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A study to forecast the waves at Digha

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ABSTRACT. In this article instrumental records of waves and wind speeds at Digha have been utilised to forecast wave characteristics, namely, the wave period and wave height. The spectrum of waves as devised by Neumann, has been used to establish an empirical relationship between the wind speed and the period of maximum wave height, also this has been used to derive a relationship between the wind speed and the optimum value of wave frequency. Co-cumulative power spectra have been drawn to study the distribution of energy, and the significant range of periods of the sea waves at different wind speeds. From the observed records, values of maximum wave heights at corresponding wind speeds have been determined. An empirical relationship between the maximum wave height in deep sea, and the speed of the generating wind is established. Significant wave heights at corresponding wind speeds were also calculated, and a formula relating significant wave height to the wind speed has been provided.

1. Introduction

The study of wave characteristics are generally based on two methods :

(1) A mathematical method based on the physical laws of fluid mechanics, and

(2) Statistical methods based on wave observations and their relationship with wind speed and coastal geometry.

For scientific investigation of the nature of waves and tides at Digha an instrument, namely, Frequency Modulated Pressure Recorder was installed to record the waves and tides. With this instrument pen records of waves and tides for ten minutes were taken at an interval of 2 hr from 6 A.M. to 6 P.M. These records (1968 to 1970) have been analysed and used to obtain the wave spectrum. The wind speeds were taken by a recording type anemometer at an interval of 1 hr from 6 A.M. to 6 P.M.

In this article the wave spectrum and the wind speeds were used to forecast the characteristics of waves at Digha, by Neumann's method (1953).

2. Generation of waves

Waves in the open ocean grow by the transfer of energy from the wind blowing over the ocean to the underlying water, but the mechanism by which this is achieved is not yet fully understood. There are several theories on the mechanism of energy transfer, notably developed by Jeffreys (1925), Sverdrup and Munk (1947), Eckart (1953), Phillips (1957), but we still do not understand many aspects of wave formation. In general, the formation of the waves depends upon the fetch and the velocity and the duration of the wind. Any of the factors can set a limit to the amplitude of waves. Even winds of great strength and of long duration cannot produce large waves, if the fetch or the distance over which the wind blows is limited. Thus limitation would be influenced by the meteorological situation, and by the configuration of the land which determines the water available for wave generation. Hence, the exposure of a coast is a very significant factor on the size of the waves reaching the beach.

The wind waves under study include only the gravity waves whose frequencies are of the order of the 1 to 10^{-1} cycles per second. The long period waves, such as, swells, tides, surfs etc are excluded.

3. Wave characteristics

Waves in open ocean are difficult to analyse as the wind blowing over the enormous expanse of a large ocean changes its direction and velocity from place to place. However, the wave characteristics, which are significant, may be divided into three main groups. They are :

(*i*) length, velocity, period and height;

(*ii*) steepness and form ; and

(*iii*) energy of the waves.

(*i*) *Length, velocity, period and height* — According to classical hydrodynamics the three parameters of an ideal wave are wave length (L), velocity (C) and period (T). They are related by $L = CT$. Wave length and period are related by

$L = (g/2\pi)\,T^2$. If we use metric units, then $L = 1{\cdot}6\,T^2$; otherwise $L=5{\cdot}12\,T^2$ if the wave length is expressed in feet and T is in seconds.

The wave velocity and length are related by $C = \sqrt{gL/2\pi}$ for deep water waves, while $C = \sqrt{(gL/2\pi)\tanh(2\pi d/L)}$ for shallow water waves, where g is acceleration due to gravity, and d is the depth of water. In deep water d/L is large and $\tanh\,(2\pi d/L)$ approaches to 1.

(*ii*) *Steepness and form* — The ratio of wave height and wave length (H/L) is a measure of wave steepness. When the steepness exceeds 1/7 the waves generally become unstable and breaks. The form of an ideal wave is sinusoidal or trochoidal. The trochoidal wave has a flatter trough and sharper crest but trough and crest for sinusoidal wave are symmetrical. The asymmetry of the trochoidal wave increases and the crest becomes sharper with increasing wave steepness.

(*iii*) *Energy* — When the wind blows over the surface of the sea, it imparts energy to the sea and this energy is transferred into wave motion. The energy of the wave is a measure of the magnitude of the forces which are at work on the beach.

