrophic balance in the atmosphere, (Adem 1967), and in the oceans the horizontal balance of Ekman (Adem 1968).

The integration has been done for an atmosphere with a constant tropospheric height H = 11 km

and a pressure  $p^*$ , density  $\rho^*$ , temperature  $T^*$ , velocity  $\overline{V^*}$  and a lapse rate  $\beta$  functions of :

$$p^{*} = p^{*} (\psi, \phi, z, t)$$

$$\rho^{*} = \rho^{*} (\psi, \phi, z, t)$$

$$T^{*} = T^{*} (\psi, \phi, z, t)$$

$$\overline{V^{*}} = \overline{V^{*}} (\psi, \phi, z, t)$$

$$\beta = \beta (\psi, \phi, t)$$

where  $\psi$  represents the longitude,  $\phi$  the latitude, z the height and t the time.

As the variations of thermal energy in the tropospheric ocean-continent system due to changes in the tropospheric height are negligible (Adem 1968), it is convenient to take in consideration, (Buendia 1974) the variations of thermal energy due to changes of lapse rate by the eddy transport in middle and higher latitudes and by convection in lower latitudes, which is in agreement with Stone and Carlson (1979).

### 2. Equation for the troposphere-ocean-continent system

Under the hypothesis already mentioned, the principle of conservation of energy in the troposphere and the oceans after averaging them and considering the effects of disturbances that are not detected by the instruments or by numerical methods (Miller 1951) have the  $n \in x^*$  form respectively.

$$\begin{split} & \int_{0}^{H} \bar{\rho}^{*} \; \frac{\partial}{\partial t} \left( C_{v} \bar{T}^{*} \right) dz + \int_{0}^{H} V_{g}^{*} \; \rho^{*} \cdot \nabla_{H} \left( C_{v} \bar{T}^{*} \right) dz + \\ & + \int_{0}^{H} \bar{\rho}^{*} \; \overline{w}^{*} \frac{\partial}{\partial z} \left( C_{v} \bar{T}^{*} \right) dz + \int_{0}^{H} \nabla_{H} \; \times \\ & \times \left( \rho^{*} C_{v} \overline{V_{g}^{*'} T^{*'}} \right) dz + \\ & + \int_{0}^{H} \frac{\partial}{\partial z} \left( \bar{\rho}^{*} C_{v} \bar{T}^{*'} W^{*'} \right) dz = \\ & = \int_{0}^{H} \bar{\varrho}^{* I} dz + \int_{0}^{H} \bar{\varrho}^{* L} dz + \int_{0}^{H} \bar{R}^{*} dz \quad (1) \\ & \int_{-h}^{0} \left\{ \bar{\rho}_{s}^{*} \frac{\partial}{\partial z} \left( C_{vs} \bar{T}_{sm}^{*} \right) + V E_{H}^{*} \bar{\rho}_{s}^{*} \cdot \nabla_{H} \left( C_{vs} \bar{T}_{sm}^{*} \right) \right. \\ & \left. + \bar{\rho}_{s}^{*} w_{s}^{*} \; \frac{\partial}{\partial z} \left( C_{vs} \bar{T}_{sm}^{*} \right) \right\} dz + \\ & \left. + \int_{-h}^{0} \nabla_{H} \cdot \left( \bar{\rho}_{s}^{*} C_{vs} \overline{V} E_{H}^{*'} T'_{sm}^{*} \right) dz + \\ & \left. + \int_{-h}^{0} \frac{\partial}{\partial z} \left( \overline{w_{s}^{*'} \bar{\rho}_{s}^{*} C_{vs} T_{sm}^{*'} \right) dz \right] \end{split}$$

$$= \int_{-h}^{0} (\overline{\varrho}_s^{*I} + \overline{\varrho}_s^{*L} + \overline{R}_s^{*}) dz \qquad (2)$$

Here  $C_v$  and  $C_{vs}$  are the specific heats at constant volume of the atmosphere and the oceans respectively,  $\nabla_{II}$  is given in spheric coordinates,

