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Simple formulae for the estimation of wet bulb temperature and precipitable water

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ABSTRACT. A simple formula $T_w = T\ (0\cdot45+0\cdot006\ h\ \sqrt{p/1060})$ has been developed for computing wet bulb temperature (T_w) on electronic computers from dry bulb temperature (T), relative humidity (h) and pressure (p). The results obtained with this equation are compared with observed values and are found to be in good agreement.

A simple relation of the form $T_w=c\sqrt{W}$ has also been derived between wet bulb temperature and precipitable water (W). The results of W obtained with this equation are compared with the observed values and are found to be in good agreement.

1. Introduction

A knowledge of the moisture content in the atmosphere is of great importance in all meteorological studies. The parameter generally used for this study is water vapour. But, for the estimation of water vapour content of the entire column of the atmosphere (*i.e.*, precipitable water vapour content (W) at a particular point requires upper air data. As the upper air data is being recorded at a limited number of stations over India, it would be useful to have alternate simple methods for the estimation of W. In biometeorological studies, the wet bulb temperature is of direct use (Haldane 1905, Leonard Hill 1911-14). These studies require continuous recording (over a day) of the wet bulb temperature. The autographic recordings of the element are, however, not available from 1967 onwards.

Moreover, there is no simple formula for the estimation of wet bulb temperature on electronic computers from other available meteorological elements. In the present study, a simple formula based on dry bulb temperature, relative humidity and pressure has therefore been derived for the estimation of wet bulb temperature on electronic computers. A simple relationship between wet bulb temperature and precipitable water has also been obtained.

2. Derivation of simple formula for the estimation of T_w

2.1. Correlation study between $(T—T_w)$ and different meteorological elements showed high correlation with h (when compared with e). Therefore a linear regression equation of the form $(T—T_w) = a+bh$, was first fitted. For this, monthly 24 hourly mean data for the year 1959 for 31 stations well distributed over India were considered (*i.e.*, 31 data points for each month). The correlation coefficient (c.c.) between $(T—T_w)$ and h is denoted by r, the regression coefficient (r.c.) by b and the regression constant by a. r, a and b in different months are shown in Table 1(a).

It is seen from Table 1 (a) that a and b are different for different months, while a is nearly 100 times to b in all the respective months. It is also seen that the values of r in winter months are relatively small. In order to improve the c.c. (*i.e.*, r) in winter months an alternate form (T_w/T) has been considered in the place of $(T—T_w)$. Correlation study between (T_w/T) and h has also shown high c.c. Therefore, a regression equation of the form $T_w/T=a'+b'h$, has been tried using all the 12 months data (*i.e.*, 372 data points). The results of c.c. between (T_w/T) and h, the r.c. b' and the regression constant a' (where, a' and b' are same for all months) obtained from this analysis are respectively $0\cdot95$, $0\cdot006$ and $0\cdot43$. However, it is seen from Rao's (1945) work that T_w decreases with decreasing pressure (p) for same value of T and h and also this decrease is more associated with h rather than with T. Therefore, a pressure correction has been made to h in the above equation. The above equation has been again solved by multiplying h with $\sqrt{p}$ (here p varies nearly from 1000 to 750 mb) as, $\sqrt{p}$ increases the accuracy of the estimates compared when h is multiplied with p (where p is the annual normal pressure in mb which is constant for a station). The c.c. value varies from $0\cdot83$ to $0\cdot98$ and $0\cdot95$ to $0\cdot98$ respectively when h is multiplied with p and $\sqrt{p}$.