The energy per second per unit length of wave front (E) is,

$$E = \tfrac{1}{2} g\, \rho a^2 C$$

where a = wave amplitude = $\frac{1}{2}$ wave height
ρ = density
g = acceleration due to gravity
C = group velocity.

Alternatively the energy (E) in ft. lbs per foot of wave crest per wave length may be given by

$$E = \frac{\omega.\, L.H^2}{8}$$

where ω is the weight of 1 cubic foot of sea water, L is the wave length and H is the wave height in feet. The above formula may be expressed as

$$E = 41\, H^2\, T^2$$

where T is the wave period.

4. Wave spectrum method of forecasting waves

In this method the ocean wave spectra corresponding to different wind speeds are to be developed from the intermittent instrumental records. The instrument remained operative for a small fraction of time (*i.e.*, 10 min. at an interval of 2 hr). Assuming that the wave conditions were constant this record was taken as a representative one for the period of 2 hrs. From a typical instrumental record, the method of determining the maximum wave height (H_1'), significant wave height (H_s) and the zero crossing period (N_z) is given in Appendix 1 (A) and 1 (B).

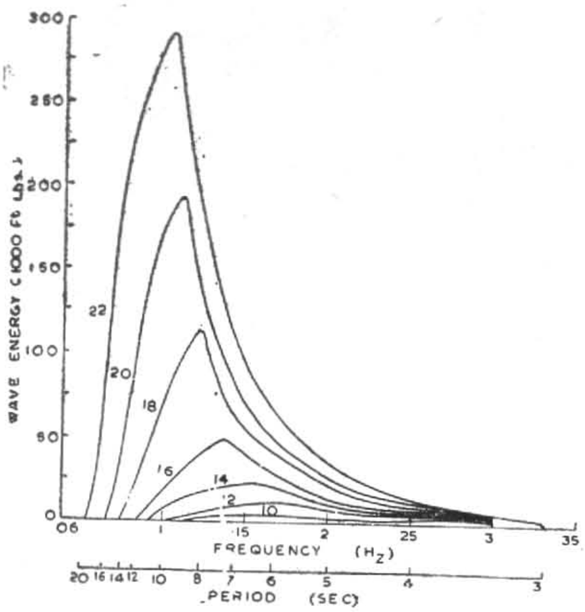

Fig. 1. Wave spectra for fully arisen sea at Digha at wind speeds 10, 12, 14, 16, 18, 20, & 22 kt respectively

The different wave records for a particular wind speed which remained almost constant for more than two hours, have been selected for developing the wave spectra. From these wave records wave heights H_1 and wave periods T_z for the corresponding wind speeds have been derived. These wave heights (H_1) have been corrected for the frequency response of the instrument and for its attenuation with depth. Then these corrected wave heights H_1' have been converted into deep water values of wave heights (H_0) by the method as shown in Appendix 1 (C).

The energy of the wave per unit length of wave front for the prevailing wind is calculated from the corresponding deep water wave height (H_0) and the time period T_z. Then the wave spectra (showing the relation between the wave period and the wave energy) for the particular wind speed is developed as shown in Appendix 1 (D.) In this way wave spectra for different wind speeds of 10, 12, 14, 16, 18, 20 and 22 kt for the sea at Digha were drawn and shown in Fig. 1.

From the wave spectrum we observe that the wave energy is concentrated in a relatively narrow band, the range of periods within this band determine the actual wave pattern. With increasing wind speed the area under the curve increases rapidly but the frequency of maximum energy shifts towards lower wave frequency. Successful prediction of the sea state have been based on the assumption that the sea waves can be considered to be the end result of superposing many simple sine waves moving in different directions. These basic wave trains have different wave lengths and periods as well as directions, so that by considering their mutual interferences it is possible to describe a complicated surface configuration, and their changes, in statistical terms.

The empirical relationship of Neumann (1953) for the optimum frequency and the speed of the generating wind is $F_{max} = 2\cdot475/U$ and $T_{max}=0\cdot405\ U$, where F_{max}, T_{max} stand for the maximum frequency and period, while U is the wind speed in knots.

From the wave spectrum given in Fig. 1 for different wind speeds the optimum value of frequency, or the most energetic period of waves, have been computed and shown in Table 1. These values were plotted graphically in Fig. 2 with wind speed as the abscissae and the most energetic period as the ordinate. A line of best fit was drawn through these points and the relationship derived from Fig. 2 may be expressed as —

$$T_{max} = 0\cdot456\ U \text{ or } T_{max} = 0.246V \qquad (1)$$

$$\text{and } F_{max} = \frac{2.193}{U} \text{ or } F_{max} = 4\cdot059V \qquad (2)$$

where U is the wind speed in kt and V is the wind speed in kmph.