 $\int_{0}^{H} \overline{\rho}^{*} \frac{\partial}{\partial t} (C_{v} T^{*}) dz \text{ is the local change in thermal energy per unit area } \int_{0}^{H} \overline{\rho}^{*} V_{g}^{*} \cdot \nabla_{H} (C_{v} T^{*}) dz$ is the change in thermal energy by advection due to geostrophic wind,  $\int_{0}^{H} \overline{w}^{*} \overline{\rho}^{*} \frac{\partial}{\partial z} (C_{v} T^{*}) dz$  is

the variation per unit area of thermal energy due to convection,  $\int_{0}^{H} \bigtriangledown_{H} \cdot (\bar{\rho}^{*} C_{v} V_{g}^{*'} T^{*'}) dz$  is the change in thermal energy per unit area of horizontal turbulent transport of sensible heat due to migratory anticyclones and cyclones,  $\int_{0}^{H} \frac{\partial}{\partial z} (\rho^{*} \overline{C_{v}} T^{*'} W^{*'}) dz$ is the variation in thermal energy per unit area by vertical turbulent transport of sensible heat,  $\int_{H} \bar{Q}^{*I} dz$  is the variation in thermal energy per unit area due excess of radiation,  $\int_{0}^{H} Q^{*L} dz$ the gain in energy by condensation of water vapour and  $\int_{0}^{H} \overline{R}^{*} dz$  is the change of thermal energy due to the work by or on the system, which is negligible for intervals of a month or a season.

The first term on the left hand side of the equation of conservation of energy in the oceans (Eqn. 2), is the storage of energy of the ocean per unit area, the second term represents the change in energy due to advection of the ocean currents, the third is the variation in thermal energy due to upwellings, the fourth term represents the change of thermal-energy per unit area due to horizontal turbulent transport by migratory vortex and the last term is the variation in thermal energy by turbulent vertical transport of sensible heat. The first term on the right hand side is the change thermal energy due to excess of radiation, the second term is the variation in thermal energy due to the loss of latent heat of evaporation from the surface, and the third term is the change of thermal energy due to the work done by or on the system. being neglegible for the oceans.

The first law of thermodynamics on the continents (Adem 1965), after considering the storage of continental energy, in this study, takes the form:

$$\int_{-hc}^{0} \overline{\rho_{c}}^{*} \frac{\partial}{\partial t} \left( C_{vc} \, \overline{T}_{sc}^{*} \right) dz = \int_{-hc}^{0} \overline{\varrho_{c}}^{*1} dz +$$
$$+ \int_{-hc}^{0} \overline{\varrho_{c}}^{*L} \, dz + \int_{-hc}^{0} \overline{\varrho_{c}}^{*S} \, dz$$
(3)

where  $\bar{\rho}_c^*$  represents the density of the continent,  $C_{vc}$  the specific heat capacity of the continent at constant volume,  $T_{sc}^*$  the temperature of the continental surface,  $h_c$  is the depth of the continental layer,  $\int_{-hc}^{0} \bar{\rho}_c^* \frac{\partial}{\partial t} (C_{vc} T_{sc}^*) dz$  is the continental storage of thermal energy. The three terms on the right hand side of the Eqn. (3), represent the change in the thermal energy per unit area by the excess of radiation, the loss of thermal energy from the ground by latent heat of evaporation and the loss of thermal energy by sensible heat, respectively.