TABLE 1

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
	(a) Variation of r, a and b in different months											
r	—0·60	—0·81	—0·87	—0·96	—0·97	—0·97	—0·87	—0·92	—0·86	—0·86	—0·73	—0·53
a	10·35	12·45	13·36	16·16	17·51	18·96	16·67	16·59	16·48	14·53	12·53	10·46
b	—0·101	—0·120	—0·131	—0·160	—0·173	—0·187	—0·164	—0·162	—0·160	—0·140	—0·123	—0·102
	(b) Variation of r_1, r_2, a' and b' in different months											
r_1	0.95	0.96	0.97	0.97	0.98	0.98	0.98	0.97	0.97	0.96	0.96	0.96
r_2	0·83	0·89	0·93	0·96	0·98	0·98	0·96	0·94	0·89	0·87	0·87	0·85
a'	0·449	0·449	0·450	0·450	0·451	0·451	0·451	0·450	0·450	0·449	·0449	0·449
$b' \times 10^3$	5·94	5·99	6·01	6·06	6·11	6·19	6·14	6·10	6·00	6·00	5·97	5·93

TABLE 2

Percentage of the deviations between calculated and actual T_w

Range (°C)	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual
$\leqslant \pm 0·5$	59·4	50·0	65·7	87·5	90·7	81·3	84·4	78·2	81·3	81·3	84·4	75·1	68·3
±0·6 to ±0·9	31·3	40·6	25·0	6·3	6·3	18·8	15·6	21·9	18·8	18·8	12·5	18·8	25·0
$\geqslant 1·0$	9·4	9·4	9·4	[illegible]·3	3·1	0·0	0·0	0·0	0·0	0·0	3·1	6·3	6·3
n	32	32	32	32	32	32	32	32	32	32	32	32	32

Note: 1. n = No. of data points

2. Stations used in the computations — (1) Trivandrum, (2) Kodaikanal, (3) Madras, (4) Bangalore, (5) Bombay, (6) Hyderabad, (7) Ahmedabad, (8) Nagpur, (9) Calcutta, (10) Gauhati, (11) New Delhi, (12) Shillong, (13) Allahabad, (14) Jodhpur, (15) Pathankot and (16) Srinagar.

TABLE 3

Percentage of the deviations between calculated and actual W

Range (gm/cm²)	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
$\leqslant \pm 0·25$	90	90	100	90	70	90	70	80	80	100	100	100
±0·26 to 0·50	10	10	00	10	30	10	30	20	10	00	00	00
$\geqslant 0·50$	00	00	00	00	00	00	00	00	10	00	00	00
n	10	10	10	10	10	10	10	10	10	10	10	10

Note : 1. n = No. of stations (or data points)

2. Stations used in the computations — (1) Trivandrum, (2) Madras, (3) Visakhapatnam, (4) Bombay, (5) Nagpur (6) Ahmedabad, (7) Calcutta, (8) New Delhi, (9) Jodhpur and (10) Allahabad.

The c.c. between T_w/T and $h\sqrt{p}$ is denoted by r_1, the r.c. by b' and regression constant by a'. r_1, a' and b' in different months are shown in Table 1(b). (The c.c. between T_w/T and hp is denoted by r_2. r_2 in different months are also shown in the table). The final equation obtained from this analyis is given as :

$$T_w = T\,(0.45 + 0006h\sqrt{p/1060}) \qquad (1)$$

where T_w=wet bulb temperature (°C)

T=dry bulb temperature (°C) and

h=relative humidity (%)

(T_w, T and h may be daily or hourly values).

This is a simple equation for computations not only on electronic computers but also by manual (by drawing a simple nomogram).