It was observed that the correct period for the maximum wave height and its frequency for the sea at Digha differ only slightly from Neumann's relation. For practical purposes Eqns. (1) and (2) may be used for the coast near Digha.

5. Determination of significant range of periods of waves for different wind speeds

Fig. 3 provides a graphical representation of the co-cumulative power spectra for sea waves at wind speed of 10, 12, 14, 16, 18, 20 and 22 knots respectively. This figure provides an idea of the distribution of energy in sea waves for different wind speeds at Digha. The ordinate is in units of wave energy, which can be converted into wave heights, as the wave energy is a function of wave height. The abscissae is in the frequency scale or in units of wave period, so that wave height and period for the range of the spectrum may be inferred from Fig. 3. The significant range of periods may be estimated from this graph by ignoring the upper 5 per cent and lower 3 per cent of the curve. Upper and lower limits of wave periods for the sea at Digha, as computed from this graph, are given in Table 2.

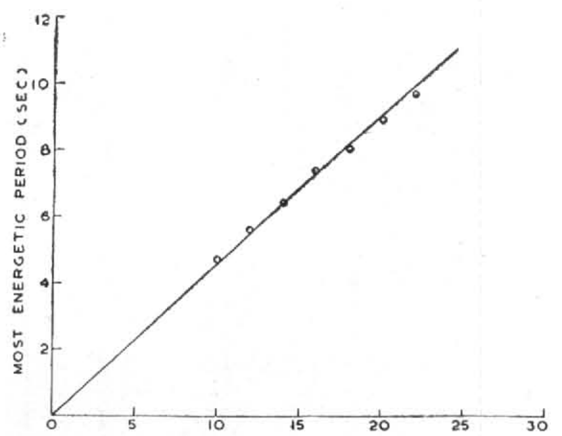

Fig. 2. Relationship of most energetic period in sec with the speed of the generating wind in kt

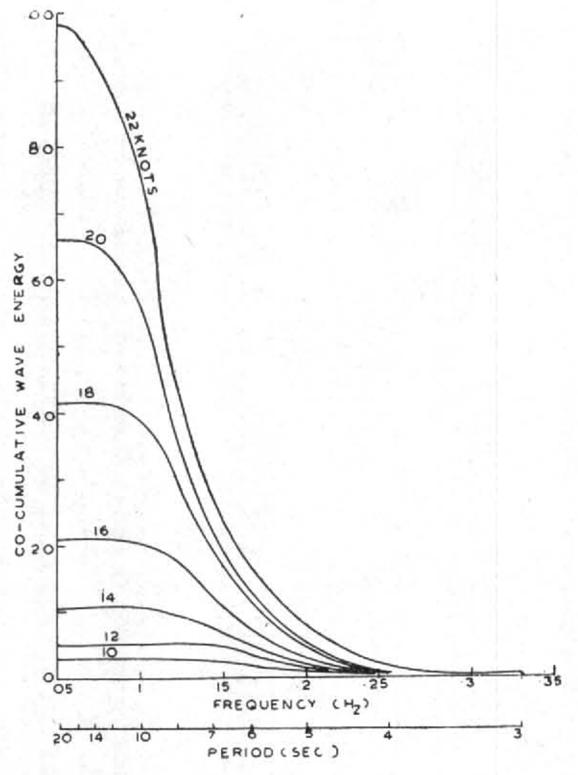

Fig. 3. Co-cumulative power spectra for the sea waves at Digha at wind velocities between 10 and 22 kt

6. Maximum and significant wave height and their variation with wind speed

An effort was made in order to establish an empirical relationship between the wave height at deep sea in Digha with the speed of the prevailing wind. The method of forecasting waves by use of wave spectra and statistics proposed by Peirson, Neumann and James is a new approach. The earlier method developed by Sverdrup-Munk has also been used extensively. The merit

TABLE 1

Period of the most energetic wave and the optimum frequency at different wind velocities for the sea at Digha

Wind speed U (kt)	Wind speed V (kmph)	Most energetic period T_{max} (sec)	Optimum frequency F_{max}
10	18·5	4·7	0·212
12	22·2	5·6	0·178
14	25·9	6·4	0·156
16	29·6	7·4	0·135
18	33·3	8·1	0·123
20	37·0	9·0	0·111
22	40·7	9·7	0·103