Making scale analysis to the equation of continental energy conservation (Eqn. 3), as done by Charney (1948) to the movement equation, we find that the continental energy storage is of the same order of magnitude as the excess of radiant energy on the continental surface and the loss of thermal energy from the surface by turbulent vertical transport by sensible and latent heats of evaporation, which is in agreement with :

$$\rho_{c} = 2g \text{ cm}^{-3} \sim 10^{\circ} \text{ (Bryson and Dittberner 1976)} \\ h_{e} = 1200 \text{ cm} \sim 10^{3} \text{ (Bryson and Dittberner 1976)} \\ C_{ve} = 1.04 \times 10^{7} \text{ erg g}^{-1} \, {}^{\circ}\text{K}^{-1} \sim 10^{7} \text{ (Bryson and Dittberner 1976)} \\ \Delta T_{se} = 10^{\circ} \text{ K} \sim 10^{1} \text{ (Adem 1967)} \\ \Delta t = 2.65 \times 10^{6} \text{ seg} \sim 10^{6} \\ \int_{-h_{e}}^{0} \bar{\rho}_{e}^{*} \frac{\partial}{\partial t} \left( C_{ve} \, \bar{T}_{se}^{*} \right) dz = \frac{10 \times 10^{3} \times 10^{7} \times 10^{9}}{10^{6}} \end{cases}$$

 $\sim 10^5$ 

 $\int_{-h_c}^{0} \overline{Q}_c^{*I} dz = \overline{E}_s = 1.0465 \times 10^5 \text{ erg/seg/cm}^3$ 

 $\sim 10^5$  (Buendia-Morales-Camissasa 1980)

 $\int_{-h_c}^{0} \{ \bar{\varrho}_c^* L + \bar{\varrho}_c^* S \} dz = \bar{G}_3 + \bar{G}_2$ = 0.15 cal min<sup>-1</sup> cm<sup>-2</sup> ~ 10<sup>5</sup> (Sellers 1975)

In the thermal energy equation of the troposphere, considering first approximation, the lapse rate  $\beta$  in the atmosphere is given by :

$$\beta = dT^*/dz \tag{4}$$

Then the temperature  $T^*$ , the pressure  $p^*$ , and the density  $\rho^*$  of the atmosphere to the height z, are given by :

$$T^{*}(\psi, \phi, z, t) = T(\psi, \phi, t) - \beta(\psi, \phi, t) \{z - H\}$$

$$p^{*} = \overline{\rho} \left[ 1 + \beta \frac{(H - z)}{T} \right]^{g/R\beta}$$

$$\rho^{*} = \overline{\rho} \left[ (1 + \beta \frac{(H - z)}{T} \right]^{g/R\beta - 1}$$
(5)

Using the Eqns. (4) and (5) in the principle of conservation of energy applied to the troposphere, (Eqn. 1), we obtain :

$$F_{2} \frac{\partial \bar{T}}{\partial t} + F_{5} \frac{\partial \beta}{\partial t} + F_{3} \cdot \nabla_{H} \bar{T} + F_{6} \cdot \nabla_{H} \bar{\beta} - KF_{4} \cdot \nabla_{H} \bar{T}$$
$$- K F_{2} \nabla_{H}^{2} \bar{T} - KF_{7} \cdot \nabla_{H} \bar{\beta} - KF_{5} \cdot \nabla_{H}^{2} \bar{\beta}$$
$$= \bar{F}_{T} + \bar{G}_{2} + \bar{G}_{5}$$

where  $F_2$ ,  $F_3$ ,  $F_4$ ,  $F_5$ ,  $F_6$ ,  $F_7$ , K,  $\overline{E}_T$ ,  $\overline{G_2}$  and  $\overline{G_5}$  are given by the following expressions :

$$F_{2} = C_{v} \int_{0}^{H} \bar{\rho}^{*} dz, \quad F_{3} = C_{v} \int_{0}^{H} \bar{\rho}^{*} \ \overline{V}_{g} dz$$

$$F_{4} = C_{v} \int_{0}^{H} \nabla_{H} \rho^{*} dz, \quad F_{5} = C_{v} \int_{0}^{H} (H - z) \bar{\rho}^{*} dz$$