2.2. Using Eq. (1), the values of T_w have been computed for 16 stations in India, at two main hours of observations (0830 and 1730 IST). For this study 1960 normal data published by the India Met. Dep. has been utilised. The results obtained are compared with the observed data. Table 2 shows the percentage of occasions of the differences between calculated and actual T_w in the ranges $< \pm 0.5$, ± 0.6 to ± 0.9 and $> \pm 1.0$°C. This table shows that out of 416 data points (combined for 0830 and 1730 hours of observations 17 (4 per cent) and 85 (20 per cent) points respectively are in the ranges $\geqslant \pm 1.0$ and ± 0.6 to ± 0.9°C. However, in the case of 1730 hours observations the 12 values that are having deviation > 1.0°C are from only two stations, Srinagar -7 and Pathankot -5. In the case of Pathankot the computed values are very close to the values obtained from the hygrometric table, *i.e.*, not even a single value is deviating by 1·0°C. Also in the case of Srinagar only two values are seen to be really deviating by 1·0°C from observed when compared with hygrometric table values. Rao (1945) showed that even the theoretical calculations will deviate from the observed values by about $\pm$·5°C because of several inherent assumptions. Eq. (1) can conveniently be used for the estimation of T_w except in the negative range of T_w (in this range, even the actual observations have their own limitations). No attempt has therefore been made to extend the formula to this range also.

3. Derivation of the relationship between T_w and W

3.1. The total moisture content of the atmosphere is expressed by the precipitable water vapour in the atmosphere. This is defined as the depth of liquid water that would result by condensing all the vapour in a vertical column of the atmosphere over one square centimetre cross-section.

Correlation study between T_w and W^k showed high correlation with $\sqrt{W}$. Therefore, an equation of the form given below was fitted :

$$T_w = c\sqrt{W} \text{ (or } W = c'\,T_w^2) \qquad (2)$$

where, T_w=wet bulb temperature (°C)

W=precipitable water vapour (gm/cm²)

and c=regression coefficient (or $c'=1/c^2$).

(T_w and W may be daily or hourly values).

For solving the constant c monthly mean precipitable water vapour content of the entire column of the atmosphere (over a cm²) values presented by Ananthakrishnan *et al.* (1965), based on the radiosonde data of 12 stations for 6-year period (1956-1961) and the corresponding surface wet bulb temperature values estimated from hygrometric tables from dry bulb and dew point temperatures (in °C) data have been used. The correlation coefficient between T_w and $\sqrt{W}$ is denoted by r'. The values of r' and c, thus obtained for different months are as follows:

	Jan	Feb	Mar	Apr	May	Jun
c	11·4	11·8	12·8	13·0	12·2	11·5
r'	0·98	0·98	0·98	0·97	0·95	0·96
	Jul	Aug	Sep	Oct	Nov	Dec
c	10·6	10·6	10·6	11·4	11·4	11·4
r'	0·96	0·97	0·97	0·98	0·98	0·98

A smooth curve can be drawn by plotting the values of c against the mid-value of the respective months. Then from this curve the respective value of 'c' for that day are taken for the computation of daily values of W.

3.2. Using Eq. (2), the values of W have been computed for 10 stations in India. For this purpose the normal data of CLIMAT TEMP based on radiosonde data for the period 1951-70 have been used. T_w values have been estimated from hygrometric tables in the same manner as stated earlier. The results of W obtained by using Eq. (2) are compared with that obtained by using the method suggested by Ananthakrishnan *et al.* (1965) (These results have been taken as observed for the comparison purpose). Table 3 shows the percentage of occasions that the deviations fall in the ranges $<\pm 0{\cdot}25$, $\pm 0{\cdot}25$ to $\pm 0{\cdot}50$ and $> \pm 0{\cdot}50$ gm/cm². This table shows that out of 120 data points only 15 values are having deviation more than 0·25 gm and only in one case the deviation is more than 0·50 gm. The higher deviations are mainly confined to monsoon season. As the magnitude of W is high in these months, the deviation amounts to only <5 per cent error. Eq. (2) therefore, can be used accurately for the estimation of precipitable water vapour content of the atmosphere.

4. Conclusions

(*i*) Using Eq. (1) the wet bulb temperature can be estimated easily from electronic computers.

(*ii*) Wherever the upper air data is not available precipitable water vapour content of the atmosphere can be estimated quite accurately by using Eq. (2) from the surface wet bulb temperature alone (by drawing a simple nomogram).

(*iii*) Eq. (2) is of considerable use for the synoptician to see, the day to day changes in the precipitable water vapour content of the atmosphere.

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