TABLE 2

Significant range of wave period at different wind velocities at Digha

Wind speed U (kt)	Wind speed V (kmph)	Range of period (sec) Lower T_l	Range of period (sec) Upper T_u
10	18·5	3·1	7·2
12	22·2	3·1	8·3
14	25·9	3·2	9·1
16	29·6	3·3	9·9
18	33·3	3·4	10·7
20	37·0	3·6	12·5
22	40·7	3·9	14·3

TABLE 3

Deep sea wave height from the wave record (with the help of Fig. 9) and the significant wave height for Digha

Observed data			Calculated data		D/L_o	H/H_o	H_o
Wind speed (kt)	(D) (ft)	Hmax (ft)	(T) (sec)	Signi. wave ht. (ft)			(ft)
10	13·4	2·0	4·7	1·2	0·119	0·92	2·17
12	16·2	2·7	5·6	1·67	0·100	0·93	2·90
14	19·1	3·45	6·4	2·24	0·091	0·95	3·63
16	21·1	4·15	7·4	2·95	0·075	0·96	4·32
18	23·5	5·4	8·1	3·75	0·07	0·97	5·56
20	25·8	7·0	9·0	4·6	0·062	0·99	7·07
22	28·9	8·1	9·7	5·55	0·06	1·00	8·10

D = Mean water depth, Hmax = Recorded max. wave height, T = Time period, Ho (Calculated)

of the two method has been reviewed by Bretschneider (1957). Neumann arrives at the relation ship for fully arisen sea of H proportional to $V^{5/2}$, where H is the wave height and V is the wind speed. Sverdrup and Munk utilised H proportional to V^2 for fully arisen sea.

By the use of Buchingham's π theorem (1914) and dimensionless analysis one can show that the wave height for a fully arisen sea should be porportional to the square of the wind speed. A re-analysis of the data presented by Neumann has been made by Bretschneider (1957) based on Sverdrup-Munk relation, *i.e.*, $gH_s/V^2 = 0{\cdot}26$. From his analysis it is sufficiently convincing that the data of Neumann agree better for H proportional to V^2 than for H proportional to $V^{5/2}$. In this study we preferred the relationship H proportional to V^2 rather than H proportional to $V^{5/2}$.

Now to derive the relationship between the maximum wave height in deep sea and the wind speed, the maximum wave height was determined from each of the wave spectra corresponding to maximum energy for the prevailing wind speed. These maximum wave heights (deep sea value) and corresponding wind speeds thus found out and given in Table 3 have been utilised to draw a curve as shown in Fig. 4 taking square of the wind speed along the abscissae and the deep sea value of maximum wave height along the ordinate. From this curve (Fig. 4) an empirical relationship between the maximum wave height and the wind speed was estabished, *i.e.*,

$$H_0 = 1{\cdot}75 \times 10^{-2}\ U^2 \qquad (3)$$

where, H_0 = the maximum wave height in ft (deep sea value) and

U = the wind speed in kt.

If we use metric units then this relationship is

$$H_0 = 1{\cdot}555 \times 10^{-1}\ V^2 \qquad (3a)$$

where, H_0 = the maximum wave height in cm in deep sea and

V = the wind speed in kmph.

The gradient wind may be defined as the wind which blows when the pressure gradient, Coriolis force, and the centrifugal force are balanced, and when there is no acceleration. The wind speeds were multiplied by 3/2 to obtain the equivalent gradient wind speed at the place concerned by Darbyshire (1956). Here also the equivalent gradient wind is derived in the same way by multiplying the wind speed by 3/2.

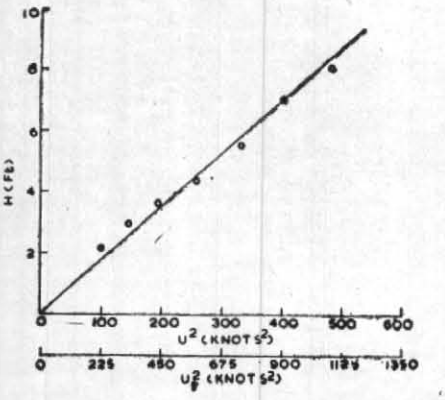

Fig. 4. Relationship between wave height in deep sea and wind speed for the sea at Digha