$$F_{6} = C_{v} \int_{0}^{H} (H - z) \rho^{*} V_{g}^{*} dz$$

$$F_{7} = C_{v} \int_{0}^{H} (H - z) \nabla_{H} \bar{\rho}^{*} dz$$

$$K = \frac{-\bar{\rho}^{*} C_{v} \overline{V^{*'} T^{*'}}}{\bar{\rho}^{*} C_{v} \nabla_{H} \overline{T}^{*}} \sim 10^{10} \text{ cm}^{2} \text{ seg}^{-1}$$

$$\overline{E}_T = \int_0^H \overline{Q^*} I dz \qquad \overline{G}_5 = \int_0^H \overline{Q^*} L dz$$

$$\overline{G}_2 = (C_v \rho^* \overline{W^{*'} T^{*'}})_{Z=0}$$

Developing Eqn. (6) according to Buendia et al. (1979), we obtain :

$$F_{2}\frac{\partial \overline{T}}{\partial t} + F_{8}^{*} \left(\frac{1}{r_{0}^{2}\cos\phi}\right) \left[J(\overline{T},\overline{\rho})\right] + F_{9}^{*} \left(\frac{1}{r_{0}^{2}\cos\phi}\right) \left[J(\overline{T},\overline{\rho})\right] + F_{9}^{*} \left(\frac{1}{r_{0}^{2}\cos\phi}\right) \left[J(\overline{p},\overline{\rho})\right] - K(F_{11}^{*},\overline{\rho})\right] + F_{10}^{*} \left(\frac{1}{r_{0}^{2}\cos\phi}\right) \left[J(\overline{p},\overline{\rho})\right] - K(F_{11}^{*},\nabla_{H}\overline{p} + F_{12}\nabla_{H}\overline{T} + F_{13}\nabla_{H}\overline{\rho}) \cdot \nabla_{H}\overline{T} - K(F_{5}/\overline{p},\nabla_{H}\overline{p} + F_{14}^{*},\nabla_{H}\overline{T} + F_{15}\nabla_{H}\overline{\rho}) \cdot \nabla_{H}\overline{\rho} + F_{5}\frac{\partial \overline{\rho}}{\partial t} - F_{2}K\nabla^{2}\overline{T} - F_{5}K\nabla^{2}\overline{\rho} = \overline{p}T + \overline{G}_{2} + \overline{G}_{5}$$
(7)

where the coefficients  $F_2, F_5, F_8^*, F_9^*, F_{10}^*, F_{11}^*, F_{12}^*, F_{13}, F_{14}^*$  and  $F_{15}$ , are the same as given by Buendia-Del Valle (1979),  $F_2 \frac{\partial \overline{T}}{\partial t}$  and  $F_5 \frac{\partial \overline{\beta}}{\partial t}$  represent the tropospheric energy storage,

$$\frac{1}{r_0^{\mathfrak{s}} \cos \phi} \left\{ F_{\mathfrak{s}}^* J(\overline{T}, \overline{\rho}) + F_{\mathfrak{s}}^* J(\overline{T}, \overline{\beta}) + F_{\mathfrak{s}}^* J(\overline{T}, \overline{\beta}) + F_{\mathfrak{s}}^* J(\overline{\rho}, \overline{\beta}) \right\}$$

is the variation in thermal energy by advection due to the geostrophic wind in the troposphere,

$$-K \left(F_{11}^* \bigtriangledown_H \overline{\rho} + F_{12} \bigtriangledown_H T + F_{13} \bigtriangledown_H \overline{\beta}\right) .$$
  
$$\bigtriangledown_H \overline{T} - K \left(F_5 / \overline{p} \bigtriangledown_H \overline{p} + F_{14}^* \bigtriangledown_H \overline{T} + F_{15} \bigtriangledown_H \overline{\beta}\right) \bigtriangledown_H \overline{\beta} - F_2 K \bigtriangledown^2 \overline{T}, -F_5 K \bigtriangledown^2 \overline{\beta},$$

represent the change of thermal energy by horizontal turbulent transport.

Each of the principle of conservation of energy in the troposphere, oceans and continents is applied the operator

$$\frac{1}{2 \pi \tau} \int_{t}^{t+\tau} \int_{0}^{2\pi} (\ ) \ dt \ d\psi$$

to study the thermal energy variations in intervals of a month or a season in a zonally averged form. The technique used to solve the equations, are the perturbations method (Adem 1962). The temperature, pressure, lapse rate and density of these are given by :

$$\overline{T} = T_0 + T$$
$$\overline{p} = p_0 + p'$$
$$\overline{\beta} = \beta_0 + \beta'$$
$$\overline{\rho} = \rho_0$$

Neglecting the second order terms, the equation of tropospheric energy conservation (Eqn. 7) takes the form :

$$(F_2)_0 \frac{\partial T'}{\partial t} + (F_5)_0 \frac{\partial \beta'}{\partial t} - (F_2)_0 \frac{Kd^2 T'}{d\phi^2} - (F_5)_0 \frac{Kd^2 \beta}{d\phi^2} = \overline{E}_T + \overline{G}_2 + \overline{G}_5$$
(8)

The lapse rate is obtained from Eqn. (4), which is given by :

$$\beta' = \frac{2(T_a' - T_m')}{H}$$
(9)

where,

$$T_m = \frac{1}{H} \int_0^H T^* \, dz = T_{m_0} + T_m'$$

Taking the excess of radiation in the troposphere  $\overline{E}r$  given by Buendia-Del Valle (1979) and Eqn. (9) in the Eqn. (8), we obtain finally :

$$2K \left\{ \left( \frac{(F_{5})_{0}}{H} \right) - (F_{2})_{0} \right\} \frac{d^{2}T_{m}'}{d\phi^{2}} + \left[ 2 \frac{(F_{5})_{0}}{H} - (F_{2})_{0} \right] \frac{\partial T_{a}'}{\partial t} + 2 \left[ (F_{2})_{0} - \frac{(F_{5})_{0}}{H} \right] \cdot \frac{\partial T_{m}'}{\partial t} + K \left[ (F_{2})_{0} - 2 \frac{(F_{5})_{0}}{H} \right] \frac{d^{2}T_{a}'}{d\phi^{2}} = \overline{a}_{2} + \overline{a}_{5} + F_{8} + \epsilon (F_{8}' + b_{3}I) + a_{2}I + T_{8}' (F_{10} + \epsilon F_{10}') + (F_{11} - F_{9}) T_{a}'$$
(10)

Here the coefficients  $F_i$  and  $F_i'$  are given by Buendia Del Valle (1979), representing  $\epsilon$  the cloudiness given by Landsberg (1945),  $a_2$  and  $b_3$  are absortion coefficients of radiant energy in short wave for the troposphere and the cloudiness respectively given by London (1957), I is the insolation at the top of the troposphere, given by Sellers (1975),  $\overline{a_2}$  the gain of thermal energy by turbulent vertical transport given by Budyko (1963),  $\overline{a_5}$  the gain of thermal energy by condensation of water vapour given by Jacobs (1951) and  $T_a'$  the air temperature near the ground is also a known value given by Stephen, G. and Stephen, H. (1979).

The surface temperature  $T'_s$  is obtained from the principle of conservation of thermal energy applied to the ocean and continent [Eqns. (2) and 3)], where this is given by :

$$\overline{T}_{s}' = L \ \overline{T}_{sm}' + (1 \rightarrow L) \ \overline{T}_{sc}' \tag{11}$$

Here L represents the ratio of ocean in each circle of latitude with respect to itself,  $T_{sm} = T_{smo} + T_{sm'}$  with  $T_{sm'} << T_{smo}$  and  $T_{sc} = T_{sco} + T_{sc'}$  with  $T_{sc'} << T_{sco}$ .