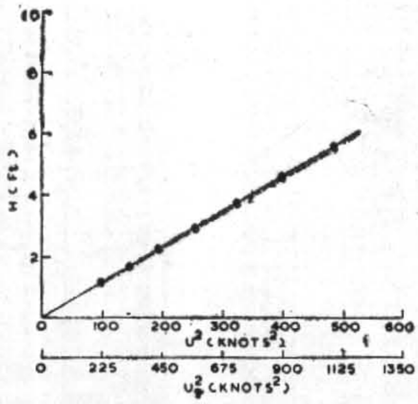

Fig. 5. Relationship between significant wave height and wind speed for the sea at Digha

If the gradient wind is taken then Eq. (3) may be written as

$$H_0 = 0 \cdot 0077\ U_g^2 \qquad (4)$$

where U_g = the gradient wind in kt.

This relationship, *i.e.*, Eq. (4) agrees closely with Darbyshire's (1956) relation for the open sea which is

$$H_{max} = 0 \cdot 0076\ U_g^2$$

Eqns. (3) or (3a) and (4) may be used for the sea at Digha as an empirical relationship between the maximum wave height and the wind speed.

Studies were also performed to establish an empirical relationship for the significant wave height of the sea at Digha with the speed of the generating wind. The waves in the spectrum are frequently referred to as significant waves and according to Sverdrup and Munk' significant wave height is defined as the average height of the highest 1/3 of the waves. Significant wave heights are calculated for each records of maximum wave height at the corresponding wind speed and shown in Table 3. In Fig. 5 we take square of the wind speed (U^2) along the abscissae and calculated significant wave height along the ordinate. Thus from Fig. 5 an empirical relatioship for the significant wave height with the wind speed is drawn and given in Eq. (5).

$$H_s = 1 \cdot 16 \times 10^{-2}\ U^2 \qquad (5)$$

where H_s = the significant wave height in ft and U = the wind speed in kt.

If we use metric units then this relation is

$$H_s = 1 \cdot 031 \times 10^{-1}\ V^2 \qquad (5a)$$

where, H_s = significant wave height in cm and V = the wind speed in kmph.

If the gradient wind is taken then the above relationship is given by the following equation

$$H_s = 0 \cdot 00515\ U_g^2 \qquad (6)$$

where H_s is in ft, U_g (the gradient wind) in kt. Eqns. (5) or (5a) and (6) may be considered as the empirical relationships between the significant wave height and the strength of the generating wind for the sea at Digha.

7. Conclusion

In this study we consider the wind speed as the main force for generating waves. When the magnitude and direction of the wind speed are known, this study will enable us to know the state of the sea at Digha at any time. This will enable us to obtain the necessary information regarding the characteristics of the sea waves at Digha while designing any structure for sea defence in that area.

Acknowlegement

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APPENDIX 1

Wave Energy Spectra

To develop wave engergy spectra from the records of sea waves and wind speeds the following procedure has been adopted.

(A) To determine the required parameters from the instrumental records

(*i*) *Wave height* (H_1) — Fig. 6 gives a typical wave record for 10 minutes of the sea at Digha. At first the mean water level line is drawn by eye estimation. Each vertical divisions of the wave recording paper corresponds to a water level variation of 0·2 ft. The highest crest height (A) from the mean water level is then determined accordingly. Similarily the depth (C) of the lowest trough from the mean water level is measured. The wave height parameter H_1 is then calculated from the distances of the highest crest A and the lowest trough C irrespective of whether they are the parts of the same wave or not. This H_1 is equal to $A+C$.

(*ii*) *Zero crossing period* (T_z) — This is derived from the known length of record of 10 min. duration by counting the number of times the record crosses the mean water level in both the upward and downward directions; occasions when a crest or trough just touches the mean water level are counted as one crossing. This number N_z (total) is divided by 2 to give the number of zero crossings in the upward directions.

Hence, T_z = duration of records in sec/$N_z \div 2$
$= 600/N_z \div 2$ sec.