To obtain the surface temperature of the ocean  $T_{sm}'$ , the equation of energy conservation for this fluid is developed in a similar way, like that for the troposphere, with the (Eqn. 1), in such a way that if we neglect the thermal energy changes due to

the advection of oceanic currents, to the horizontal turbulent transport, to the vertical transport by upwellings and vertical turbulent of sensible heat to the depth h of the thermocline because these changes are one order of magnitude smaller than the changes in thermal energy by storage, by vertical turbulent transport of sensible heat and latent heat of evaporation from the surface, in the same way as the changes produce by solar and terrestrial radiation. Some of the not-considered terms which have only a local importance, have been neglected and, considered in integrations when not zonally averaged.

In this way, the energy conservation equation of the oceans is reduced to :

$$\rho_s^* C_{vs} \int_{-h}^0 \frac{\partial \bar{T}_{sm}^*}{\partial t} dz = \bar{E}_s - \bar{G}_2 - \bar{G}_3$$

where,

 $T_{sm}^* = \bar{T}_h + (1 + z/h) (T_{sm} \to T_h)$ (12)  $T_h \text{ is a constant [(Buendia-Del Valle (1979), Del Valle (1978)].}$ 

$$\bar{E}_{s} = \int_{-h}^{0} \bar{Q}_{s}^{*T} dz = (Q + q)_{0} [1 - (1 - \mu)\epsilon].$$

$$(1 \to a) + F_{13}' + F_{12}^{*} + F_{13}^{*} \bar{T}_{a}' + F_{12}'$$

$$+F_{14}' + F_{14} T_{sm}'$$

$$\bar{G}_{3} = \int_{-h}^{0} \bar{Q}_{s}^{*L} dz$$

$$\bar{G}_{2} = (C_{vs} \rho_{s}^{*} \overline{T_{sm}^{*'} W_{s}^{*'}})_{z=0}$$
(13)

The coefficients  $F_i'$  and  $F_i^*$  are given by Buendia-Del Valle (1979),  $(Q+q)_0$  represents the possible radiation received from the ocean surface, given by Budyko (1974),  $\mu$  is a coefficient of regression given also by Budyko (1974) and  $\alpha$  is the albedo of the ocean given by Possey and Clapp (1964).

Using the excess of radiation on the ocean surface, Eqn. (13) and Eqn. (12) in the principle of energy conservation of the ocean, Eqn. (11), we obtain :

$$\begin{split} \bar{T}_{sm}' &= [D_s \left( T_{sm}' \right)_i + F_{12}^* + \epsilon F_{12}' \longrightarrow G_2 \longrightarrow G_3 + \\ &+ F_{13}' + F_{14}' + F_{13}^* T_a' + (1 - \alpha) \left( Q + q \right)_0 \\ &\left\{ (1 - (1 - \mu) \epsilon) \right\} ] / F_{15}' \end{split}$$

Here  $(T_{sm}')_i$  is the ocean surface temperature of a month before the calculated temperature. The observed ocean surface temperature is given by U.S. Navy (1969) and

$$D_s = -\frac{C_{vs} \rho_s^* h}{2 \bigtriangleup t} \quad F_{15}' = D_s - F_{14}$$

From the equation of conservation of energy in the continents [Eqn. (3)] is obtained.

$$T_{sc} = [d_{sc} (T_{sc}')_i - (1 - \alpha) \{1 - (1 - \mu) \epsilon\}. (Q + q) - F_{12}^{*} - \epsilon F_{12}' + F_{13} + F_{14}' - F_{13} T_{a}' - \overline{G}_{2c} - \overline{G}_{3c}] / (d_s - F_{14})$$
(15)

where  $(T_{sc'})_i$  is the surface continental temperature of a month before the calculated temperature and  $d_s = C_{vc} \ \bar{\rho}_{e}^* h_c / 2 \Delta t$ , the continental surface temperature as given by Haurwitz and Austin (1944).

## CONTINENTAL THERMAL ENERGY STORAGE

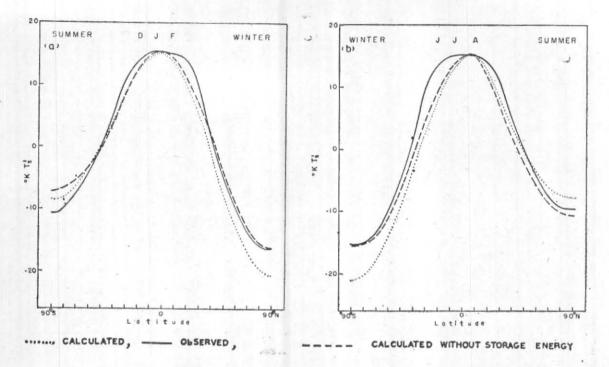


Fig. 1. The dotted line represents the calculated tropospheric temperature for winter in the Northern Hemisphere and summer in the Southern Hemisphere (1a), and for the summer in the Northern Hemisphere & winter in the Southern (1 b). The continuous line is the observed tropospheric temp, in 500 mb and the broken line is the calculated tropospheric temp, without the storage energy of the continents and  $T_{s'}=T_{a'}$ .

#### 3. Results and conclusions

The solution of the equation of thermal energy conservation in the troposphere, using a frontier condition a null zonal wind in the poles, where

$$\frac{dT_{m'}}{d\phi}\Big|_{\frac{-\pi}{2}} = \frac{dT_{m'}}{d\phi}\Big|_{\frac{\pi}{2}} = 0, \text{ as used by}$$

Adem (1963) and Buendia-Del Valle (1979), is given in Eqn. (10).

The Fig. 1 shows the calculated tropospheric temperatures  $T_m'$  (dotted line) for winter in the Northern Hemisphere and summer in the Southern Hemisphere (Fig. 1a) and for the summer in the Northern Hemisphere and winter in the Southern (Fig. 1b). The continuous line represents the observed tropospheric temperatures for the 500 mb and the broken'line represents the result by Buendia-Del Valle (1979) with no considerations of continental energy storage and making the surface temperature equal to the air temperature near the surface (z=0).

For the winter hemispheres the theoretical results for the tropospheric middle temperature are slightly less than those obtained by Buendia-Del Valle (1979), because in the winter the continental energy storage as well as the transport of thermal energy from the continental to the atmosphere, is diminished for the summer hemispheres, the opposite situation is observed, with the exception for high latitudes of the Southern Hemisphere, where low response of the energy storage to the solar radiation is observed.

Fig. 2 represents the ocean surface temperature calculated from Eqn. (15), (broken line), and the observed temperature is represented by a continuous line. It can be observed as in some other papers Adem (1964a), Buendia (1974) and Buendia del Valle (1979), that the oceanic storage of energy is predominant, getting very close to the values observed.

Fig. 3 shows the observed continental surface temperature (dotted line), the calculated temperature considering the continental energy storage in the principle of energy conservation, (Eqn. 16) with a different limiting air temperature  $T_{a'}$  to the continental surface temperature  $T_{so}$  (discontinuous) line), the dotted and dashed line represents the continental surface temperature without the energy storage and with  $T_{so} \neq T_{a'}$ , the line of crosses is the continental temperature calculated without energy storage and  $T'_{s} = T'_{a}$  following the Adem (1968) hypothesis. A notable difference is observed

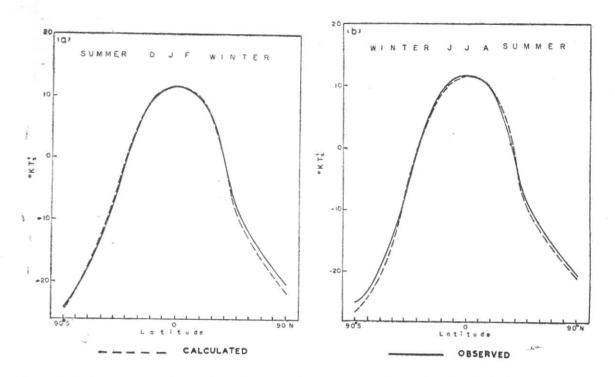


Fig. 2. The broken line represents the calculated ocean surface temperatures, the continous line is the observed ocean surface temperatures for the same periods of figure 1