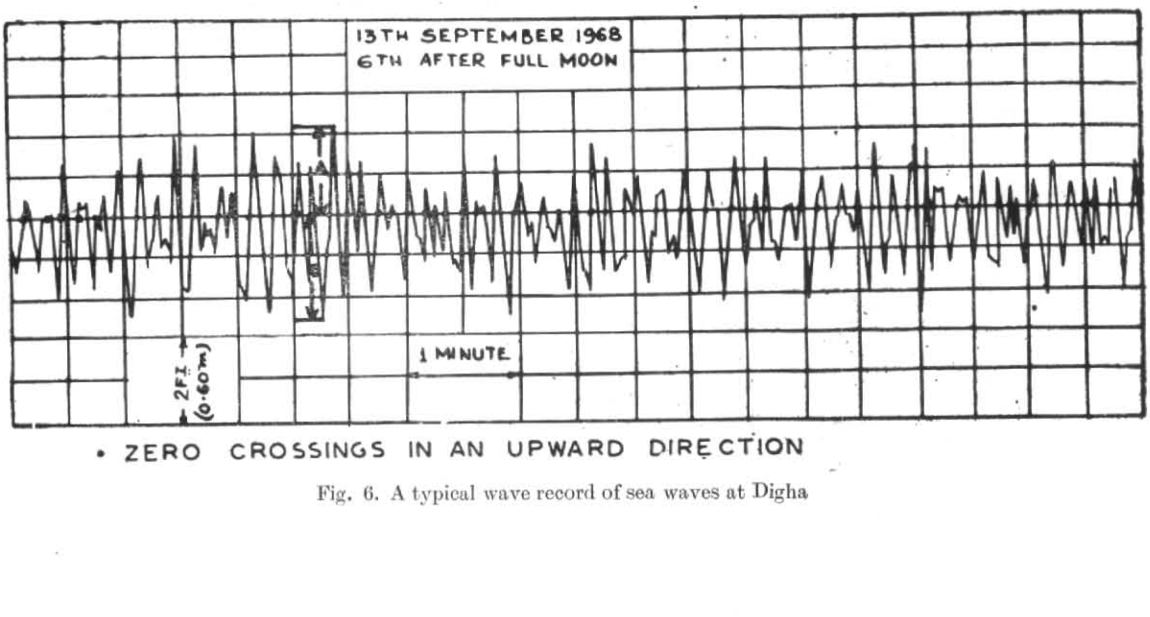

Fig. 6. A typical wave record of sea waves at Digha

(B) Processing of the above parameters

(*i*) *Corrected wave height* (H_1') — As the wave height sensor was an under water pressure measuring unit, the recorded values of wave heights are to be corrected for attenuation of waves with depth and for the frequency response of the instrument. So the parameters H_1 is to be corrected for (*a*) the frequency response of the recording instrument and for (*b*) the attenuation of waves with depths to give the true height H_1'. The probe is placed at 13·7 ft below the mean sea level. The depth of water is obtained by adding or subtracting the water level recorded in tide records from 13·7 ft as the case may be. Knowing the wave period T_z and the depth of water above the probe the corresponding correction factor is obtained from the curve in Fig. 7, which is adopted from Draper L. attenuation of sea waves with depth (*vide* La Houlle Blanche, No. 6, December 1957).

H_1 is to be divided by the corresponding factor obtained from Fig. 7 to get the corrected wave height H_1'.

(*ii*) *Significant wave height* (H_s) — Significant wave height is defined (according to Sverdrup-Munk) as average height of the 1/3 of the highest waves. The calculation of H_s from H_1' is a simple statistical process and depends only on the number of waves. The relationship for narrow band frequencies may be expressed as follows :

$$H_s = H_1' 2\cdot(2\phi)^{-1/2}(1+0\cdot289\phi^{-1} - \cdot0247\phi^{-2})^{-1}$$

This relationship may be expressed graphically in Fig. 8. This figure is a modification of Fig. 2 in analysis of records of sea waves by M.J. Tucker. The effect of wide frequency band is to reduce the value of H_s thus determined.

(C) To convert the observed wave height to its corresponding deep water value

The wave heights are recorded at a distance of 1 km from the shore within the shallow area. Waves change in length and height as they enter shallow water. Let H_0 be the wave height and L_0 be the wave length at deep sea where the water depth d_0 is greater than the wave length. If the observed wave height and length be H_1 and L_1 respectively at a place near the shore where the water depth is d, then these values of wave length and wave height can be related to its corresponding deep water values with the help of Fig. 9 given by Russel and Macmillan (1952).

(D) To draw the wave spectra for a particular wind speed

Let the wave spectra for a particular wind speed say, 16 kt, *i.e.*, 18·4 mph or 29·6 kmph is to be drawn. The waves striking the shore at Digha are

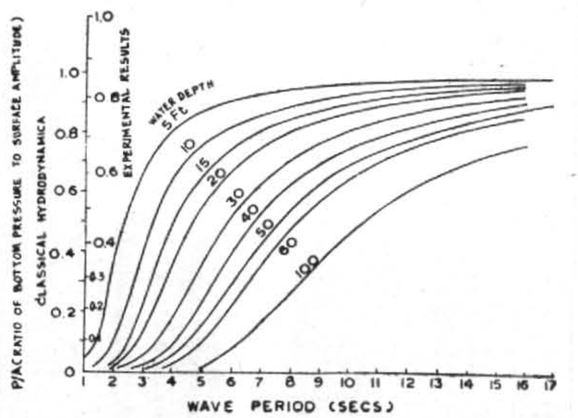

Fig. 7. The relationship between pressure fluctuations on the sea bottom, surface wave height, wave period and depth of water. The "experimental results" scale is adapted from Draper L. attenuation of sea waves with depth "La Houlle Blanche" No. 6, Dec 1957 and refers to waves recorded on a gently sloping beach.

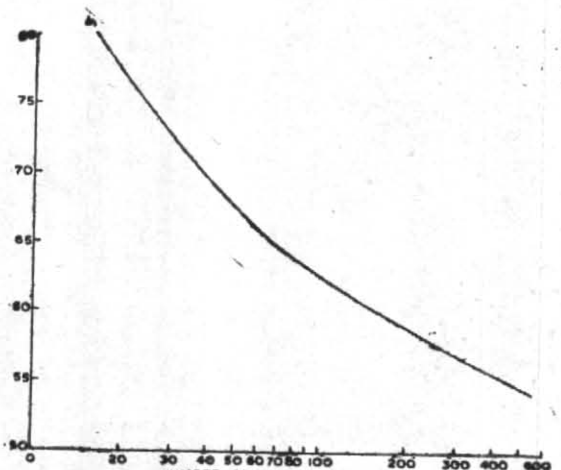

Fig. 8. Determination of H_s

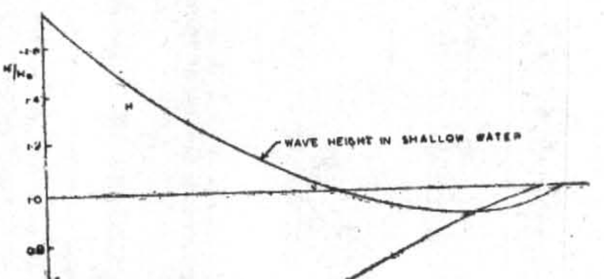

Fig. 9. The change of wave length and wave height in relation to ratio of depth to wave length [*after* Russel and Macmillan 1952]

normally generated by southern wind—winds blowing from S to SW direction. So the records for the wind speed varying between 15·5 to 16·5 kt and blowing from S to SW for more than two hours are taken. For example, the wind speed on 6 April 1968 varied between 17·8 and 18·4 mph from 1400 to 1600 hours. The wave record on that date of 1600 hr is taken. From this wave record T_z—the zero crossing period, H_1'—the corrected wave height are measured following the method stated in (A). The zero crossing period is found to be 5·82 sec and the wave height 3·97 ft. The wave length 173·4 ft (L_0) is calculated from the relationship of wave period T_z and wave length. The mean water depth D over the sensor unit was noted from the tide records, obtained simultaneously with the wave records. From this D/L_0 is determined and it is in this case 0·08; the corresponding deep sea value of wave height H_1' is obtained from Fig. 9. The value of H_0 in the particular case is 4·18 ft. The energy for the wave period of 5·82 and wave height 4·18 ft is calculated with the formula stated in 3(*iii*). This wave energy per foot of wave crest per wave length becomes 24259 ft lbs.

Similarly, the other wave records correspond to this particular wind speed of 16 kt are found out from all the wave records taken during the period from April 1968 to March 1970. From each of this record the zero crossing period T_z, wave heights H_1' and H_0 and the wave energy are calculated in the similar way. From these data, wave spectra for 16 kt is prepared taking the wave period along the abscissae and the corresponding wave energy as the ordinate and a best fit smooth curve is drawn through these points. This is shown in Fig. 1. The energy wave spectra for other wind speeds of 10, 12, 14, 18, 20 and 22 kt are similarly drawn and shown in Fig. 1.