between the last result and the observed temperatures, because when  $T_{s_{c}}' = T_{a}'$ , the same amount of radiant energy is emitted by the atmosphere to the surface than emitted by the surface to the atmosphere less the emitted by the surface to the atmosphere through the atmospheric window, suffering the surface a heating in lower latitudes for the summer hemisphere, decreasing the heating in the rest of the winter hemisphere. With the consideration  $T_s' \neq T_a'$  we have a normal energy interchange with losses of thermal energy from the surface by turbulent vertical transport of latent heat of evaporation and sensible heat, a prescribed variables in this paper, observing that the solution (dotted and dashed line), gets colder and closer to the observed temperatures (dotted line) because when  $T_s' > T_a'$  the continental surface lose more energy.

The energy storage is a function of the heat received one month before the season in the same way to the oceanic energy storage Adem (1963) the calculated temperature, gets colder and closer to the observed in lower latitudes and the summer hemisphere. For the winter hemisphere is observed a continental energy storage smaller and practically could be considered a balance between the excess of radiant energy and the loss of thermal energy from the surface by turbulent vertical transport of sensible and latent heat of evaporation, but Adem's hypothesis is not valid for all the seasons.

For this reason, in the climatic models the continental energy storage has to be considered. In this way the euqation of thermal energy conservation applied to the continents is a forecasting equation, and also is in balance between the continental energy storage and the loss of thermal energy by vertical turbulent transport of sensible and latent heat of evaporation, the excess of radiation in the surface and the variations of thermal energy by downward or upward conduction of the lower boundary of the continental layer. This energy change will be parameterized in future paper.

Another advantage of this result is the empirical parameterisation of the loss of thermal energy by turbulent vertical transport of sensible and latent heats of evaporation and the albedo which are function, with some other variables like the continental temperature calculated for average climatological conditions. It will be more sensitive to the changes of thermal energy in the troposphere-ocean-continent system, because the regression coefficients used by Adem (1968) do not respond in a convenient way to the changes of thermal energy due to the big differences found

## CONTINENTAL THERMAL ENERGY STORAGE

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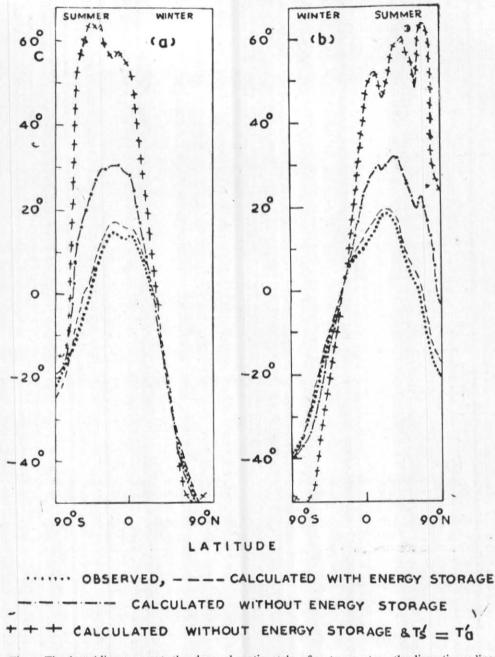


Fig. 3. The dotted line represents the observed continental surface temperature, the discontinous line is the calculated temperatures considering the energy storage. The dotted & dashed line represents the calculated continental surface without energy storage and with  $T_{sc} \neq T_{a'}$ , the line of crosses is the continental temperature calculated without energy storage and  $T_{s'} = T_{a'}$ , for the same periods of figure 1

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between the observed and calculated temperatures, when the continental energy storage is not considered.

That's why some failures are observed in the accurate forecasting of temperatures by the weather bureau and Adem y Jacobs (1968).

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